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Diamond Semiotic Short Studies

A collection of papers on semiotics, polycontexturality and diamond theory

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Diamond Semiotics

An interplay of semiotic and graphematic diamonds

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Abstract

Some preliminary remarks about an interplay of semiotic and graphematic diamonds are sketched. A reconstruction of Alfred Toth's semiotic constructions of diamonds with the help of different notations is introduced. A distinction between the diamond properties of basic semiotic configurations and the composition of semiotic configurations as micro- and macro-analysis is proposed. The as-abstraction for semiotic connections is introduced and a mechanism to complement semiotic figures is proposed.

1. Mathematical semiotics and diamonds

1.1. Semiotics, again?

Thanks to the recent work of the semiotician Alfred Toth about mathematical semiotics and its application to polycontextural and kenogrammatic concepts, like chiasms and diamonds, a chapter of *semiotization* of diamonds and a diamondization of semiotics has to be added to the project of *Short Studies.*

This is a very first response to the profound work of Alfred Toth. It takes me back to the 70s/80s when I got involved in this headaching adventure of confronting Bense's semiotics with Gunther's polycontextural logic and kenogrammatics, both, at this time, quite in *status nascendi*, especially Gunther's project.

Semiotics is defined by Peirce and is elaborated in extenso by Bense and Toth as a triadic-trichotomic system of semiosis, i.e. as a scheme of generating signs. Obviously, it has not to be confused with other sign theoretical projects, like semiology (de Saussure, Barthes) or the pre-war Semiotik for formal systems by Manfred Schröter and Hans Hermes .

Diamonds are not triadic-trichotomic but genuinely tetradic, chiastic, antidromic and 4-fold. Hence, diamonds are not semiotical

Are semiotic diamonds semiotical?

First diamondization: internal or micro
The semiotic sign relation is a product of semiosis which can be modeled as a categorical composition of elementary sign relations. Hence, a diamondization of semiotics is a diamondization of the semiotic composition operation of elementary sign relations. This kind of diamondization shall be called *internal* (micro) diamondization in contrast to the *external* (macro) diamondization of the composition of full sign systems.

Basic work to the study of diamonds of elementary semiotic compositions had been published by the semiotician Alfred Toth.
Toth gives a solution for the diamondization of sign systems with the help of the *inversion* operation (INV) he introduced.

Second diamondization: external or macro
A second kind of diamondization is introduced with the diamondization of the composition of signs as it occurs, i.e. in the constructions of iterative and accretive compositions of sign schemes, e.g. *superposition* and *superisation* of signs.

Transpositions, dualizations, inversions and compositions are semiotic operations, diamondization consists of difference, saltisitions, bridges and complementarity.

1.2. Toth's semiotic diamonds

The semiotic composition operation of the elementary semiotic mappings, like (I->M), (M->O), (I->O), between the objects I, M, O, is *commutative* and *associative*. And obviously the *identity* mapping *id* is realized for the objects I, M, O.
Hence, semiotic composition can be studied as a mathematical category in the sense of category theory with objects I, M, O and its mappings (arrows) between the objects.

In concreto, it still has to be analyzed how the semiotic *matching conditions* for compositions are realized.
In the example above, the question is, how is "2.1" in (3.1 -> 2.1) as a *codomain* and in (2.1 -> 1.3) as a *domain* defined?

The new question which arises for abstract diamond theory is: How are the difference relations and hence the hetero-morphisms defined *in concreto*?

The papers of Toth are filling this gap with his *semiotic* modeling of diamonds.

Toth is suggesting an answer to the question, how to interpret the *difference* relations, with the introduction of the operation of *inversion* INV of a concrete sign scheme. Hence, Toth's interpretation of diamonds is joining together semiotic and diamond thematizations and notational systems.

Inversion
Where is this operation INV from?

The sign relation ZR is defined as a relation of *monadic*, *dyadic* and *triadic* relations:

ZR = (a, (a==>b), (a==>b==>c).

Sign values for ZR are:
a = {1.1, 1.2, 1.3}
b = {2.1, 2.2, 2.3}
c = {3.1, 3.2, 3.3}

ZR = <3.x, 2.y, 1.z> with x,y,z∈{1,2,3} and x<=y<=z.

It is clear, that the semiotic inversion operation INV is a semiotic operation based on the elementary operations of transpositions and is not leading out of the semiotic domain.
INV is defined by:

INV(a.b c.d e.f) = (e.f c.d a.b).

In contrast, dualization is defined as:

DUAL(a.b c.d e.f) = (f.e d.c b.a).

The *abstract sign scheme* gets an interpretation by the introduction of the instances:
I = interpretant, M=medium and O=object.

Hence, the semiotic triad occurs as morphisms between the instances I, O, M and their combinations, called graph theoretic sign models.

1. (I-->O -->M)	4. (O-->M-->I)
2. (M-->O-->I)	5. (I-->M-->O), (M-->I-->O)
3. (I-->M--> O)	6. (O-->I-->M).

It is proposed by the Stuttgarter School of semiotics (Bense, Walter) that those triadic sign schemes can be composed by their dyadic relations (mappings). Most of the semiotic work is in German, thus it is easily possible that I will miss the correct terminology.

Example:
2. (M ==> O) (O ==> I) = (M ==> O • O==> I)
Hence, triadic-trichotomic sign relations are compositions of *dyadic*-dichotomic relations. This is a strong thesis, and I don't see the necessity of such a reduction.
Even more problematic, Elisabeth Walter (1979, S. 79), speaks of a *lattice theoretical* union of "(M ==> O) (O ==> I) = (M ==> O.O==> I)". (Toth, (2008b), p.11)
Bense (1976) is mentioning a category theoretical composition of the triadic sign conceived as a transition from the set theoretic and relational definition to a more adequate presentation (Darstellung) of semioticity .

In the following, I will first follow this strategy, then I will focus on the composition of triadic-trichotomic sign structures as such.
The diamondization of the internal relations of signs might be called *micro*-analysis, the focus on the latter *macro*-analysis of semiotic diamonds.

These 6 graph theoretic sign models of I, O, M, get an interpretation by their corresponding numeric value occupancies.
Example
3.1 (I-->O-->M)
(3.1 2.1 1.1) (3.1 2.3 1.3)
(3.1 2.1 1.2) (3.2 2.2 1.2)
(3.1 2.1 1.3) (3.2 2.2 1.3)
(3.1 2.2 1.2) (3.2 2.3 1.3)
(3.1 2.2 1.3) (3.3 2.3 1.3)
(Thot, p. 2, 2008a)

The new question which arises now is: How are the *difference* relations of the semiotic diamond and hence the hetero-morphisms defined *in concreto*? More precisely, how are the difference relations between the domains/codomains of morphisms and hetero-morphisms of semiotic composition defined? And is the inversion INV operation strong enough to define the differentness of the new hetero-morphisms?

A semiotic diamond by Toth

Semiotic diamond for (3.1 2.1 1.3) with
INV (3.1 2.1 1.3) = (1.3 2.1 3.1)
diff(2.1_{ω}) = (1.3)
diff (2.1_{α}) = (3.1)

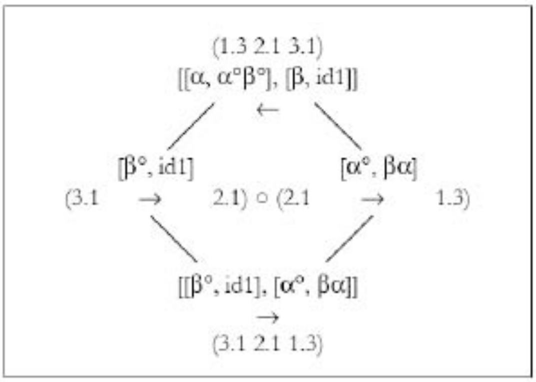
$(3.1_{\alpha} \rightarrow 2.1_{\omega}) \diamond (2.1_{\alpha} \rightarrow 1.3_{\omega})$: composition

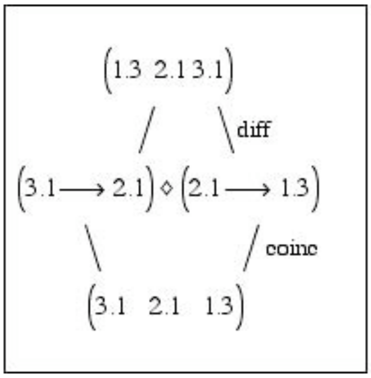
$\underline{(3.1_{\alpha} \rightarrow 1.3_{\omega}) \mid (1.3 \longleftarrow 3.1)}$: acception | rejection

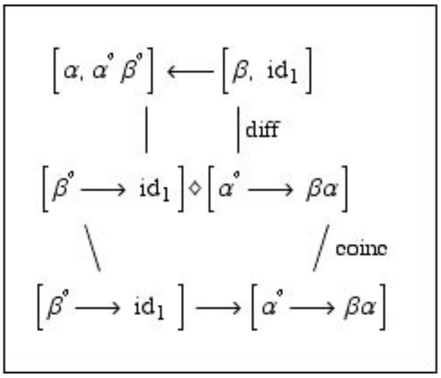
$(3.1\ 2.1\ 1.3) \mid (1.3_{2.1}\ 3.1)$: diamond result

In general, the sign class (3.a 2.b 1.c) and its inversion INV(3.a 2.b 1.c) = (1.c 2.b 3.a) are the components for the composition of semiotic diamonds (Thot, p.1, Saltatorien, 2008b).

Toth's presentation of the micro-structure of a semiotic diamond and additional notational explications.


(1.3 2.1 3.1)
[[α, α°β°], [β, id1]]
←
[β°, id1]
[α°, βα]
(3.1 → 2.1) ○ (2.1 → 1.3)
[[β°, id1], [α°, βα]]
→
(3.1 2.1 1.3)


(1.3 2.1 3.1)
diff
(3.1 ⟶ 2.1) ◇ (2.1 ⟶ 1.3)
coinc
(3.1 2.1 1.3)


[α, α° β°] ⟵ [β, id1]
diff
[β° ⟶ id1] ◇ [α° ⟶ βα]
coinc
[β° ⟶ id1] ⟶ [α° ⟶ βα]

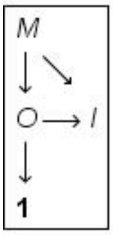
1.3. General micro-structure of semiotic diamonds

Example $M \longrightarrow O \longrightarrow I$

Semiotic composition:

$$(M \longrightarrow O) \circ (O \longrightarrow I) \Longrightarrow (M \longrightarrow I).$$

Conceptual graph for signs



Semiotics (Peirce, Bense, Toth) is fundamentally mono – contextural and it is blind for its monocontexturality, *i.e.* the *uniqueness* property, **1**, is not part of the definition of semiot

Diamond composition:

$$(M_\alpha \longrightarrow O_\omega) \diamond (O_\alpha \longrightarrow I_\omega) \Longrightarrow (M_\alpha \longrightarrow I_\omega) \,\Big|\, (O_\omega \longleftarrow O_\alpha)$$

Diamond relations as rules

Semiotic composition rule

$$\frac{(M \longrightarrow O) \circ (O \longrightarrow I)}{M \longrightarrow I}$$

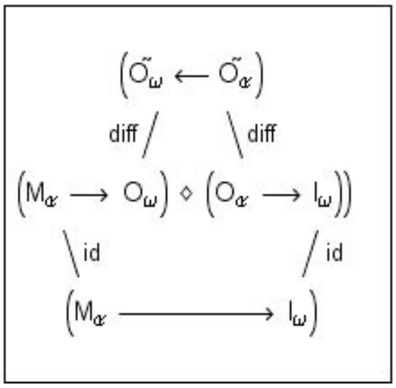
Null

Diamond composition rule $(M_\alpha \longrightarrow O_\omega) \diamond (O_\alpha \longrightarrow I_\omega)$

$$(M_\alpha \longrightarrow I_\omega) \,\Big|\, (\tilde{O}_\omega \longleftarrow \tilde{O}_\alpha)$$

$\tilde{O}_\alpha \equiv \text{diff}(O_\alpha)$

$\tilde{O}_\omega \equiv \text{diff}(O_\omega)$.

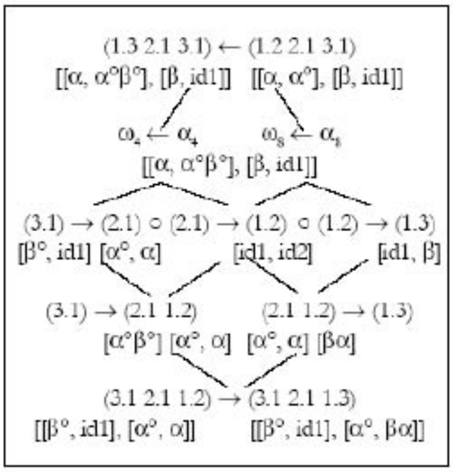


1.4. Compositions of semiotic diamonds

Diamond composition rule $\left(A_\alpha \longrightarrow B_\omega\right) \diamond \left(B_\alpha \longrightarrow C_\omega\right) \diamond \left(C_\alpha \longrightarrow D_\omega\right)$

$$\frac{\left(A_\alpha \longrightarrow B_\omega\right) \diamond \left(B_\alpha \longrightarrow C_\omega\right) \diamond \left(C_\alpha \longrightarrow D_\omega\right)}{\left(A_\alpha \longrightarrow D_\omega\right) \,\Big|\, \left(\tilde{B}_\omega \longleftarrow \tilde{B}_\alpha\right) \Big|\Big| \left(\tilde{C}_\omega \longleftarrow \tilde{C}_\alpha\right)}$$

Toth's Example (SemDiamanten, p. 14, 2008b)



A full definition of diamonds in diamond theory requires at least 3 basic morphisms with 2 corresponding basic compositions.
Categories are defined by 2 morphisms and 1 composition. Between 3 categorical morphisms the property of associativity holds naturally. All other properties are inherited by the basic definition of a category.
It could be said that the gaps between hetero-morphisms occurs automatically with the extension of single diamonds to compound diamonds. But the jump operation, between different hetero-morphisms is not automatically given by the extension.
Categories
Structure: *composition* and *identities*, for the
Properties: *unit* and *commutativity* axioms. All based on
Data: arrows, with source and target. Fulfilling the matching conditions for arrows.
Saltatories
Data:inverted arrows
Properties: *diversity* and *jump-law*
Structure: *saltisition* and *differences*.
Diamonds
complementarity
interplay
bi-objects.

Thus, not only hetero-morphisms, with *antidromic* directionality and differences, are new and not covered by a semiotic modeling but saltisitions (jump-operations) too. Further unknown operation to semiotics are *bridge* and *bridging.*

Toth's application of the diamond strategies (diamonds, chiasms) to semiotics has discovered some important new features, structures and dynamics in the field of semiotics per se.

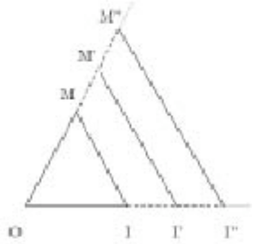
The following paragraphs will present a kind of a reconstruction of Toth's approach to *semiotic diamonds* and some further ideas to a diamondization of semiotics.

1.5. Semiotic operations

It seems that a more genuine semiotic approach to diamondize the semiotic sign relation might be introduced not as an *internal* reflection inside the sign definition but as an operation between signs as such. That is, the binary (bivariate) operation of *adjunction*, *superisation* and *iteration*. (Obviously, it has to be 3 types of combinations.)

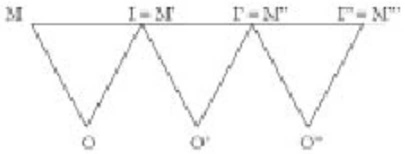
Adjunction

"**Adjunktion** ist eine Zeichenoperation mit reihendem, verkettendem Charakter" (Bense und Walther 1973, S. 11).



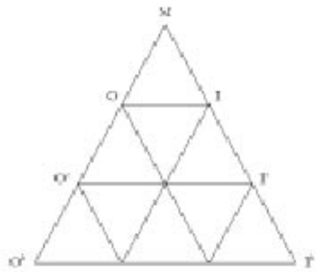
Superisation

"**Superisation** ist ein Zeichenprozess im Sinne der zusammenfassenden Ganzheitsbildung einer Menge von einzelnen Zeichen zu einer 'Gestalt', einer 'Struktur' oder einer 'Konfiguration'" (ibid., S. 106).



Iteration

"**Iteration** ist eine Operation, die alle Teilmengen des Zeichenrepertoires gewinnt, als Potenzmengenbildung darstellbar ist" (ibid., S. 46).



The global sign relation or sign scheme seems to be ruled by an interplay of iteration, superisation and adjunction. Those operators are defining the *complexity* and *complication* of a composed sign structure.
Unfortunately, the semiotic literature is not giving much information about their definition and how their internal mechanism is working. (Cf. Toth, pp.14-15, 2008a)

2. General macro-structure of semiotic diamonds

2.1. Dyads and triads (n-är vs. n-adisch)

The above considerations about formalization strategies for semiotics are supposing the possibility of composing the triadic-trichotomic sign relation out of dyadic sign relations. This is a common approach and might go back to Elisabeth Walter.
Are triads, composed by dyads, still those highly privileged objects Peirce tried to introduce into mathematics with his trichotomic mathematics and trichotromic semiotics? Schroeder has written his famous book about relations on the base of dyadic relations. And from this point of view it is easy to proof that all n-adic relations can be reduced to binary relations. But Peirce wasn't in love with dyads but with triads.

Metacritics

Metatheoretical comments to Toth's mathematical semiotics as well as to the complex Bense/Walter is simply this: it is mathematical. Mathematics is not triadic-trichotomic, hence all the applications of set theory, logic, category theory, etc. are artificial. As long as this situation would be critically reflected in the semiotic studies it would be adequate as a kind of modeling, simulation and formalization. But this is not the case! All such simulative applications comes with Bense's scholastic authority. Hence, this strategy is sabotaging its own intention of developing a complex system of semiotics.

Nothing is changed with the involvement of the so called "qualitative mathematics" (Kronthaler, Toth). Not only because the dichotomy of quantitative/qualitative is Aristotelian but as a consequence, most operators introduced are "quantitative" per se. Even worse, there is no such thing as a 'quantitative mathematics'. There is nothing "quantitative" with group and set theory and much less with category theory.

2.2. Semiotic operations

It seems that a more genuine semiotic approach to diamondize the semiotic sign relation might be introduced not as an *internal* reflection inside the sign definition but as an operation between signs as such. That is, the unary or binary (bivariate) operation of *adjunction*, *superisation* and *iteration* as a starting point leads to interesting diamond constructions. (Obviously, it has to be 3 types of combinations.)

Adjunction, superisation and iteration are operations on the general sign model (O, I, M). Such unary operations, like successor operations, can be set into a binary formulation, i.e. as adding an element to an existing element or complexion. This addition, concatenation or composition can be specified as adjunctive, superitative and iterative. All 3 types of compositions have to fulfil their matching conditions to enable the specific composition. Such a specification is considering the internal structure of the sign, depending on its I, O, M constellations (cf. Toth's "Makrosemiotische Zeichenzusammenhänge").
But operations as compositions or combinations can be understood as diamonds, i.e. as having the acceptional composat and its rejectional sign as results.

General diamond combination scheme

$$\mathrm{Op}_{diam}\left(sign_1, sign_2\right) = sign_3 \mid \overline{sign_4}$$

$$sign_1 = \left(O, I, M\right)_1$$

$$sign_2 = \left(O, I, M\right)_2$$

$$\left(sign_1 \diamond sign_2\right) = \left[sign_3 \mid \overline{sign_4}\right]$$

$$\frac{sign_1 \diamond sign_2}{sign_3 \mid \overline{sign_4}}$$

2.3. Semiotic monadic sign compositions

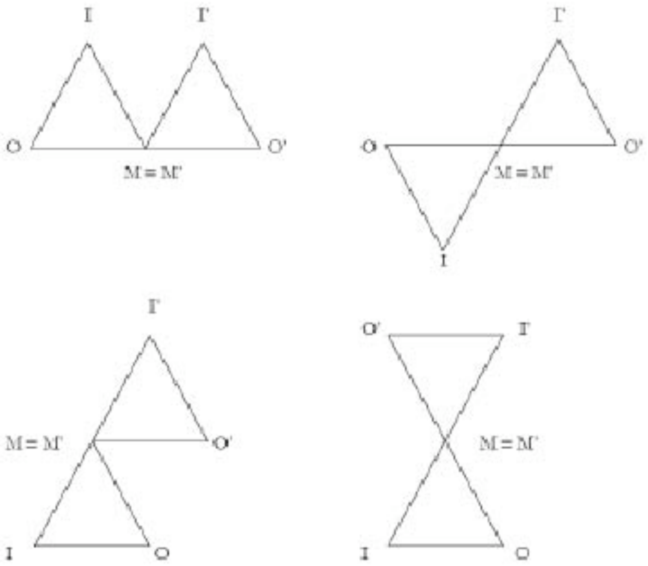
Matching conditions for monadic sign compositions:

$X \equiv X'$, with $X, X' \in \{I, O, M, I', O', M'\}$

Semiotic examples are taken from: (Toth, 2008 *a*)

Example: 3.1.2.1. M ≡ M' (Toth, p. 20, 2008a)

3.1.2.1. M = M'



$$\frac{(O, I, M) \circ (M', I', O'), \text{ with } MC = M \equiv M'}{(O, I, MM', I', O')}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type 1}\\ M\equiv M'}}\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix}}{\begin{bmatrix} I & \square & I' \\ O & MM' & O' \end{bmatrix}, \begin{bmatrix} \square & \square & I' \\ O & MM' & O' \\ \square & I & \square \end{bmatrix}, \begin{bmatrix} I' & \square \\ MM' & O' \\ I & O \end{bmatrix}, \begin{bmatrix} O & \square & I \\ \square & MM' & \square \\ I & \square & O \end{bmatrix}}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type 1}\\ M\equiv M'}}\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix}}{\begin{bmatrix} O & \longrightarrow & O' \end{bmatrix}, \begin{bmatrix} O & \longrightarrow & O' \\ I & \longrightarrow & I' \end{bmatrix}, \begin{bmatrix} I \longrightarrow & I' \end{bmatrix}, \begin{bmatrix} I & \longrightarrow & I'' \\ O & \longrightarrow & O' \end{bmatrix}}$$

Example : 3.1 .2 .2. *M'* ≡ *M*

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type 1}\\ M'\equiv M}}\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix}}{\begin{bmatrix} I' & \square & I \\ O' & M'M & O \end{bmatrix}, \begin{bmatrix} \square & \square & I \\ O' & M'M & O \\ \square & I' & \square \end{bmatrix}, \begin{bmatrix} O & \square \\ M'M & I \\ O' & I' \end{bmatrix}, \begin{bmatrix} I & \square & O \\ \square & MM' & \square \\ O' & \square & I' \end{bmatrix}}$$

$$\frac{\Pi^{(2)}_{\substack{\text{Type 1}\\ M' \equiv M}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} O' & \longrightarrow & O \end{bmatrix}, \begin{bmatrix} O' & \longrightarrow & O \\ I' & \longrightarrow & I \end{bmatrix}, \begin{bmatrix} I' \longrightarrow & I \end{bmatrix}, \begin{bmatrix} I' & \longrightarrow & I \\ O' & \longrightarrow & O \end{bmatrix}}$$

Example : 3.1 .2 .3 $M \equiv O'$

$$\frac{\Pi^{(2)}_{\substack{\text{Type 1}\\ M \equiv O'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} I & \square & I' \\ O & MO' & M' \end{bmatrix}, \begin{bmatrix} \square & \square & I' \\ O & MO' & M' \\ \square & I & \square \end{bmatrix}, \begin{bmatrix} I' & \square \\ MO' & M' \\ I & O \end{bmatrix}, \begin{bmatrix} M' & \square & I' \\ \square & MO' & \square \\ I & \square & O \end{bmatrix}}$$

ETC: to 3.1.2.18. I'≡I (Toth, 2008a, p.32)

2.3.1. Semiotic complementations

Given monadic compositions complementations/supplementations/derivations of the compositions are naturally produced and interpreted as a kind of semiotic deductions.
A reasonable complementation operation has to rely on the *as-abstraction* to build its matching conditions. The existing concept of matching conditions is object-based, i.e. the coincidence of domain and codomain is not yet ruled by the as-abstraction, which is a mechanism to redefine and re-frame the functionality of the encountered objects.

This approach of complementations leads to the question how the different matching types are inter-related.
How, e.g. can Type $A \equiv C$ be generated from Type1, $M \equiv M'$?

Toth (2008a) gives a complex classification of different semiotic constellations. Complementations of one type is generating other types, hence a complementation-based inter-relation between different semiotic constellations becomes accessible for further studies.

Transformation of semiotic structural types by the as-abstraction
As an example, *Type1 M≡M'*, if *M≡M'*, the matching condition for *M≡M'* is naturally realized. If *I* becomes *I"* and *I'* functions as *M"* and *M≡M'* becomes *O"* then the concept of matching conditions is involved into a different kind of interpretation than the *is-abstraction.* Thus, the object-related or set-theoretical interpretation of objects as domains and codomains is not flexible enough to deal such a situation. In other words, the simple categorical identity relation for objects *id* has to be replaced by a more complex operation.

Hence, a reframing of M≡M' of Type1 is generating new matching possibilities and thus a transformation of Type1 M≡M' to other existing or new semiotic constellation types.
The additional matchings for Type1M≡M' are:
$\{I \equiv I',\ M \equiv O'' \equiv M',\ M'' \equiv I'\}$, which are generating $\begin{bmatrix} II'' & \square & M''I' \\ O & MO''M' & O' \end{bmatrix}$.

Example

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type 1}\\ M\equiv M'}}\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I\longrightarrow & & M\end{pmatrix}}{\begin{bmatrix} I & \square & I^1 \\ O & MM^1 & O^1\end{bmatrix}}$$

First supplement of $\begin{bmatrix} I & \square & I^1 \\ O & MM^1 & O^1\end{bmatrix}$

$\text{MC}: M \equiv M^1 \Longrightarrow \{I \equiv I^2,\ M \equiv M^1 \equiv O^2,\ M^2 \equiv I^1\}:$

$$\frac{\text{supplem}_1\left(\begin{bmatrix} I & \square & I^1 \\ O & MM^1 & O^1\end{bmatrix}\right)}{\begin{bmatrix} (I)\,I^2 & \square & (I^1)\,M^2 \\ \square & (MM^1)\,O^2 & \square \\ (O) & \square & (O^1)\end{bmatrix}}$$

Second supplement of $\begin{bmatrix} I & \square & I^1 \\ O & MM^1 & O^1\end{bmatrix}$

$\text{MC}: M \equiv M^1 \Longrightarrow \{O \equiv I^2,\ M \equiv M^1 \equiv M^2,\ O^1 \equiv O^2\}:$

$$\frac{\text{supplem}_2\left(\begin{bmatrix} I & \square & I^1 \\ O & MM^1 & O^1\end{bmatrix}\right)}{\begin{bmatrix} (I) & \square & (I^1) \\ \square & (MM^1)\,M^2 & \square \\ (O)\,I^2 & \square & (O^1)\,O^2\end{bmatrix}}$$

Compositions of supplements

a. $\text{supplem}_3 = \text{supplem}_1 \oplus \text{supplem}_2:$

$$\frac{\text{supplem}_3\left(\begin{bmatrix} (I)\,I^2 & \square & (I^1)\,M^2 \\ \square & (MM^1)\,O^2 & \square \\ (O) & \square & (O^1)\end{bmatrix} \oplus \begin{bmatrix} (I) & \square & (I^1) \\ \square & (MM^1)\,M^2 & \square \\ (O)\,I^2 & \square & (O^1)\,O^2\end{bmatrix}\right)}{\begin{bmatrix} (II^2) & \square & (M^2\,I^1\,I^2) \\ \square & (MM^1\,O^2)\,M^3 & \square \\ (O)\,I^3 & \square & (O^1\,O^2)\,O^3\end{bmatrix}}$$

The supplementation *supplem*$_4$ seems to saturate the possibilities of complementing figure Type1 $M \equiv M'$. A saturation is achieved if all possible semiotic knots are connected. A saturated semiotic figure or constellation can then be reduced or it can be augmented in complexity by iterative and accretive operations. The labelling of the semiotic knots with I, M, O might be at first just arbitrary and only depending on the as-abstraction, e.g. $(MM^1\ O^2\ M^3)$ reads as " $(MM^1\ O^2)$ as M^3". Thus, different labelling decisions are possible.

Redundant supplementation

A saturated figure, like supplem$_3$, might iteratively augmented by supplem$_4$ as a *redundant* iteration of an existing sub-figure, say $(I^3,\ M^3,\ O^3)$.

Redundant supplementation is based on a semiotic operation, which isn't common in semiotic literature. It is well based on the polycontextural *as-abstraction.* Hence, it enables to thematize semiotic objects *"X as Y is Z"* , e.g. I as I^1 is (II^1), I as O^1 is (IO^1), I as M^1 is (IM^1) and O as O^1 is (OO^1), etc.

Hence, again, the category theoretic identity operation *id* of classical semiotics is transformed to a difference operation. Identity, i.e. the is-abstraction, derives naturally from the as-abstraction with "X as X is X".

$$\frac{op_{redun}\begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ O & \longrightarrow & M \end{pmatrix}}{\begin{bmatrix} II^1 & \square \\ OO^1 & MM^1 \end{bmatrix}, \begin{bmatrix} IO^1 & \square \\ OM^1 & MI^1 \end{bmatrix}, \begin{bmatrix} IM^1 & \square \\ OI^1 & MO^1 \end{bmatrix}, \ldots, \begin{bmatrix} IM^1 & \square \\ OO^1 & MI^1 \end{bmatrix}}$$

2.4. Diamond monadic sign compositions

$$\frac{(O, I, M) \diamond (M', I', O'),\ \text{with}\ MC = M \equiv M'}{(O, I, M) \circ (M', I', O') \,\big|\, (M,\ M')}$$

$$\frac{(O \longrightarrow I \longrightarrow M) \diamond (M' \longrightarrow I' \longrightarrow O'),\ M \equiv M'}{(O \longrightarrow I \longrightarrow M) \circ (M' \longrightarrow I' \longrightarrow O') \,\big|\, (M \longleftarrow M')}$$

$$\frac{(O \longrightarrow I \longrightarrow M) \diamond (M' \longrightarrow I' \longrightarrow O'),\ M \equiv M'}{(O \longrightarrow M) \circ (M' \longrightarrow O') \,\big|\, (M \longleftarrow M')}$$

$$\frac{(O \longrightarrow I \longrightarrow M) \diamond (M' \longrightarrow I' \longrightarrow O'),\ M \equiv M'}{(O \longrightarrow O') \,\big|\, (M \longleftarrow M')}$$

$$\frac{\Pi^{(2)}_{\substack{\text{Type 1} \\ M \equiv M'}}\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix}}{\begin{bmatrix} I & \square & I' \\ O & MM' & O' \end{bmatrix} \,\big|\, (M \longleftarrow M')}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type 1}\\ M\equiv M'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} \square & \square & I' \\ O & MM' & O' \\ I & \square & \square \end{bmatrix} \Bigg| \left(M \longleftarrow M'\right)}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type 3}\\ M\equiv M'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} \square & I' \\ MM' & O' \\ I & O \end{bmatrix} \Bigg| \left(M \longleftarrow M'\right)}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type 4}\\ M\equiv M'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} O' & I' \\ MM' & \square \\ I & O \end{bmatrix} \Bigg| \left(M \longleftarrow M'\right)}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type } A\\ \textbf{MC } M\equiv M',\\ O\equiv O'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} I & \square \\ MM' & OO' \\ I' & \square \end{bmatrix} \Bigg| \left(M \longleftarrow M'\right) \Big\| \left(O \longleftarrow O'\right)}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type } B\\ \textbf{MC } M\equiv M',\\ O\equiv O'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} I & MM' \\ OO' & I' \end{bmatrix} \Bigg| \left(\overline{O} \longleftarrow \overline{O}'\right) \Big\| \left(\overline{M} \longleftarrow \overline{M}'\right)}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type } c \\ \textbf{MC } M \equiv M', \\ O \equiv O'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} MM' & I' \\ M & OO' \end{bmatrix} \Big| \left(\overline{M} \longleftarrow \overline{M}'\right) \| \left(\overline{O} \longleftarrow \overline{O}'\right)}$$

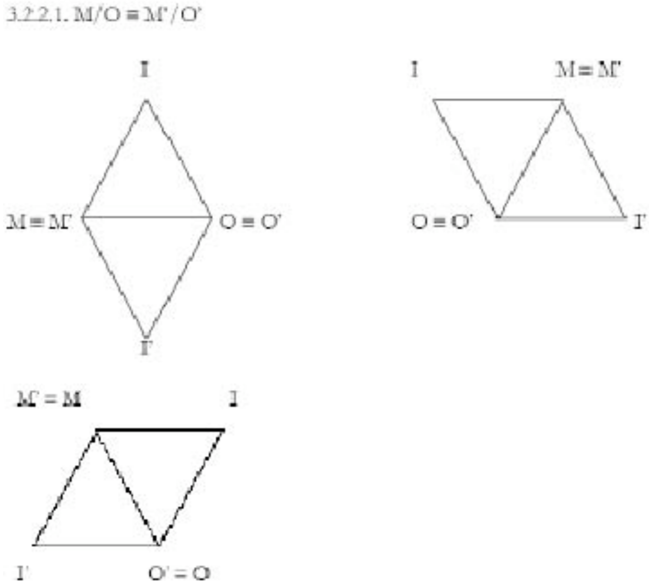
2.5. Dyadic sign compositions

2.5.1. Dyadic semiotic sign compositions

Matching conditions for dyadic sign compositions:

MC: $X / Y \equiv X' / Y'$, with $X, X' Y, Y' \in \{I, O, M, I', O', M'\}$

Example: 3.2.2.1. M/O ≡ M'/O' (Toth, p. 33, 2008a)



$$\frac{\Pi^{(2)}_{\substack{O \equiv O' \\ M \equiv M'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} \square & I \\ MM' & OO'' \\ \square & I'' \end{bmatrix}, \begin{bmatrix} I & MM'' \\ OO' & I' \end{bmatrix}, \begin{bmatrix} MM' & I \\ I' & O'O \end{bmatrix}}$$

2.5.2. Diamond dyadic sign compositions

$$\frac{(M \longrightarrow I \longrightarrow O) \diamond (O' \longrightarrow I' \longrightarrow M'), \text{ with MC} = M / O \equiv M' / O'}{(M \longrightarrow I \longrightarrow O) \circ (O' \longrightarrow I' \longrightarrow M') \,\Big|\, (O \longleftarrow O') \,\Big\|\, (M \longleftarrow M')}$$

$$\frac{(O \longrightarrow I \longrightarrow M) \diamond (M' \longrightarrow I' \longrightarrow O'), \text{ with MC} = M / O \equiv M' / O'}{(O \longrightarrow I \longrightarrow M) \circ (M' \longrightarrow I' \longrightarrow O') \,\Big|\, (M \longleftarrow M') \,\Big\|\, (O \longleftarrow O')}$$

$$\frac{(O, I, M) \diamond_{M,O} (M', I', O'), \text{ with MC} = M / O \equiv M' / O'}{(O, I, M) \circ_{M,O} (M', I', O') \,\Big|\, (M \longleftarrow M') \,\Big\|\, (O \longleftarrow O')}$$

a. $$\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix} \diamond_{O,M} \begin{pmatrix} O' & & \square \\ \downarrow & \searrow & \\ I' & \longrightarrow & M' \end{pmatrix}, \text{ with } M/O \equiv M'/O'$$

---

b. $$\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ M & \longrightarrow & I \end{pmatrix} \diamond_{O,M} \begin{pmatrix} O' & \longrightarrow & I' \\ \downarrow & \nearrow & \\ M' & & \end{pmatrix} \Bigg\| \left(M \longleftarrow M'\right) \| \left(O \longleftarrow O'\right)$$

---

c. $$\left[\begin{matrix} \left(O \equiv O'\right) & \longrightarrow & I \\ \downarrow & x & \square \\ \left(M \equiv M'\right) & \longrightarrow & I' \end{matrix}\right] \Bigg\| \left(M \longleftarrow M'\right) \| \left(O \longleftarrow O'\right)$$

$$\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix} \diamond_{O,M,I} \begin{pmatrix} O' & & \square \\ \downarrow & \searrow & \\ I' & \longrightarrow & M' \end{pmatrix}, \text{ with } M/O \equiv O'/I'$$

$$\left[\begin{matrix} \left(M \equiv O'\right) & \longrightarrow & I \\ \downarrow & x & \square \\ \left(O \equiv I'\right) & \longrightarrow & M' \end{matrix}\right] \Bigg\| \left(M \longleftarrow O'\right) \| \left(O \longleftarrow I'\right)$$

$$\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix} \diamond_{O,M,I} \begin{pmatrix} O' & & \square \\ \downarrow & \searrow & \\ I' & \longrightarrow & M' \end{pmatrix}, \text{ with } O/I \equiv O'/I'$$

$$\left[\begin{matrix} \left(O \equiv O'\right) & \longrightarrow & M \\ \downarrow & x & \square \\ \left(I \equiv I'\right) & \longrightarrow & M' \end{matrix}\right] \Bigg\| \left(O \longleftarrow O'\right) \| \left(I \longleftarrow I'\right)$$

$$\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix} \diamond_{O,I,M} \begin{pmatrix} O' & & \square \\ \downarrow & \searrow & \\ I' & \longrightarrow & M' \end{pmatrix}, \text{ with } O/I \equiv M'/I'$$

$$\left[\begin{matrix} \left(O \equiv M'\right) & \longrightarrow & M \\ \downarrow & x & \square \\ \left(I \equiv I'\right) & \longrightarrow & O' \end{matrix}\right] \Bigg\| \left(O \longleftarrow M'\right) \| \left(I \longleftarrow I'\right)$$

$$\textbf{Type 1 = 1:}\ \text{MC}: M \equiv M', O' \equiv M'', O'' \equiv M''' \ (\text{Toth, p. 63, 2008 a}) \begin{pmatrix} O & \square \\ \downarrow & \searrow \\ I \longrightarrow & M \end{pmatrix}$$

$$\left[\begin{array}{ccccccccc} I & \square & \square & I' & \square & \square & I'' & \square & \square \\ \downarrow & \searrow & \square & \downarrow & \searrow & \square & \downarrow & \searrow & \square \\ O & \longrightarrow & M \equiv M' & & \longrightarrow & O' \equiv M'' & & \longrightarrow & O'' \equiv M''' \end{array}\right] \left| \begin{array}{c} (O' \longleftarrow M'') \parallel \\ (M \longleftarrow M') \\ \square \end{array}\right.$$

$$\textbf{Type 1 = 1:}\ \text{MC}: M \equiv M', O' \equiv M'', O'' \equiv M''' \qquad \Pi^{(3)}_{\diamond_{O,\,I,\,M}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}$$

$$\left[\begin{array}{ccc} \square & I & \square \\ O' & M'M & O \\ \square & I'M'' & \square \\ O'' & \square & I'' \end{array}\right] \Bigg| \ (M \longleftarrow O'')$$

$$\textbf{Type 1 = 1:}\ \text{MC}: M \equiv M', O' \equiv M'', O'' \equiv M''' \qquad \Pi^{(3)}_{\diamond_{O,\,I,\,M}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}$$

$$\frac{(O \longrightarrow M,\ M' \longrightarrow O',\ M'' \longrightarrow O'' \ |\ (M \longleftarrow O'')}{(O \longrightarrow O''\ |\ (M \longleftarrow O'')}$$

Type 4 = 4:

MC: $I' \equiv O''$, $M \equiv M'$, $M' \equiv M''$, $O \equiv I'$ (Toth, *p*. 65, 2008 *a*)

$$\Pi^{(4)}_{\textbf{Type 4=4}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}$$

$$\left[\begin{array}{ccc} O' & I'O'' & I'' \\ MM' & \square & M'M'' \\ I & OI'' & O' \end{array}\right] \left| \begin{array}{l} (I' \longleftarrow O'') \parallel \\ (M \longleftarrow M') \parallel \\ (M' \longleftarrow M'') \parallel \\ (O \longleftarrow I') \end{array}\right.$$

Type 1 = 2 :

MC : $M \equiv M'$, $O' \equiv O''$, $M'' \equiv M'''$ (Toth, *p*. 65, 2008 *a*)

$$\Pi^{(4)}_{\text{Type } 1=2} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix} \quad \frac{\left[\begin{array}{ccccccccccccc} I & \square & \square & I' & \square & \square & I'' & \square & \square & I''' & \square & \\ \downarrow & \searrow & \square & \downarrow & \searrow & \square & \uparrow & \searrow & & \uparrow & \searrow & \\ O & \longrightarrow & M & M' & \longrightarrow & O' & O'' & \longrightarrow & M'' & M''' & \longrightarrow & O \end{array}\right.}{}$$

$$\begin{pmatrix} \square & I & I' & \square & \square \\ O & MM' & O'O'' & M''M''' & O''' \\ \square & I'M'' & \square & I''' & \square \end{pmatrix} \Bigg| \begin{array}{l} (M'' \longleftarrow M''') \,\|\, (M \longleftarrow M') \,\| \\ (O' \longleftarrow O'') \end{array}$$

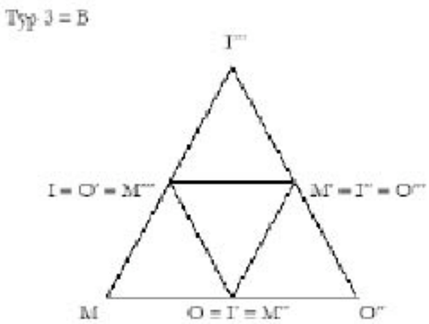
Type *A* ≡ *A* = *A* ≡ *B* : (Toth, 2008 *a*, *p*. 68)

MC*$_I$*: $I' \equiv I''$, $M \equiv M'$, $O \equiv O' \equiv O''$

$$\Pi^{(4)}_{\text{Type }(A \equiv A = A \equiv B)\ \text{MC}_I} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix}$$

$$\frac{}{\begin{bmatrix} \square & II'' & M'' \\ MM' & OO'O'' & I'' \\ \square & I' & \square \end{bmatrix}} \Bigg| \begin{array}{l} (I \longleftarrow I'') \;\| \\ (M \longleftarrow M') \;\| \\ (O' \longleftarrow O'') \;\| \\ (O \longleftarrow O') \;\| \\ (O \longleftarrow O'') \end{array}$$

Type 3 ≡ *B* : (Toth, 2008 *a*, *p*. 71)



$$\mathbf{MC}_k : I \equiv O' \equiv M''',\ M' \equiv I''' \equiv O''',\ O \equiv I' \equiv M''$$

$$\frac{\Pi^{(4)}_{\text{Type}\,(3\equiv B)\,\text{MC}_k}\begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ M & \longrightarrow & O \end{pmatrix}}{\begin{bmatrix} \square & I''' & \square \\ IO'M'' & \square & M'I''O'' \\ M & OI'M'' & O'' \end{bmatrix} \left| \begin{matrix} (O' \longleftarrow M'') \parallel \\ (I \longleftarrow O') \parallel \\ (I'' \longleftarrow O'') \parallel \\ (M' \longleftarrow I'') \parallel \\ (I' \longleftarrow M'') \parallel \\ (O \longleftarrow I'). \end{matrix} \right.}$$

2.6. Local-/global-heteromorphisms

Sign functions, understood as a composition of elementary sign relations are producing their *local* diamond structures according to the matching conditions of the compositions of elementary sign morphisms.
Sign systems, understood as combinations of sign functions are producing their *global* diamonds in accordance to the matching conditions of their combined signs.
As an interpretation, this semiotic difference of *local* and *global* or *micro/macro* diamond structures can be connected with the distinction of *inner* and *outer* environments of semiotic diamond systems.

$$\frac{\textbf{Micro – analysis of } (\boldsymbol{I},\ \boldsymbol{O},\ \boldsymbol{M})\ \ (I_\alpha \to O_\omega) \circ (O_\alpha \to M_\omega)}{(I_\alpha \to M_\omega) \mid (\tilde{O}_\omega \longleftarrow \tilde{O}_\alpha)}$$

$$\tilde{O}_\alpha \equiv \text{diff}(O_\alpha)$$
$$\tilde{O}_\omega \equiv \text{diff}(O_\omega).$$

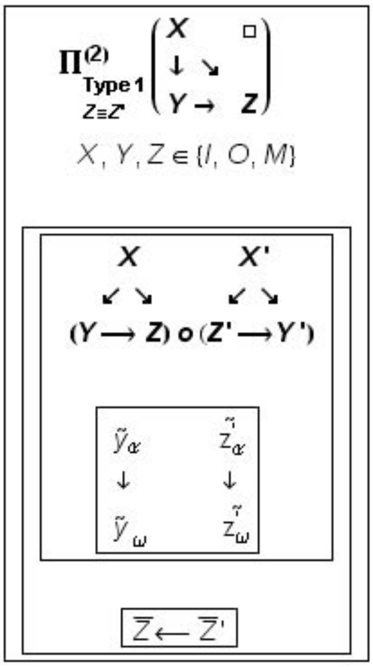
Macro – analysis of Type 1, MC = *M* ≡ *M'*

$$\frac{\Pi^{(2)}_{\text{Type 1}\ M\equiv M'}\begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ O \to & & M \end{pmatrix}}{\begin{bmatrix} I & \square & I' \\ O & MM' & O' \end{bmatrix} \mid (\overline{M} \leftarrow \overline{M'})}$$

Micro / Macro – analysis of Type 1, MC = *M* ≡ *M'*

$$\frac{\Pi^{(2)}_{\text{Type 1}\ M\equiv M'}\begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ O \to & & M \end{pmatrix}}{\begin{bmatrix} I & \square & I' \\ O & MM' & O' \end{bmatrix} \left| \begin{matrix} \tilde{O}_\omega \longleftarrow \tilde{O}_\alpha \parallel \\ \tilde{\tilde{O}}_\omega \longleftarrow \tilde{\tilde{O}}_\alpha \end{matrix} \right| \overline{M} \leftarrow \overline{M}'}$$

Generalized Notations: Boxes and brackets

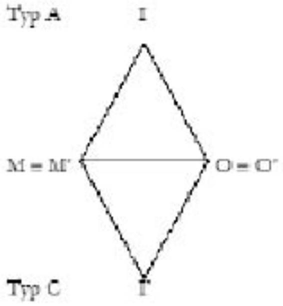


$$\Pi^{(2)}_{\text{Type 1}, Z\equiv Z'} = \text{Diam}\big(\text{diam}(X, Y, Z), \text{diam}(X', Z', Y')\big)$$

$$\left[\begin{array}{l}\textbf{Matrix} = \\ \Pi^{(2)}_{\text{Type 1}, Z\equiv Z'} \\ \left[\begin{array}{l}\text{Diamond} \\ \left[\begin{array}{l}\text{diamond} \\ \left[(X, Y, Z)\right. \\ \left[\begin{array}{l}\text{diamond} \\ \left[(X', Z', Y')\right]\end{array}\right.\end{array}\right.\end{array}\right.\end{array}\right] = \left[\begin{array}{l}\textbf{Matrix} \\ \left[\begin{array}{l}\text{Diamond} \\ \left[\begin{array}{l}\left[(X, Y, Z)\right| \ (y_\alpha, y_\omega) \\ \left[(X', Z', Y')\right| (\dot{z}_\alpha, \dot{z}_\omega)\end{array}\right]\end{array}\right.\end{array}\right] = \left[\begin{array}{l}\textbf{Matrix} \\ \left[\begin{array}{l}\left[(X, Y, Z)\right| (y_\alpha, y_\omega) \\ \left[(X', Z', Y')\right| (\dot{z}_\alpha, \dot{z}_\omega)\end{array}\right.\end{array} \middle| (\mathbf{z}_\omega \longleftarrow \mathbf{z}_\alpha)\right]$$

Diamond micro / macro – analysis of Type *A*, MC = *M* ≡ *M'*, *O* ≡ *O'* (*p*.63)

$$\frac{\Pi^{(2)}_{\substack{\text{Type }A \\ \text{MC } M\equiv M', \\ O\equiv O'}} \begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ M \longrightarrow & & O \end{pmatrix},}{\begin{bmatrix} I & \square \\ MM' & OO' \\ I' & \square \end{bmatrix} \Big| (M \longleftarrow M') \| (O \longleftarrow O')}$$



$$\left[\begin{array}{l}\textbf{Matrix} = \\ \Pi^{(2)}_{\text{Type } A,\, M \equiv M',\, O \equiv O'} \\ \left[\begin{array}{l}\text{Diamond} \\ \left[\begin{array}{l}\text{diamond} \\ [(I, M, O) \\ \left[\begin{array}{l}\text{diamond} \\ (M', I', O')\end{array}\right.\end{array}\right.\end{array}\right.\end{array}\right] = \left[\begin{array}{l}\textbf{Matrix} \\ \left[\begin{array}{l}\text{Diamond} \\ \left[\begin{array}{l}[(I, M, O) \,|\, (M_\alpha, M_\omega) \\ \left[(M', I', O') \,|\, (I_\alpha, I_\omega)\right.\end{array}\right.\end{array}\right.\end{array}\right] = \left[\begin{array}{l}\textbf{Matrix} \\ \left[\begin{array}{l}[(I, M, O) \,|\, (M_\alpha, M_\omega) \\ \left[(M', I', O') \,|\, (I_\alpha\, I_\omega)\right.\end{array}\right.\end{array} \;\middle|\; \boldsymbol{(M, M') \parallel (O, O')}\right]$$

Diamond micro / macro – analysis of Type 4 = 4 (iter, acc)

Type 4 = 4 : (iter)

MC : $I' \equiv O''$, $M \equiv M'$, $M' \equiv M''$, $O \equiv I'$ (Toth, 2008 *a*, *p*. 65)

$$\Pi^{(4)}_{\text{Type } 4=4} \begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ M \longrightarrow & & O \end{pmatrix}, \text{ macro}$$

$$\begin{bmatrix} O' & I'O'' & I'' \\ MM' & \square & M'M'' \\ I & OI' & O' \end{bmatrix} \left| \begin{array}{l} (I' \longleftarrow O'') \parallel \\ (M \longleftarrow M') \parallel \\ (M' \longleftarrow M'') \parallel \\ (O \longleftarrow I') \end{array} \right.$$

Type 4 = 4 is ***iterating*** system $\left(I', M', O'\right)$ as

$op_{mirr}\left(I', M', O'\right) = \left(M', I', O'\right)$ •

Accretive distribution of system $\left(I^1, M^1, O^1\right)$ as

$op_{acc}\left(op_{mirr}\left(I^1, M^1, O^1\right)\right) = \left(M^2, I^2, O^2\right)$

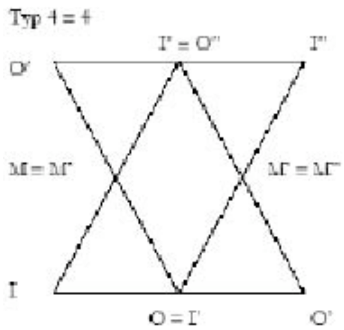
$X' = X^1,\ X'' = X^2,\ X''' = X^3,\ X \in \{M, I, O\}$

Type 4 = 4 : **(acc)**

MC: $I^2 \equiv O^3,\ M \equiv M^2,\ M^1 \equiv M^3,\ O \equiv I^1$

$$\Pi^{(4)}_{\textbf{Type 4=4}}\begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ M & \longrightarrow & O \end{pmatrix},\ \text{macro}$$

$$\left[\begin{matrix} O^2 & I^2\,O^3 & I^3 \\ MM^2 & \square & M^1\,M^3 \\ I & OI^1 & O^1 \end{matrix}\right] \left|\ \begin{matrix} \left(I^2 \longleftarrow O^3\right) \| \\ \left(M \longleftarrow M^2\right) \| \\ \left(M^1 \longleftarrow M^3\right) \| \\ \left(O \longleftarrow I^1\right) \end{matrix}\right.$$

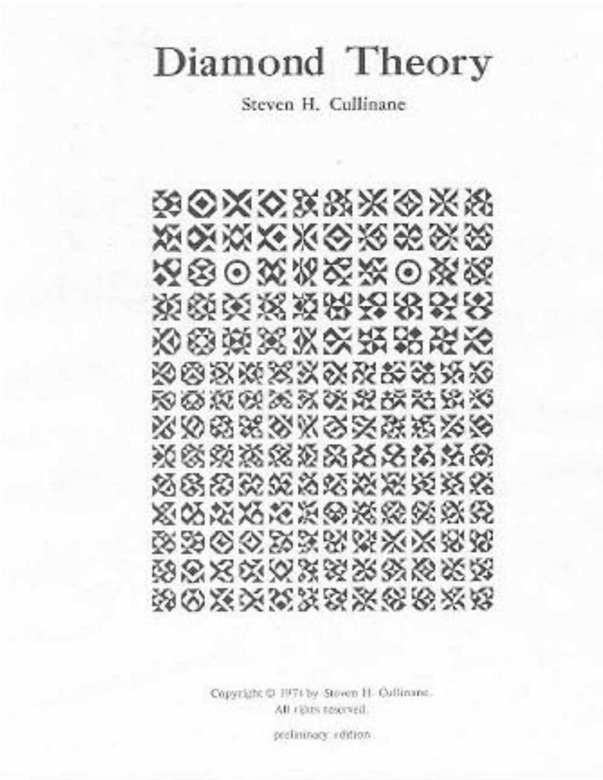


$$\left[\begin{matrix} \textbf{Matrix} = \\ \Pi^{(4)}_{\textbf{Type 4=4 (iter)}} \\ \left[\begin{matrix} \text{Diamond} \\ \left[\begin{matrix} \text{diamond}_1 \\ \left[\left(I, M, O\right)\right] \\ \left[\begin{matrix} \text{diamond}_2 \\ \left(I', M', O'\right) \end{matrix}\right] \\ \left[\begin{matrix} \text{diamond}_3 \\ \left(M', O', I'\right) \end{matrix}\right] \\ \left[\begin{matrix} \text{diamond}_4 \\ \left(M'', O'', I''\right) \end{matrix}\right] \end{matrix}\right] \end{matrix}\right] \end{matrix}\right] = \left[\begin{matrix} \textbf{Matrix} \\ \left[\begin{matrix} \text{Diamond} \\ \left[\begin{matrix} \left[\left(I, M, O\right) \mid \left(M_\alpha, M_\omega\right)\right. \\ \left[\left(I', M', O'\right) \mid \left(M'_\alpha, M'_\omega\right)\right. \\ \left[\left(M', O', I'\right) \mid \left(O'_\alpha, O'_\omega\right)\right. \\ \left[\left(M'', O'', I''\right) \mid O''_\alpha, O''_\alpha\right) \end{matrix}\right] \end{matrix}\right] \end{matrix}\right] = \left[\begin{matrix} \textbf{Matrix} \\ \left[\begin{matrix} \left[\left(I, M, O\right) \mid \left(M_\alpha, M_\omega\right)\right. \\ \left[\left(I', M', O'\right) \mid \left(M'_\alpha, M'_\omega\right)\right. \\ \left[\left(M', O', I'\right) \mid \left(O'_\alpha, O'_\omega\right)\right. \\ \left[\left(M'', O'', I''\right) \mid O''_\alpha, O''_\alpha\right) \end{matrix}\right] \end{matrix} \left|\ \begin{matrix} \left(I' \longleftarrow O''\right) \| \\ \left(M \longleftarrow M'\right) \| \\ \left(M' \longleftarrow M''\right) \| \\ \left(O \longleftarrow I'\right) \end{matrix}\right.\right]$$

Generalizations of the matrix or bracket notation should easily be accessible to develop a general notational system for semiotic diamonds.

Diamond Theory (Steven H. Cullinane) has many faces:

http://finitegeometry.org/sc/gen/dth/DiamondTheory.html
http://diamondtheorem.com/



2.7. Composition of semiotic diamonds

A *first* analysis of the composition operation for semiotic diamonds results into an iterative and accretive or serial and parallel form of “2-dimensional” composition. These operators might correspond to the semiotic operations “*adjunction*”, “*superisation*” and ”*iteration*”. As a further operation, a general reflection or mirror operation is necessary. Hence, a figure like *Type 1≡A* might be written as a combination of iteration, "op_{iter}" accretion, op_{acc}, and mirroring, op_{mir}.

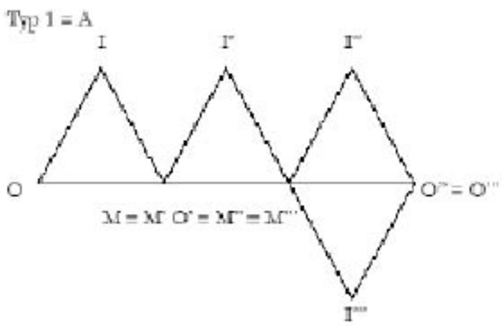
Each operation has to fulfill the matching conditions, MC, and is prepared to be involved into diamondization. Matching conditions are reflecting, externally, the composability of compositions, therefore they are represented as hetero-morphisms of saltatories. Both together, the categorical compositions with "op_{iter}", op_{acc}, and op_{mir} and the heteromorphic representations of their combinations are building the diamond structure of the semiotic composition.

Semiotic Figure Type 1 ≡ A (Toth, 2008 a, p.79)

$$(op_{acc}(op_{mir}(op_{iter}(op_{iter}(op_{iter}([O, I, M])))))),$$

reflecting the matching conditions:

$$MC = \{M \equiv M', O' \equiv M'' \equiv M''', O'' \equiv O'''\}$$



Diamondization of Figure Type 1 ≡ *A*:

Type 1 ≡ *A*:
MC: $M \equiv M',\ O' \equiv M'' \equiv M''',\ O'' \equiv O'''$

$$\Pi^{(4)}_{\mathbf{Type\ 1 \equiv \mathit{A}}} \begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ O \longrightarrow & & M \end{pmatrix},\ \text{macro}$$

$$\begin{bmatrix} I & I' & I'' & \square \\ O & MM' & O'M''M''' & O''O''' \\ \square & \square & \square & I''' \end{bmatrix} \left| \begin{array}{l} (O'' \longleftarrow O''') \ \| \\ (M'' \longleftarrow M''') \ \| \\ (O' \longleftarrow M'') \ \| \\ (M \longleftarrow M') \end{array} \right.$$

Matching conditions

op_{iter} : MC = $(M \equiv M',\ O' \equiv M'' \equiv M''',\ O'' \equiv O''')$

op_{acc} : MC = $(M'' \equiv M''',\ O'' \equiv O''')$

op_{mir} : MC := $(O'' \equiv O''',\ M'' \equiv M''')$ •

3. Goguen's semiotics: From Peirce to Pierce

A closer connection between *trichotomic semiotics* and mathematical *category theory* is accessible with Joseph Goguen's work to the semiotics of Human-Computer-Interface (HCI) theory. This chapter is a preliminary hint to a new *Short Story* about the interplay of semiotics, category theory and *diamond theory* in respect to a theory of presentation, representation and evocation. It might then be seen as a late contribution to my old (German) project **"Kalkül und Kreativität" (1998-2002).**

Goguen, Semiotic Morphism, 1996/2004
Definition 1
A **sign system**, or **semiotic system** or **semiotic theory**, consists of:
1. a **signature**, which declares sorts, subsorts and operations (including constructors and selectors);
2. a subsignature of **data sorts** and **data functions**;
3. **axioms** (e.g. equations) as constraints;
4. a **level ordering** on sorts, including a maximum element called top; and
5. a **priority ordering** on constructors at the same.

The non-data sorts classify signs and their parts, just as in grammar, the "parts of speech" classify sentences and their parts. There are two kinds of operation: constructors build new signs from old signs as parts, while selectors pull out parts from compound signs. Data sorts classify a special kind of sign that provides values serving as attributes of signs. Axioms act as constraints on what count as allowable signs for this system. Levels indicate the whole/part hierarchy of a sign, with the top sort being the level of the whole; priorities indicate the relative significance of subsigns at a given level; social issues play a dominant role in determining these.

Definition 2
A **semiotic morphism** $M : S_1 \text{-->} S_2$ from a semiotic system S_1 to another S_2 consists of the following partial mappings:
1. from sorts of S_2 to sorts of S_2, so as to preserve the subsort relations,
2. from operations of S_2 to operations of S_2, so as to preserve their source and target sorts,
3. from levels of S_1 sorts to levels of S_2, so as to preserve the ordering relation, and
4. from priorities of S_1 constructors to priorities of S_2 constructors, so as to preserve their ordering relations,

so as to strictly preserve all data elements and their functions.

It is easy to prove that this definition of composition obeys the following identity and associative laws, in which $A : R \text{-->} S$, $B : S \text{-->} T$ and $C : T \text{-->} U$,

$A ; 1_S = A$

$1_S ; B = B$

$A ; (B ; C) = (A ; B) ; C$

where 1_S denotes the identity morphism on S. These three laws are perhaps the most fundamental for a calculus of representation, since they imply that semiotic theories and their morphisms form what is called a "category" in the relatively new branch of mathematics called category theory [Mac Lane, 1998].

The basic ingredients of a **category** are *objects*, *morphisms*, and a *composition* operation that satisfies the above three laws, and that is defined on two morphisms if and only if they have *matching* source and target.

Three of the simplest categorical concepts are isomorphism, sum and product.
A morphism $A : R \text{-->} S$ is an **isomorphism** if and only if there is another morphism $B : S \text{-->} R$ such that $A ;B = 1_R$ and $B ;A = 1_S$, in which case B is called the inverse of A and denoted A-1; it can be proved that the inverse of a morphism is unique if it exists. The following laws can also be proved, assuming that $A : R \text{-->} S$ and $B : S \text{-->} T$ are both isomorphisms (and no longer assuming that B is the inverse of A).

$1_R{}^{-1} = 1_R$
$(A^{-1})^{-1} = A$
$(A ; B)^{-1} = B^{-1} ; A^{-1}$

Because sign systems and their morphisms form a category, these three laws apply to representations.
http://www.cs.ucsd.edu/users/goguen/4mari/4mari.html

"The creative process is to some extent *unpredictable* and *uncontrollable*; this is more true of artistic creation than of design, but it holds for both. The best designs often seem both surprising and obvious, and they also often seem to come suddenly out of nowhere, but usually after a lot of hard work."
http://www-cse.ucsd.edu/users/goguen/papers/sm/node5.html#SECTION3-2

A sign theory of representation, as it is the case for the Peirceian semiotics, might well be conceptualized in its basic structure by the concepts and laws of mathematical category theory.

Also signs are introduced by Peirce as '*representamen',* sign events haven't to be restricted to the process of *representation*. Innovative and creative sign events are not representational. They are not re-presenting something existing but are creating something, which has not to be in any sense an ontological "*something*", which isn't yet existing.
Qualitative categories like "*surprise*", "*suddenly of nowhere*", "*unpredictable*", etc. are not covered by category theory and category based semiotics. Categorical and semiotic compositions are conservative and save, not leaving the framework of their definition.

Neither is the *New* covered by the category of Becoming (Hegel, Lawvere).
"Thus I believe to have demonstrated the plausibility my thesis that category theory will be a necessary tool in the construction of an adequately explicit science of knowledge." (Lawvere, 1994, p. 55)
F. William Lawvere: Tools for the Advancement of Objective Logic: Closed Categories and Toposes, in: (Eds.) John McNammara, Gonzales E. Reyes, the logic foundation of cognition, Oxford 1994, pp. 43-56

A new approach beyond the *magic of inspiration* and the *mechanization of creation* is proposed by the general diamond strategies.

"The possibility of being surprised is exhibited as the main argument for the existence of protentions. It is an observable fact, Husserl says, that we all can be at every moment suddenly surprised; now, in order one to be surprised, she must experience an unexpected event; therefore we have at every moment some expectation about the most immediate future. We anticipate the future whenever we are having intentional acts." (Julio Ostalé)
http://staff.science.uva.nl/~michiell/docs/Corrected%20Handout.pdf

"The harmonic My-Your-Our-Class conceptualization has to be augmented by a class which is incorporating the place for the other, the unknown, the difference to the harmonic system. That is, the NotOurClass is thematized positively as such as the class for others, called the OthersClass. Hence, the OthersClass can serve as the place where intruders, attacks, disturbance, etc. can be observed and defended. But also, it is the place where the new, inspiration, *surprise* and challenge can be localized and welcomed." (Kaehr, 2007)

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Toth’s semiotic diamonds

Analyzing construction principles for semiotic diamonds

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Abstract

A a detailed comparison of Toth’s semiotic diamonds (Diamanten) and the diamonds of diamond category theory is presented. It turns out that Toth’s Diamanten are based on inversions of acceptional morphisms and are not constituting any rejectional morphisms. A proper definition of the matching conditions is missing. A comparison of the matching conditions for Diamanten and diamonds gives easy criteria for separation of the approaches. As a result, semiotic Diamanten are not working as semiotic models of categorical diamonds. Nevertheless, semiotic Diamanten are a novelty in semiotics and are opening up new fields of semiotic studies.

Sketch of Toth’s semiotics

The semiotic composition operation of the elementary semiotic mappings, like (I→M), (M→O), (I→O), between the objects I, M, O, is *commutative* and *associative*. And obviously the *identity* mapping *id* is realized for the objects I, M, O.
Hence, semiotic composition can be studied as a mathematical category in the sense of category theory with objects I, M, O and its mappings (arrows) between the objects.

Categorical interpretation of the semiotic sign scheme

$$
\begin{array}{l}
i.i \;=\; \text{idi},\;\; i = 1, 2, 3 \\[6pt]
1.2 \;=\; \alpha \quad\equiv\; 1 \xrightarrow{\alpha} 2 \\
1.3 \;=\; \beta\alpha \;\equiv\; 1 \xrightarrow{\beta\alpha} 3 \\
2.3 \;=\; \beta \quad\equiv\; 2 \xrightarrow{\beta} 3 \\[6pt]
2.1 \;=\; \alpha^{\circ} \quad\equiv\; 1 \xleftarrow{\alpha^{\circ}} 2 \\
3.1 \;=\; \alpha^{\circ}\beta^{\circ} \equiv\; 1 \xleftarrow{\alpha^{\circ}\beta^{\circ}} 3 \\
3.2 \;=\; \beta^{\circ} \quad\equiv\; 2 \xleftarrow{\beta^{\circ}} 3
\end{array}
$$

$$
\begin{array}{l}
1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3,\;\; 1 \xrightarrow{\beta\alpha} 3 \\[10pt]
1 \xleftarrow{\alpha^{\circ}} 2 \xleftarrow{\beta^{\circ}} 3,\;\; 1 \xleftarrow{\alpha^{\circ}\beta^{\circ}} 3
\end{array}
$$

In concreto, it still has to be analyzed how the semiotic *matching conditions* for compositions are realized.
In the example above, the question is, how is “2.1” in (3.1 → 2.1) as a *codomain* and in (2.1 → 1.3) as a *domain* defined?

The new question which arises for abstract diamond theory is: How are the difference relations and hence the hetero-morphisms defined *in concreto*?

I had the idea that the papers of Toth are filling this gap with his *semiotic* modeling of diamonds. But it turned out that there is a cricial difference between ‘semiotische Diamanten” and categorical diamonds.
Therefore, there is still an open question about a semiotic concretization of categorical diamonds.

Toth is *suggesting* an answer to the question, how to interpret the *difference* relations, with the introduction of the operation of *inversion* INV of a concrete sign scheme. Hence, Toth’s interpretation of diamonds tries to join together semiotic and diamond thematizations and notational systems.

Inversion

Where is this operation INV from?
The sign relation ZR is defined as a relation of *monadic*, *dyadic* and *triadic* relations:

ZR = (a, (a⟹b), (a⟹b⟹c).

Sign values for ZR are:
a = {1.1, 1.2, 1.3}
b = {2.1, 2.2, 2.3}
c = {3.1, 3.2, 3.3}

ZR = <3.x, 2.y, 1.z> with x,y,z∈{1,2,3} and x≤y≤z.

It is clear, that the semiotic inversion operation INV is a semiotic operation based on the elementary operations of transpositions and is not leading out of the semiotic domain.
INV is defined by:

INV(a.b c.d e.f) = (e.f c.d a.b).

In contrast, dualization is defined as:

DUAL(a.b c.d e.f) = (f.e d.c b.a).

The *abstract sign scheme* gets an interpretation by the introduction of the instances:
I = interpretant, M=medium and O=object.

Hence, the semiotic triad occurs as morphisms between the instances I, O, M and their combinations, called graph theoretic sign models.

1. (I⟶O ⟶M)	4. (O⟶M⟶I)
2. (M⟶O⟶I)	5. (I⟶M⟶O), (M⟶I⟶O)
3. (I⟶M⟶ O)	6. (O⟶I⟶M).

It is proposed by the Stuttgarter School of semiotics (Bense, Walter) that those triadic sign schemes can be composed by their dyadic relations (mappings).
Most of the semiotic work is in German, thus it is easily possible that I will miss the correct terminology.

Example:

2. (M ⟹ O) (O ⟹ I) = (M ⟹ O • O⟹ I)
Hence, triadic-trichotomic sign relations are compositions of *dyadic*-dichotomic relations. This is a strong thesis, and I don’t see the necessity of such a reduction.
Even more problematic, Elisabeth Walter (1979, S. 79), speaks of a *lattice theoretical* union of “(M ⟹ O) (O ⟹ I) = (M ⟹ O.O⟹ I)". (Toth, (2008b), p.11)

Bense (1976) is mentioning a category theoretical composition of the triadic sign conceived as a transition from the set theoretic and relational definition to a more adequate presentation (Darstellung) of semioticity .

In the following, I will first follow this strategy, then I will focus on the composition of triadic-trichotomic sign structures as such.
The diamondization of the internal relations of signs might be called *micro*-analysis, the focus on the latter *macro*-analysis of semiotic diamonds.

These 6 graph theoretic sign models of I, O, M, get an interpretation by their corresponding numeric value occupancies.
Example
3.1 (I⟶O⟶M)
(3.1 2.1 1.1) (3.1 2.3 1.3)
(3.1 2.1 1.2) (3.2 2.2 1.2)
(3.1 2.1 1.3) (3.2 2.2 1.3)
(3.1 2.2 1.2) (3.2 2.3 1.3)
(3.1 2.2 1.3) (3.3 2.3 1.3)
(Thot, p. 2, 2008a)

The new question which arises now is: How are the *difference* relations of the semiotic diamond and hence the hetero-morphisms defined *in concreto*? More precisely, how are the difference relations between the domains/codomains of morphisms and hetero-morphisms of semiotic composition defined? And is the inversion INV operation strong enough to define the differentness of the new hetero-morphisms?

Response to Toth’s remark

My question: *“Is the inversion INV operation strong enough to define the differentness of the new hetero-morphisms compared to morphisms?”*

Toth’s remark
"In einer kürzlich veröffentlichten Kritik bemerkte Rudolf Kaehr zurecht, dass in dergestalt eingeführten semiotischen Diamanten die Heteromorphismen nichts anderes seien als *Spiegelungen* dyadischer semiotischer Funktionen (Kaehr 2008, S. 3).

Kaehr übersieht allerdings, dass die *Umkehrungen* dyadischer Funktionen nur *formal*, aber nicht *inhaltlich* Spiegelungen sind. Z.B. bedeutet (2.1 ⇒ 3.1) die rhematische Interpretation eines Abbildes, aber die umgekehrte Funktion (3.1 ⇒ 2.1) muss, wie bereits Bense (1981, S. 124 ff.) bemerkte, nicht zum selben Icon zurückführen. Es kann sich hier also um einen echten semiotischen *Heteromorphismus* handeln.” [my emph]

Response
As far as I understand, Toth is pointing to the fact, that an inversion (Umkehrung, Spiegelung) of a morphism like (2.1 ⇒ 3.1) is not re-establishing the identity of “2.1” *in concreto*.
In other words, the formal *inversion* of a dyadic function is not identical with its mirroring with regard to its *content*. Hence, with an inversion INV (2.1 ⇒ 3.1) = (3.1 ⇒ 2.1), the role of “2.1” might change from a rhematic interpretation of an object to a different interpretation of an object, not necessary the same icon.

The same is true for the inversion of morphisms in a category! But, nevertheless, category theory is studying morphisms and their inversions and duality *in abstracto*. The same is intended for diamond theory. Morphisms and heteromorphisms in diamonds are considered as *abstract*.

That’s a reason too, while Toth’s approach to diamonds is important: he offers a concrete interpretation. Hence, it also could be understood as a *model* for abstract diamonds. On the other hand, diamonds are instrumental to solve some intriguing problems of mathematical semiotics.

With my introduction of diamonds and their hetero-morphisms I tried to distinguish them from both, inversions and duals (opposites) of morphisms. It is well known, that an inverted, also dual, morphism is still a morphism for which all the properties and laws for morphisms (identity, commutativity, associativity) of the category holds. While for hetero-morphisms, different laws are involved. And that's the reason why they are called *hetero-morphism* and are belonging to *saltatories* and not to categories.

What I called *antidromic* direction of arrows for hetero-morphisms is not simply an *inversion* of the arrow of a morphism, and therefore still a morphism, but a new abstraction based on "*difference*" relations of the "targets" and "sources" of composed morphisms, i.e. of *compositions*. Hetero-morphisms are conceived as abstractions from the operation of *composition* and not from morphisms between objects. Even if those difference relations are considered as inversions, they have to take place at the right place, i.e. at the target of the first and at the domain of the second morphism of the composition of the morphisms.

This in full harmony to what I developed from the very beginning of the introduction of the diamond concept (diamond category theory, diamond theory, etc.).

The name *"hetero-morphisms*" might be misleading, but the formal definition is what counts and not its label. There is a different use of the term 'hetero-morphism' in my paper *"Categories and Contextures".*

Because diamonds are introduced in an *abstract* way, i.e. depending on the *alpha/omega-structure* of composition of morphisms only, it is of importance to find reasonable examples as *concretizations*. An other interesting concretization is the attempt to introduce diamond *relations* as a diamondization of relation theory.

In this sense of *concretization*, I see the importance of Toth's approach, albeit he is missing the train.

Unfortunately, after I finally understood Toth's concept "semiotischer Diamanten" (semiotic diamonds) it turned out that Toth's "*Diamanten*" are strictly different from my 'diamonds'.

Hence, Toth has given some interesting interpretations for his "Diamanten" in the context of semiotics but not to my diamonds. That is, my hope for a semiotic concretization of the abstract mathematical concept of 'diamond category theory' has still to wait to be achieved.

Nevertheless, there are also *creative* misunderstandings! In this sense, both concepts, the "Diamanten" and the diamonds, are interesting topics. Toth's "Diamanten" are, thus, a (creative) misinterpretation of my diamonds.

Hence, despite the examples for 'semiotische Diamanten', the relevant question still is: How are *semiotic* diamonds defined? Where are the properties, rules and laws? And, first of all, how are the *difference* relations introduced? There is surely an inversion operation in semiotics but there is no abstraction corresponding to the diamond difference relations or operations.

Semiotics is still depending on is-abstractions in contrast to as-abstractions in diamond theory.

Diamonds, everywhere?

Independent of the question of the motivation to mention that my teacher Gotthard Gunther had introduced diamonds long before my own humble trial, Toth's move confirms clearly and without sophisticated manoeuvres of my interpretations what exactly he understands by a "Diamant" (diamond). Correctly, Toth understands by a Diamant the formal structure of a special square: ◇. Unfortunately, not all such diamonds are diamonds in the sense of the *diamond category theory* I introduced recently. What counts in a mathematical theory are the definitions, properties and rules. The same holds for diamond (category) theory.

I'm quite convinced that my definitions, despite their tentativeness, are clear enough to show the difference between a '*square*' concept and my diamond concept of an interplay between *categorical* and *saltatorical* arrows (maps, morphisms).

In other words, diamonds, labeled with some mighty labels might seduce to engage into a recreational game but are by no way of any interest for my own work, which stands without any 'didactical' figures.

The other hint to Toth's diamond interpretation is given by his *historical* comments that I introduced myself the diamond approach first 1995 and then specially in 2007. I appreciate the honor I' m given for a figure, which is known, at least outside polycontexturality, since tausends of years.

In fact, I’m turning around in this carousel at least, officially, since 1973-75 (published 1978), with my *closed* proemial relationship ("geschlossene proemial Relation"), which Gunther liked very much, albeit I didn’t see much to formalize.

It was writen strategically against the Varela’s ECI and the re-entry mysticism.

A university degree was achieved by my student Klaus Grochowiak with *“Die formal Struktur der Zirkulation bei Karl Marx”*, 1976, which is an intriguing application of diamonds, albeit labelled differently, and chiasms!

Then, the very first diamond approach for communicational purposes (and money making) was developed 1992 and was called, for a joke, *“Das Existenz-Halma”*. Halma is a board game. It worked! Why not? And again, I started a series of serious work about *“Diamond Strategien"* (1995-1998).

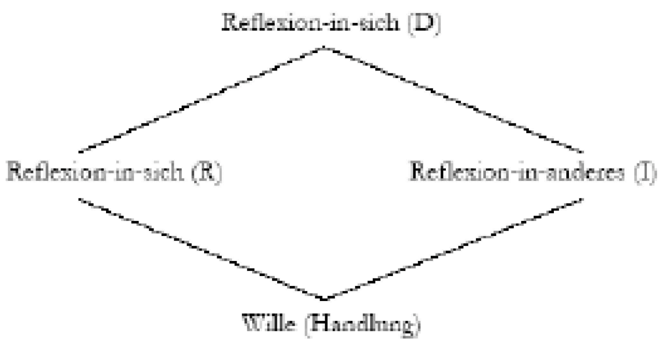
Without doubt, I’m very much aware about that, i.e. my own intellectual history. Too long to tell! It would be surprisingly strange or even highly cranky if I would have proclaimed **2007** that I discovered the diamond of category theory or diamond category theory, not knowing that I and especially Gunther introduced diamonds long before.

Again, the whole story is documented in my “*Handbuch*” (1995)! I simply would like to recall Daniel Hellerstein’s *Diamond Logic*! Which nevertheless has nothing to do with diamond theory.

As often, misunderstandings might create some interesting results.
But scholasticism of some groups is forcing me to offer some clarifications.

Nevertheless, it is a merit of Alfred Toth to have done interesting work on the base of his concept of Diamanten and he has mentioned his source of inspiration as far as he has access to it.
On the other hand, I feel motivated to typset even more constructions.

Alfred Toth, Phantasie und Technik, 1.1.2009
"Offiziell wurde der Diamant als logisches Modell erst durch Kaehr (1995) und vor allem Kaehr (2007) in die Polykontexturalitätstheorie eingeführt. Allerdings findet man bereits in Günthers “Bewusstsein der Maschinen” einen höchst interessanten Diamanten im Zusammenhang mit der reflexionstheoretischen Begründung einer Theorie des Willens im Sinne einer Theorie transzendentaler Handlungen:



Toth's general procedure

"Die semiotische Rejektionsfunktion ist nun aber keineswegs auf den Strukturtyp (e.f c.d a.b) wie im obigen semiotischen Diamanten beschränkt. Semiotische Inversion (INV) ist allgemein durch folgende zwei Anweisungsschritte erreichbar:

1. Kehre die *Reihenfolge* der konstituierenden Subzeichen einer Zeichenklasse (oder einer ihrer Transpositionen bzw. Dualisationen) um.
2. Vertausche alle semiotischen Morphismen mit ihren *Inversen* (wobei natürlich z.B. a°°= a, b°° = b und per definitionem (vgl. Toth 1993, S. 21 ff.) (ba)° = a°b° und (a°b°)°= ba gilt."

INV – Rules
I. $a^{\circ\circ} = a$, $b^{\circ\circ} = b$,
II. $(ba)^\circ = a^\circ b^\circ$,
III. $id^\circ = id$
$(a^\circ b^\circ)^\circ = (b^{\circ\circ} a^{\circ\circ}) = ba$ (IV.)

Hence, there are two steps to consider for the construction of a hetero-morphism in a semiotic diamond:
*1. **Chiasm**: Change the order of subsigns.*
*2. **Inversion**: Exchange all semiotic morphisms with their inversion.*

This can be reformulated by:
1. Apply the *inverse* operation INV to the parts (A_3, B_3) of an (acceptional) morphism ($morph_3$) .
An *acceptional* morphism is the composition of two basic morphism of a semiotic category.
2. *Substitute* the results of the *first* part, (A_3), of the morphism with the *second* part, (B_4), of the hetero-morphism, $morph_4$, to be constructed. And the same procedure with the *second* part, (B_3); substitute it with the *first* part (A_4) of the hetero-morphism $morph_4$.
And additionally:
3. *Positioning, difference, matching conditions*
3.1 The second part of the first morphism, its codomain or target, and the first part of the hetero-morphism, its domain, source, have to be matched.
3.2 The first part of the second morphism, its domain or source, and the second part of the hetro-morphism, its codomain, target, have to be matched.

A hetero-morphism is the rejectional morphism of an acceptional morphism. But this distinction has first to be established. Without the distinction between acceptional and rejectional morphism, the constructed hetero-morphism is not yet placed. To realize its correct placement, a new condition has to be fulfilled. It is the role of the *difference* relations to organize such a placement of a not yet positioned hetero-morphism.
It seems that Toth's approach is not considering this part of the construction.

Semiotic hetero – morphism construction
$\forall$ morph (A_3, B_3), (A_4, B_4) $\in$ SEMIOTICS
IF
(A_3, B_3) $\in$ Morph
$Subst_{(INV(A_3)/B_4)} \wedge Subst_{(INV(B_3)/A_4)}$
THEN
(A_4, B_4) $\in$ Het – Morph.

Schemes of inversion and exchange

***a*) intended scheme?**

$$\begin{array}{l} \boxed{\overline{C} \longleftarrow \overline{A}} \text{ : heter} \\ \quad | \; ? \qquad | \; ? \\ (A \longrightarrow B)\, o\, (B \longrightarrow C) \text{ : comp} \\ | \qquad\qquad\qquad | \\ \boxed{A \longrightarrow C} \text{ : morph} \end{array}$$

***b*) realised scheme!**

$$\begin{array}{l} \boxed{\overline{C} \longleftarrow \overline{A}} \text{ : heter} \\ | \qquad\qquad\qquad | \\ (A \longrightarrow B)\, o\, (B \longrightarrow C) \text{ : comp} \\ | \qquad\qquad\qquad | \\ \boxed{A \longrightarrow C} \text{ : morph} \end{array}$$

Example

$$\begin{array}{l} A_3 = [\alpha^\circ \beta^\circ, \alpha] \longrightarrow [\text{id1}, \beta] = B_3 \\ \quad \downarrow \text{ INV, Subst} \downarrow \\ B_4 = [\beta\alpha, \alpha^\circ] \longleftarrow [\text{id1}, \beta^\circ] = A_4 \end{array}$$

$$\begin{array}{l} \overleftarrow{[[\text{id1}, \beta^\circ], [\beta\alpha, \alpha^\circ]]} = \text{heter}_4 \\ \qquad \mathbf{x} \\ \underrightarrow{[[\alpha^\circ \beta^\circ, \alpha], [\text{id1}, \beta]]} = \text{morph}_3 \end{array}$$

Toth’s examples are showing a solution in the sense of scheme (a), i.e. a hetero-morphism (heter_4) is defined as a direct inversion of an acceptional morphism (morph_3). And morph_3 is a composition of morph_1 and morph_2.

Inversion (categorical dualisation)

It seems that the first step of ‘chiastic exchange’ is included in the definition of the inversion operation.

Example

1. Chiastic exchange ("external exchange")

$A_3 = [\alpha^\circ \beta^\circ, \alpha] \longrightarrow [\text{id1}, \beta] = B_3$

$\text{INV}(A_3 \longrightarrow B_3) = \text{INV}(B_3) \longrightarrow \text{INV}(A_3)$. This is the 'exchange' (by INV – RuleI – II)

$\qquad = \text{INV}(A_3) \longleftarrow \text{INV}(B_3)$

2. Inversions ("internal exchange")

$A_3 = [\alpha^\circ \beta^\circ, \alpha] = ([\alpha^\circ \beta] \longrightarrow \alpha)$

$B_3 = [\text{id1}, \beta] = (\text{id1} \longrightarrow \beta)$

$\text{INV}(A_3) = \text{INV}([\alpha^\circ \beta] \longrightarrow \alpha)$:

$\qquad = (\text{INV}(\alpha) \longrightarrow (\text{INV}([\alpha^\circ \beta^\circ]))$: INV – RuleI – II

$\qquad = (\alpha^\circ \longrightarrow [\alpha\beta]) = A_4$

$\text{INV}(B_3) = \text{INV}(\text{id1} \longrightarrow \beta)$

$\qquad = (\text{INV}(\beta) \longrightarrow \text{INV}(\text{id1})$: INV – RuleI – II

$\qquad = (\beta^\circ \longrightarrow \text{id1}) = B_4$ •

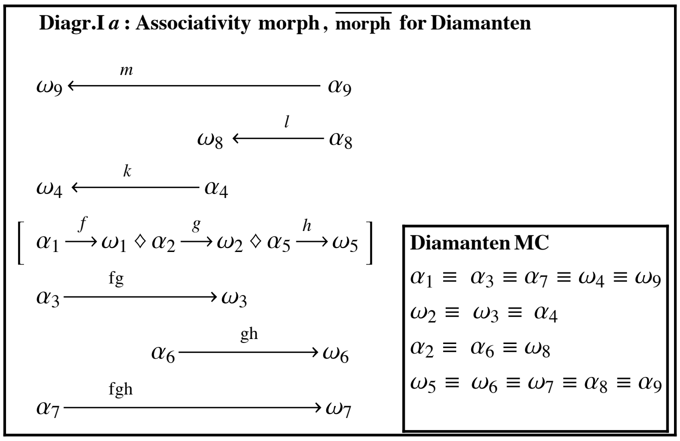
Toth’s notation for brackets [x, y] in [x, y], [z]] of a hetero-morphism could mislead to a wrong interpretation if taken as a mapping "(x)⟶ (y), but [x, y] is by definition, in this setting, a non-decomposable “object".

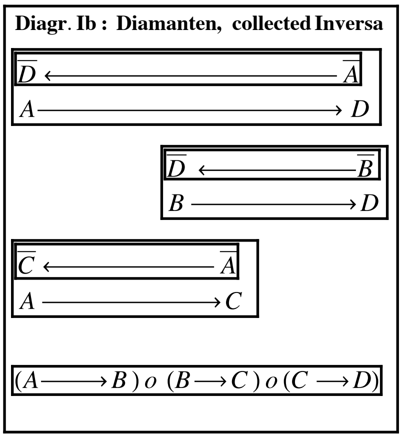
Further comparisons: Are Diamanten diamonds?

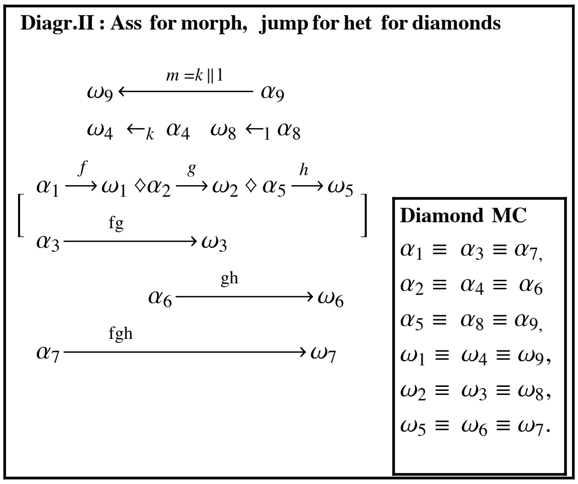
Hence, the difference to my approach is clear. Hetero-morphisms are not directly accessible by acceptional morphisms but are depending on the structure of the composition of morphisms. That is, the process of composition is reflected in hetero-morphisms, while acceptional morphisms are the positive or direct results of compositions, i.e. $morph_1 o\, morph_2 = morph_3$.

A consequence of Toth's approach seems to be the lack of jumping situation or *saltisitions* for general Diamanten. Toth's hetero-morphisms are connected and composed without any gaps to be over-jumped and hence, the associativity rule holds unrestrictedly.

The main difficulty to understand properly Toth's approach has two reasons:
1. the diagrams used are not always clear. Different readings are possible, because
2. there are no formal *matching conditions* (MC) for compositions of morphisms and hetero-morphisms defined.

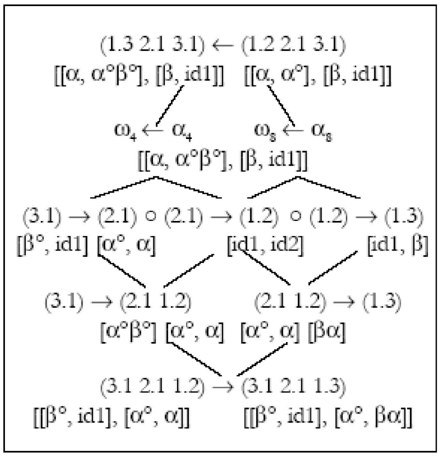






Again, because diagrams are not telling much without their formal definitions, I repeated the formal definitions of the diagrams for Diamanten and for diamonds.
And, unfortunately, programs are still not stable enough to produce proper results in different formats.

The above diagram is semiotically concretised by Toth's example for a semiotic Diamant based on 3 composed semiotic morphisms.



As a first result we might observe that Toth's hetero-morphisms are a kind of *counter-morphisms*. They might even have the properties of being: *"... genuinely tetradic, chiastic, antidromic and 4-fold."* And they are connected with morphisms by *inversion* and *chiastic* exchange. Obviously those properties are necessary but maybe not sufficient for diamonds.

What I proposed as diamonds at different places, are structures with very different laws compared to the laws of categories. That is, diamonds, which consist of a complementary interplay of categories and saltatories, are as categorical systems identitive, commutative and associative in respect to their objects, morphisms and composition. Therefore, inheriting all the laws and methods from category theory. In sharp contrast, saltatories as parts of diamonds, are ruled by difference, jumps (saltisitions) and jump-associativity, etc.
Additionally, diamonds as such, are containing *bridges* and *bridging* rules between categories and saltatories.

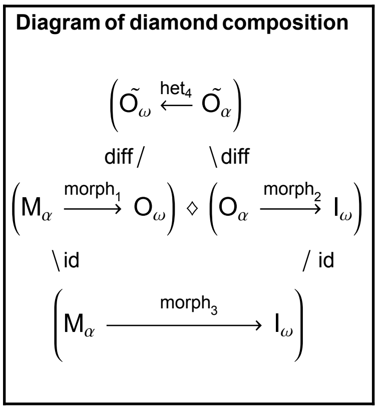
▫ *Again, diamond definitions*

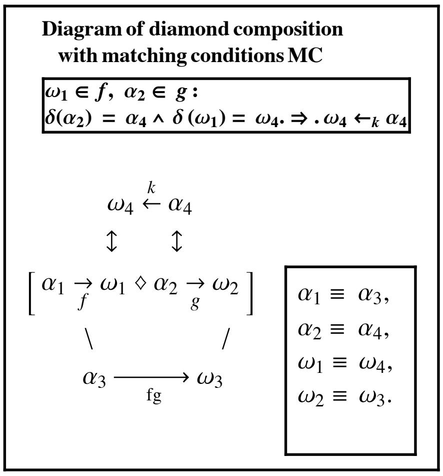
Diamond composition rule $(M_\alpha \rightarrow O_\omega) \diamond (O_\alpha \rightarrow I_\omega)$

$(M_\alpha \rightarrow I_\omega) \,\Big|\, (\tilde{O}_\omega \longleftarrow \tilde{O}_\alpha)$

$\tilde{O}_\alpha \equiv \text{diff}(O_\alpha)$

$\tilde{O}_\omega \equiv \text{diff}(O_\omega)$.





One of Toth's semiotic motivations for diamonds

"Ein sowohl für die Semiotik wie für die Kybernetik wichtiges Ergebnis ist, dass zwar die Abbildung von Zeichenklassen auf Morphismen eindeutig ist, nicht aber die Abbildung von Morphismen auf Realitätsthematiken: Während also $\alpha° \longrightarrow$ (2.1), id2 $\longrightarrow$ (2.2), $\beta \longrightarrow$ (2.3) gilt, erhalten wir für ((2.1), (2.1)), ((2.2), (2.2), (2.2)) und ((2.3), (2.3)) jedesmal id2, d.h. bei den Semiosen zwischen Trichotomischen Triaden, die ja aus Realitätsthematiken konstruiert sind, können die für Zeichenklassen eingeführten Morphismen nicht zwischen den trichotomischen Stellenwerten unterscheiden!

Der Grund dafür hängt wohl mit der Tatsache zusammen, dass dynamische (generative) Semiosen und (degenerative) Retrosemiosen im Gegensatz statischen Subzeichen und Zkln/Rthn nicht umkehrbar-eindeutig sind, weshalb in Toth (2008a) der Versuch unternommen wurde, sie mit Rudolf Kaehrs Hetero-Morphismen im Rahmen seiner polykontexturalen Diamanten-Theorie zu identifizieren (vgl. Kaehr 2007).

Sollte diese Identifikation statthaft sein, kommt denjenigen Trichotomischen Triaden, die wie etwa die Nrn. 589 und 617 ausschliesslich aus semiotischen Heteromorphismen (und idx) und vor allem denjenigen, die (neben) idx nur Morphismen und ihre korrespondierenden Heteromorphismen enthalten wie etwa den Nrn. 551, 561 und 564, besondere Bedeutung im Zusammenhang mit einer erst zu entwickelnden kybernetischen Semiotik der 2. Ordnung bzw. einer Vereinigung von Semiotik und Polykontexturalitätstheorie zu." (Toth, Kybernetik, p.663/64)

$589\,[\text{II, OO, MM}] \Longleftrightarrow [3.1\times3.2\times3.3 - 2.1\times2.2\times2.3 - 1.1\times1.2\times1.3]$
$\Longleftrightarrow [\alpha^{\circ}\beta^{\circ}\,\beta^{\circ}\ \text{id3} - \alpha^{\circ}\ \text{id2}\ \beta - \text{id1}\ \alpha\ \beta\alpha]$

$3.1\times3.2\times3.3$ $\quad$ $2.1\times2.2\times2.3$ $\quad$ $3.1\times3.2\times3.3$
$\text{T1}: 2.1\times2.2\times2.3$ $\quad$ $\text{T2}: 1.1\times1.2\times1.3$ $\quad$ $\text{T3}: 1.1\times1.2\times1.3$

$b'1 = [\beta^{\circ}, \beta^{\circ}, \beta^{\circ}]$ $\quad$ $b'2: = [\alpha^{\circ}, \alpha^{\circ}, \alpha^{\circ}]$ $\quad$ $b'3 = [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}\beta^{\circ}, \alpha^{\circ}\beta^{\circ}]$
$\bigcap b'i = \emptyset$ $\quad$ (p.595)

Playing the game

Now I would like to test Toth's construction rules (INV, exchange) in reconstructing his examples.
It seems, that the INV rule is always working if applied in the introduced sense as "INV = inv(inv, inv)" . With the help of the exchange advise, INV is reduced to "(inv, inv)".

Again, Toth's example and an explicite notation:

$[\alpha, \alpha^{\circ}\beta^{\circ}] \leftarrow [\beta, \text{id}_1] : \text{morph}_4$
/ \
$[\beta^{\circ}, \text{id}_1] \diamond [\alpha^{\circ}, \beta\alpha]$
\ /
$[\beta^{\circ}, \text{id}_1] \longrightarrow [\alpha^{\circ}, \beta\alpha] : \text{morph}_3$

explicite notation
$[\alpha \leftarrow (\alpha^{\circ}\beta^{\circ})] \leftarrow [\beta \leftarrow \text{id}_1] : \quad \text{morph}_4$
/ \
$[\beta^{\circ} \rightarrow \text{id}_1] \diamond [\alpha^{\circ} \rightarrow \beta\alpha] : \quad \text{morph}_1\ o\ \text{morph}_2$
\ /
$[\beta^{\circ} \rightarrow \text{id}_1] \longrightarrow [\alpha^{\circ} \rightarrow (\beta\alpha)] : \quad \text{morph}_3$

- ***Example1***

$(1.3\times1.2\times3.1)$
$[[\text{id1}, \beta^{\circ}], [\beta\alpha, \alpha^{\circ}]]$

$[\alpha^{\circ}\beta^{\circ}, \alpha]$ o $[\text{id1}, \beta]$

$[[\alpha^{\circ}\beta^{\circ}, \alpha], [\text{id1}, \beta]]$
$(3.1\times1.2\times1.3)$

$$
\begin{array}{l}
1.\ A_3 = [\alpha^\circ \beta^\circ, \alpha] \Longrightarrow A_4 = [\beta\alpha, \alpha^\circ] = [\beta\alpha \leftarrow \alpha^\circ] \\
\qquad\downarrow \qquad\qquad\qquad\qquad \downarrow \\
2.\ B_3 = [id1, \beta] \Longrightarrow B_4 = [id1, \beta^\circ] = [id1 \leftarrow \beta^\circ]
\end{array}
$$

INVERSION INV

ad1. $INV([\alpha^\circ \beta^\circ, \alpha]) = INV([INV([\alpha^\circ \beta^\circ]), INV([\alpha])$
$= [INV([\alpha]), INV([\alpha^\circ \beta^\circ])$
$= [[\alpha^\circ], [\beta\alpha]] = [\alpha^\circ, \beta\alpha] = [\alpha^\circ \longrightarrow \beta\alpha] =$ $\mathbf{A_4}$

ad2. $INV([id1, \beta]) = INV([INV(id1), INV(\beta)])$
$= [INV(\beta), INV(id1)]$
$= [\beta^\circ, id1] = [\beta^\circ \longrightarrow id1] =$ $\mathbf{B_4}$

- ***Example2***

$$
\begin{array}{c}
(3.1 \times 2.1 \times 1.3) \\
[[\beta^\circ, id1], [\alpha^\circ, \beta\alpha]] \\
\\
[\alpha, \alpha^\circ \beta^\circ] \quad o \quad [\beta, id1] \\
\\
[[\alpha, \alpha^\circ \beta^\circ], [\beta, id1]] \\
(1.3 \times 2.1 \times 3.1)
\end{array}
$$

$$
\begin{array}{l}
1.\ A_3 = [\alpha, \alpha^\circ \beta^\circ] \Longrightarrow A_4 = [\alpha^\circ, \beta\alpha] = [\alpha^\circ \leftarrow \beta\alpha] \\
\qquad\downarrow \qquad\qquad\qquad\qquad \downarrow \\
2.\ B_3 = [\beta, id1] \Longrightarrow B_4 = [\beta^\circ, id1] = [\beta^\circ \leftarrow id1]
\end{array}
$$

INVERSION INV

ad1. $INV[\alpha, \alpha^\circ \beta^\circ] = [\alpha^\circ, \beta\alpha]$
$= INV(INV[\alpha], INV[\alpha^\circ \beta^\circ])$
$= INV[\alpha^\circ \beta^\circ], INV[\alpha]$
$= [[\beta\alpha], \alpha^\circ] = [\beta\alpha, \alpha^\circ] = [\beta\alpha \longrightarrow \alpha^\circ]$ $\mathbf{= A_4}$

ad2. $INV[\beta, id1] = [\beta^\circ, id1]$
$= INV[INV(\beta) \rightarrow INV(id1)]$
$= [INV(id1) \rightarrow INV(\beta)]$
$= [id1 \rightarrow \beta^\circ]$ $\mathbf{= B_4}$

Are Toth’s hetero-morphism and semiotic diamonds the same constructs or constructs in the same spirit as the diamond categories introduced by my own intuitions? What could the difference be? And how could such a possible difference matter?

Toth’s construction is considering the complementarity between acceptional and rejectional morphisms based on inversion (INV) and chiastic exchange.
This corresponds to some sketches I produced myself. But I conceived them as *abbreviations* of the difference based constructions.

"Compositions as well as sautisitions (jump-operations) are ruled by identity and associativity laws. Complementarity between categories and saltatories, i.e., between acceptional and rejectional domains of diamonds, are ruled by difference operations.” (Kaehr, p.3)

▫ *Comparision: Toth's Diamanten and diamonds*

The following definitions could give a hint to understand the *difference* between the two diamond constructions.

Complementarity of Acc and Rej based on diff

$X \in$ Acc iff compl(X) $\in$ Rej

$X = g\ o\ f$:

1. $X \in$ Acc if compl(X) $\in Rej$

compl($g\ o\ f$) = compl (compl(g) o compl(f))

= compl(diff(cod(f)) o diff(dom(g)))

= compl$\left(\left(\overline{B_{cod}}\right) o \left(\overline{B_{dom}}\right)\right) = \omega_4 \leftarrow \alpha_4$.

(u : $\omega_4 \leftarrow \alpha_4$) $\in$ Rej

Hence, ($g\ o\ f$) $\in$ Acc if (u : $\omega_4 \leftarrow \alpha_4$) $\in$ Rej

($g\ o\ f$) $\in$ Acc if $\left(g\ \overline{o}\ f\right) \in$ Rej.

2. compl(X) $\in$ Rej if $X \in$ Acc

compl($\omega_4 \leftarrow \alpha_4$) = compl(compl($\omega_4$) $\leftarrow$ compl(α_4))

= compl(($A_{dom} \to B_{cod}$) $\leftarrow$ ($B_{dom} \to C_{cod}$))

= (($A_{dom} \to B_{cod}$) o ($B_{dom} \to C_{cod}$))

= ($f\ o\ g$).

3. Hence, $X \in$ Acc iff compl(X) $\in$ Rej.

In this "proof", the complementarity operation "compl" is used quite freely to do also the transitional job of completing the morphisms out of the objects. This is done by the operation of "difference" and "completition", which is completing domains and codomains to their morphisms. This points to the asymmetry of Acc- and Rej-domains.

Toth's complementarity operation between acceptional and rejectional morphisms, played by INV and exchange, is symmetrical.

The open question for the construction of semiotic diamonds still is: How is the difference relation defined, *concretely*?

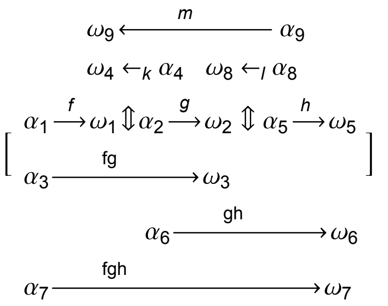
Again, Toth's approach, despite its own merits, is not answering this questions, simply because they don't exist for his Diamanten.

▫ *Similarity?*

The nearest comparision between Toth's approach to diamond and what I published myself can be found in the general complementarity between acceptional and rejectional morphisms as sketched below. Nevertheless, the definitions of the hetero-morphisms k, l, m are based on the abstractions of compositions and not on objects. That is, hetero-morphism k : $\delta(\alpha_2) = \alpha_4$ and $\delta(\omega_1) = \omega_4$.ETC!

$\omega_1 \in f,\ \alpha_2 \in g:$

$\delta(\alpha_2) = \alpha_4 \wedge \delta(\omega_1) = \omega_4 . \Rightarrow . \omega_4 \leftarrow_k \alpha_4$



Diamond MC

$\alpha_1 \equiv \alpha_3 \equiv \alpha_7,$

$\alpha_2 \equiv \alpha_4 \equiv \alpha_6$

$\alpha_5 \equiv \alpha_8 \equiv \alpha_9,$

$\omega_1 \equiv \omega_4 \equiv \omega_9,$

$\omega_2 \equiv \omega_3 \equiv \omega_8,$

$\omega_5 \equiv \omega_6 \equiv \omega_7.$

complementarity of accept, reject

reject(fg)	= *k* iff accept(*k*) = (fg)
reject(gh)	= *l* iff accept(*l*) = (gh)
reject(fgh)	= *m* iff accept(*m*) = (fgh)

Thus, the operation *reject(gf)* of the composition of the acceptance morphisms f and g is producing the *rejectance* hetero-morphism k.
And the operation *accept(k)* of the rejectance morphism k is producing the *acceptance* of the composition of the morphisms g and f.

Comparision of Diamanten and diamonds

Diamond $MC^{(3)}$

$\alpha_1 \equiv \alpha_3,$

$\alpha_2 \equiv \alpha_4,$

$\omega_1 \equiv \omega_4,$

$\omega_2 \equiv \omega_3.$

Diamant $MC^{(3)}$

$\alpha_1 \equiv \alpha_3 \equiv \omega_4$

$\omega_2 \equiv \omega_3 \equiv \alpha_4$

plus MC for compositions

Diamond $MC^{(4)}$	Diamant $MC^{(4)}$
$\alpha_1 \equiv \alpha_3 \equiv \alpha_7,$	$\alpha_1 \equiv \alpha_3 \equiv \alpha_7 \equiv \omega_4 \equiv \omega_9$
$\alpha_2 \equiv \alpha_4 \equiv \alpha_6$	$\omega_2 \equiv \omega_3 \equiv \alpha_4$
$\alpha_5 \equiv \alpha_8 \equiv \alpha_9,$	$\alpha_2 \equiv \alpha_6 \equiv \omega_8$
$\omega_1 \equiv \omega_4 \equiv \omega_9,$	$\omega_5 \equiv \omega_6 \equiv \omega_7 \equiv \alpha_8 \equiv \alpha_9$
$\omega_2 \equiv \omega_3 \equiv \omega_8,$	
$\omega_5 \equiv \omega_6 \equiv \omega_7.$	

That is,

$$\forall\, \mathbf{MC}: \quad \mathbf{MC}_{\text{diamond}} \neq \mathbf{MC}_{\text{Diamant}} \Longrightarrow \textbf{Diamanten} \neq \textbf{diamonds}$$

This detailed comparison of Toth's semiotic diamonds (Diamanten) and the diamonds of diamond category theory has shown some results:
- Toth's Diamanten are based on inversions of acceptional morphisms and are not constituting any rejectional morphisms.
- A proper definition of the matching conditions is missing.
- A comparison of the matching conditions for Diamanten and diamonds gives easy criteria for separation of both approaches.
- As a result, semiotic Diamanten are not working as semiotic models of categorical diamonds.
- Nevertheless, semiotic Diamanten are a novelty in semiotics and are opening up new fields of semiotic studies.

Sketch on semiotics in diamonds

Embedding semiotics into anchored diamonds

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Abstract

Semiotics are embedded into diamonds in a double way. Semiotics gets a internal environment as its neighbor semiotics and an external environment by its anchors. Embedding semiotics is a process of concretization of the abstract concept of Peircean semiotics.

1. Diamond embedments of semiotics

1.1. Semiotics within semiotics

Introduction of the semiotic scheme by Bense/Toth

$$ZR = ((a), (a \Longrightarrow b), (a \Longrightarrow b \Longrightarrow c))$$

Firstness is *a* relation to itself, 'monadic relation': a

Secondness is *a* binary relation, 'dyadic relation': $a \Longrightarrow b$

Thirdness is *a* ternary relation, 'triadic relation': $a \Longrightarrow b \Longrightarrow c$.

An extensive study of a *relational* introduction of the Peircean semiotics is presented in [Toth, Einführung in die semiotische Relationentheorie]. Additionally, a further development to a ternary semiotics and its relation to Gunther's ternary epistemology is proposed.

1.1.1. Diamondization of semiotics

A diamondization of the semiotic scheme is naturally realized as a diamond interpretation of the semiotic relations, i.e. morphisms.

Firstness as a doublet
A composition always is accompanied by an environment of its morphisms. Therefore, an initial object or the number 1, firstness, is diamond theoretically always doubled: as itself and as its environment, i.e.(A **|** a). That is, as a morphism, and as a hetero-morphism. A diamond initial object is not a singular object but a *doublet*. Also called *bi-object*. Furthermore, self-identity is able to distinguish its directionality as left (lo) and right (ro) order.

$\text{Diam}(ZR) = ((A \mid a),\ (A \Longrightarrow B \mid c),\ (A \Longrightarrow B \Longrightarrow C \mid b_2 \longleftarrow b_1))$

Peirce – Firstness is a relation to itself : $a \Longrightarrow a$

$\text{Diam}(a \Longrightarrow a)$:

$A \longrightarrow B \circ B \longrightarrow A$: $\text{cod}(f) = \text{dom}(g)$, hence $A = B$.

$A \longrightarrow A \mid b_1 \longleftarrow b_2$: $A = B$

$A \longrightarrow A \mid a_1 \longleftarrow a_2$: $A = B$

$A \mid a$.

Hence, **diamond – firstness** is $\boldsymbol{A} \mid \mathbf{a}$.

right | left – circularity for monads $\boldsymbol{A} \mid \mathbf{a}$:

If A^{ro} then a^{lo}, hence $A^{ro} \mid a^{lo}$

If A^{lo} then a^{ro}, hence $A^{lo} \mid a^{ro}$

Peirce – Secondness is a binary relation : $A \Longrightarrow B$.

$\text{Diam}(A \Longrightarrow B)$:

$(A \longrightarrow B \diamond B \longrightarrow A) \diamond (B \longrightarrow A \diamond A \longrightarrow B)$

$(A \longrightarrow A \mid b) \circ (B \longrightarrow B \mid a)$

$(A \mid b) \circ (B \mid a)$

$A \longrightarrow B \mid a \longleftarrow b$: $a = b$

$A \longrightarrow B \mid c$.

Hence, **diamond – secondness** is $\boldsymbol{A} \longrightarrow \boldsymbol{B} \mid \mathbf{c}$.

Alternative:

1\. $(A \Longrightarrow B)$: $(A \Longrightarrow B \Longrightarrow B)$:):

$(A \longrightarrow B \diamond B \longrightarrow B)$

$(A \longrightarrow B \circ B \longrightarrow B) \mid b_2 \longleftarrow b_1$: $b_2 = b_1$

$(A \longrightarrow B) \mid c$.

2\. $(A \Longrightarrow B)$: $(A \Longrightarrow A \Longrightarrow B)$:):

$(A \longrightarrow A \diamond A \longrightarrow B)$

$(A \longrightarrow A \circ A \longrightarrow B) \mid a_2 \longleftarrow a_1$: $a_2 = a_1$

$(A \longrightarrow B) \mid c$.

Peirce – Thirdness is a ternary relation : $A \Longrightarrow B \Longrightarrow C$.

$\text{Diam}(A \Longrightarrow B \Longrightarrow C)$:

$A \longrightarrow B \circ B \longrightarrow C$: $\text{cod}(f) = \text{dom}(g)$,

$A \longrightarrow C \mid b_1 \longleftarrow b_2$

Comments[1]

Therefore,

Firstness is a relation to itself, '*monadic* relation': a as $a \Longrightarrow a$ is a reduction of $a \Longrightarrow a \Longrightarrow a$,

Secondness is a binary or '*dyadic* relation': $a \Longrightarrow b$ is a reduction of $a \Longrightarrow b \Longrightarrow b$, $a \Longrightarrow a \Longrightarrow b$

Thirdness is a ternary or '*triadic* relation': $a \Longrightarrow b \Longrightarrow c$ is a realization of an intuition.

Diamond semiotics scheme, anchors omitted

$$\text{Diam}(ZR) = \begin{pmatrix} \emptyset \mid \varnothing \\ A \mid a \\ A \Longrightarrow B \mid c \\ A \Longrightarrow B \Longrightarrow C \mid b_2 \longleftarrow b_1 \\ A \Longrightarrow B \Longrightarrow C \Longrightarrow D \mid b_2 \longleftarrow b_1 \parallel c_2 \longleftarrow c_1 \end{pmatrix}$$

with the operators ops $= \{\Longrightarrow, \circ, \mid, \longleftarrow, \parallel\}$:

morphism $(\Longrightarrow)$, composition $(\circ)$, separation $(\mid)$,

hetero-morphism $(\longleftarrow)$, saltisition $(\parallel)$.

Diamond – Semiotics

diam – firstness:	$A \mid a$
diam – secondness:	$A \longrightarrow B \mid c$
diam – thirdness:	$A \longrightarrow C \mid b_1 \longleftarrow b_2$
diam – forthness:	$A \longrightarrow D \mid b_1 \longleftarrow b_2 \parallel c_1 \longleftarrow c_2$
diam – zeroness:	$\emptyset \mid \varnothing$

Classificatory remarks[2]

In Transit (Toth, 2008)[3]

http://www.uni-klu.ac.at/iff-tewi/inhalt/280.htm

For a start, it might be a significant result to proof the difference with the simple example of Toth' s dually invariant sign class $(3.1\ 2.2\ 1.3) \times (3.1\ 2.2\ 1.3)$, which is, as it turns out, contexturally different, i.e. $\times(3.1_3, 2.2_{1.2}, 1.3_3) = (3.1_3, 2.2_{2.1}, 1.3_3) \neq (3.1_3, 2.2_{1.2}, 1.3_3)$.

This, obviously is based on a different paradigm and its duality results for identity:

duality for Sem$^{(3,1)}$

$\text{dual}_1(\text{id}_1, \text{id}_2, \text{id}_3) = (\tilde{\text{id}}_1, \text{id}_3, \text{id}_2)$

$\text{dual}_2(\text{id}_1, \text{id}_2, \text{id}_3) = (\text{id}_3, \tilde{\text{id}}_2, \text{id}_1)$

1.2. Diamond semiotic values

1.2.1. Semiotic values

Semiotic principle for sign classes according to Bense/Toth

1. Principle of Triadic Diversity:

The general sign class structure has the form (a.b c.d e.f) with a, b, c, d, e, f ∈ {1, 2, 3} and a != b != c.

2. Principle of Degenerative Triadic Order:

The general sign class structure must have the form (3.a 2.b 1.c).

3. Principle of Trichotomic Inclusion:
(3.a 2.b 1.c) with a <= b <= c and a, b, c $\in$ {.1, .2,.3}.

"However, we may question if the three semiotic restrictions do hold. From the standpoint of relational algebra, nothing speaks in favor of the Principle of Semiotic Inclusion." (Toth, Ghost, p. 10)

1.2.2. Mediation of semiotics

Sem $^{(4)}$ - mediation table MM (Kaehr, White Papers 1978)

.

Numeric unary scheme

$$\text{Sem}^{(4,1)} = \begin{bmatrix} \text{Sem}^1 = (1, 2, 3) \\ \text{Sem}^2 = (2, 3, 4) \\ \text{Sem}^3 = (1, 2, 4) \\ \text{Sem}^4 = (1, 3, 4) \end{bmatrix}$$

$$\text{Sem}^{(4,1)} = \boxed{\begin{array}{l} 1 \longrightarrow 2 \longrightarrow 3 \longrightarrow x \\ x \longrightarrow 2 \longrightarrow 3 \longrightarrow 4 \\ 1 \longrightarrow 2 \longrightarrow x \longrightarrow 4 \\ 1 \longrightarrow x \longrightarrow 3 \longrightarrow 4 \end{array}}$$

$$\text{Sem}^{(4,1)} = \prod \text{sem}^i, \; i = 1, 2, 3, 4$$

Null

Numeric binary matrix

$Sem^{(4,1)} x Sem^{(4,1)} =$
$[(Sem^1 \; x \, Sem^1), (Sem^2 \; x \, Sem^2), (Sem^3 \; x \, Sem^3), (Sem^4 \; x \, Sem^4)]$:

$(Sem^1 \; x \, Sem^1) = (1, 2, 3, x) \, x \, (1, 2, 3, x)$
$(Sem^2 \; x \, Sem^2) = (x, 2, 3, 4) \, x \, (x, 2, 3, 4)$
$(Sem^3 \; x \, Sem^3) = (1, 2, x, 4) \, x \, (1, 2, x, 4)$
$(Sem^4 \; x \, Sem^4) = (1, x, 3, 4) \, x \, (1, x, 3, 4)$

$sem^1 \; x \, sem^1 =$

MM	1	2	3	4
1	1.1_1	1.2_1	1.3_1	1.4
2	2.1_1	2.2_1	2.3_1	2.4
3	3.1_1	3.2_1	3.3_1	3.4
4	4.1	4.2	4.3	4.4

$sem^2 \; x \, sem^2 =$

MM	1	2	3	4
1	1.1_1	1.2_1	1.3_1	1.4
2	2.1_1	$2.2_{1.2}$	$2.3_{1.2}$	2.4_2
3	3.1_1	$3.2_{1.2}$	$3.3_{1.2}$	3.4_2
4	4.1	4.2_2	4.3_2	4.4_2

$sem^3 \; x \, sem^3 =$

MM	1	2	3	4
1	$1.1_{1.3}$	$1.2_{1.3}$	1.3_1	1.4_3
2	$2.1_{1.3}$	$2.2_{1.2.3}$	$2.3_{1.2}$	$2.4_{2.3}$
3	3.1_1	$3.2_{1.2}$	$3.3_{1.2}$	3.4_2
4	4.1_3	$4.2_{3.2}$	4.3_2	$4.4_{3.2}$

$sem^4 \; x \, sem^4 =$

MM	1	2	3	4
1	$1.1_{1.3.4}$	$1.2_{1.3}$	$1.3_{1.4}$	$1.4_{3.4}$
2	$2.1_{1.3}$	$2.2_{1.2.3}$	$2.3_{1.2}$	$2.4_{2.3}$
3	$3.1_{1.4}$	$3.2_{1.2}$	$3.3_{1.2.4}$	$3.4_{2.4}$
4	$4.1_{3.4}$	$4.2_{3.2}$	$4.3_{2.4}$	$4.4_{2.3.4}$

4 – contextural semiotic matrix

$Sem^{(4,2)} =$

MM	1	2	3	4
1	$\mathbf{1.1_{1.3.4}}$	$1.2_{1.3}$	$1.3_{1.4}$	$1.4_{3.4}$
2	$2.1_{1.3}$	$\mathbf{2.2_{1.2.3}}$	$2.3_{1.2}$	$2.4_{2.3}$
3	$3.1_{1.4}$	$3.2_{1.2}$	$\mathbf{3.3_{1.2.4}}$	$3.4_{2.4}$
4	$4.1_{3.4}$	$4.2_{3.2}$	$4.3_{2.4}$	$\mathbf{4.4_{2.3.4}}$

Semiotic reduction matrix :

MM	1	2	3	4
1	1.1	1.2	1.3	1.4

categorical 4 – contextural semiotic matrix

$$\mathbf{Sem_{cat}}^{(4,2)} = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & id_{1.3.4} & \alpha_{1.3} & \alpha_{1.4} & \alpha_{3.4} \\ 2 & \alpha^{\circ}{}_{1.3} & id_{1.2.3} & \alpha_{1.2} & \alpha_{2.3} \\ 3 & \alpha^{\circ}{}_{1.4} & \alpha^{\circ}{}_{1.2} & id_{1.2.4} & \alpha_{2.4} \\ 4 & \alpha^{\circ}{}_{3.4} & \alpha^{\circ}{}_{3.2} & \alpha^{\circ}{}_{2.4} & id_{2.3.4} \end{pmatrix}$$

1.2.3. Ternary semiotics as composed by binary relations

If unary triadic signs are defined as compositions of dyadic sub-signs then their compositions to binary and ternary triadic sign systems should have a natural decomposition into their sub-signs. A natural decomposition is guided mainly by its mathematical constructions and not by any content or other interest oriented restrictions to formal procedures.
If a composition is a sign then all its permutation have to be accepted as signs.

$$Sem^{(3,1)} = \left(Sem^1, \; Sem^2, \; Sem^3\right) = \left(1_{1.3}, 2_{2.1}, 3_{2.3}\right)$$

$$Sem^{(3,1)} x Sem^{(3,1)} =$$
$$\left[\left(Sem^1 x Sem^1\right), \left(Sem^2 \, x \, Sem^2\right), \left(Sem^3 \, x \, Sem^3\right)\right]$$

$$\left(Sem^1 x Sem^1\right) = (1, 2)_1 \, x \, (1, 2)_1$$
$$\left(Sem^2 x Sem^2\right) = (2, 3)_2 \, x \, (2, 3)_2$$
$$\left(Sem^3 x Sem^3\right) = (1, 3)_3 \, x \, (1, 3)_3$$

polycontextural semiotic 3 – matrix

$$Sem^{(3,2)} = \begin{pmatrix} MM & 1_{1.3} & 2_{1.2} & 3_{2.3} \\ 1_{1.3} & \mathbf{1.1}_{1.3} & \mathbf{1.2}_1 & \mathbf{1.3}_3 \\ 2_{1.2} & \mathbf{2.1}_1 & \mathbf{2.2}_{1.2} & \mathbf{2.3}_2 \\ 3_{2.3} & \mathbf{3.1}_3 & \mathbf{3.2}_2 & \mathbf{3.3}_{2.3} \end{pmatrix}$$

short version:

$$Sem^{(3,2)} = \begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & \mathbf{1.1}_{1.3} & \mathbf{1.2}_1 & \mathbf{1.3}_3 \\ 2 & \mathbf{2.1}_1 & \mathbf{2.2}_{1.2} & \mathbf{2.3}_2 \\ 3 & \mathbf{3.1}_3 & \mathbf{3.2}_2 & \mathbf{3.3}_{2.3} \end{pmatrix}$$

$$cat^{(3)}\left(Sem^{(3,2)}\right) = \begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & 1 \longrightarrow 1_{1.3} & 1 \longrightarrow 2_1 & 1 \longrightarrow 3_3 \\ 2 & 2 \longrightarrow 1_1 & 2 \longrightarrow 2_{1.2} & 2 \longrightarrow 3_2 \\ 3 & 3 \longrightarrow 1_3 & 3 \longrightarrow 2_2 & 3 \longrightarrow 3_{2.3} \end{pmatrix} \Longrightarrow$$

$$\begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & id_{1.3} & \alpha_1 & (\beta\alpha)_3 \\ 2 & \alpha^{\circ}{}_1 & id_{1.2} & \beta_2 \\ 3 & (\alpha^{\circ}\beta^{\circ})_3 & \beta^{\circ}{}_2 & id_{2.3} \end{pmatrix} \Longrightarrow \begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & id_{1.3} & \alpha_1 & \alpha_3 \\ 2 & \alpha^{\circ}{}_1 & id_{1.2} & \alpha_2 \\ 3 & \alpha^{\circ}{}_3 & \alpha^{\circ}{}_2 & id_{2.3} \end{pmatrix}$$

categorical 3 – contextural semiotic matrix

$$Sem^{(3,2)}{}_{cat} = \begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & \mathbf{id_{1.3}} & \boldsymbol{\alpha_1} & \boldsymbol{\alpha_3} \\ 2 & \boldsymbol{\alpha^{\circ}{}_1} & \mathbf{id_{1.2}} & \boldsymbol{\alpha_2} \\ 3 & \boldsymbol{\alpha^{\circ}{}_3} & \boldsymbol{\alpha^{\circ}{}_2} & \mathbf{id_{2.3}} \end{pmatrix}$$

Special notation

$\alpha_1 \equiv \alpha$

$\alpha_2 \equiv \beta$

$\alpha_3 \equiv \beta\alpha$

1.2.4. Semiotic vs. polysemiotic sign classes

Sign classes`

$(3.1\ 2.1\ 1.1) \Longrightarrow (3.1_3\ \ 2.1_1\ \ 1.1_{1.3}) = ((3.1_3, xx,\ 1.1_3), (xx, 2.1_1, 1.1_1))$

$(3.1\ 2.1\ 1.2) \Longrightarrow (3.1_3\ \ 2.1_1\ \ 1.2_1) = ((3.1_3\ ,\ x.x\ ,\ xx), (xx\ ,\ 2.1_1\ ,\ 1.2_1))$

Toth' *s* non – sign classes

$(3.1\ 3.2\ 1.3) \Longrightarrow ((3.1_3,\ xx,\ 1.3_3), (xx,\ 3.2_2,\ xx))$

$(2.1\ 2.2\ 1.2) \Longrightarrow ((2.1_1, 2.2_1, 1.2_1), (xx,\ 2.2_2,\ xx)) \Longrightarrow (2.1_1,\ 2.2_1,\ 1.2_1)$

$(1.1\ 1.3\ 3.1) \Longrightarrow ((1.1_1,\ xx,\ xx), (1.1_3,\ 1.3_3,\ 3.1_3)) \Longrightarrow (1.1_3,\ 1.3_3,\ 3.1_3)$

Null

Bense' s 10 sign classes in a polycontextural setting

$(3.1\ 2.1\ 1.1) \Longrightarrow (3.1_3,\ 2.1_1,\ 1.1_{1,3}) \Longrightarrow ((3.1_3,\ xx,\ 1.1_3),\ (xx,\ 2.1_1,\ 1.1_1))$

$(3.1\ 2.1\ 1.2) \Longrightarrow (3.1_3,\ 2.1_1,\ 1.2_1) \Longrightarrow ((3.1_3,\ xx,\ xx),\ (xx,\ 2.1_1,\ 1.2_1))$

$(3.1\ 2.1\ 1.3) \Longrightarrow (3.1_3,\ 2.1_1,\ 1.3_3) \Longrightarrow ((3.1_3,\ xx,\ 1.3_3),\ (xx,\ 2.1_1,\ xx))$

$(3.1\ 2.2\ 1.2) \Longrightarrow (3.1_3,\ 2.2_{1.2},\ 1.2_1) \Longrightarrow ((3.1_3,\ xx,\ xx),\ (xx,\ 2.1_1,\ 1.2_1),\ (xx,\ 2.2_2,\ xx))$

$(3.1\ 2.2\ 1.3) \Longrightarrow (3.1_3,\ 2.2_{1.2},\ 1.3_3) \Longrightarrow ((3.1_3,\ xx,\ 1.3_3),\ (xx,\ 2.1_1,\ xx),\ (xx,\ 2.2_2,\ xx))$

$(3.1\ 2.3\ 1.3) \Longrightarrow (3.1_3,\ 2.1_1,\ 1.1_{1.3}) \Longrightarrow ((3.1_3,\ xx,\ 1.1_3),\ (xx,\ 2.1_1,\ 1.1_1))$

$(3.2\ 2.2\ 1.2) \Longrightarrow (3.2_2,\ 2.2_{1.2},\ 1.2_1) \Longrightarrow ((3.2_2,\ 2.2_2,\ xx),\ (xx,\ 2.2_1,\ 1.2_1))$

$(3.2\ 2.2\ 1.3) \Longrightarrow (3.2_2,\ 2.2_{1.2},\ 1.3_3) \Longrightarrow ((3.2_2,\ 2.2_2,\ xx),\ (xx,\ 2.2_1,\ xx))$

$(3.2\ 2.3\ 1.3) \Longrightarrow (3.2_2,\ 2.3_2,\ 1.3_3) \Longrightarrow ((3.2_2,\ 2.3_2,\ xx),\ (xx,\ xx,\ 1.3_3))$

$(3.3\ 2.3\ 1.3) \Longrightarrow (3.3_{2.3},\ 2.3_2,\ 1.3_3) \Longrightarrow ((3.3_2,\ 2.3_2,\ xx),\ (3.3_3,\ xx,\ 1.3_3))$.

1.2.5. A special case, (3.1 2.2 1.3)?

Toth mentions a special case in his new classification of sign classes: (3.1 2.2 1.3) x (3.1 2.2 1.3), which is *"the dual invariant sign class"*. (Toth, Ghost, p. 14)

A polycontextural micro-analysis of the duality shows a subtile *difference* which is not confirming in detail the *"dual invariance"* of Toth' s *mathematicl* semiotic approach to sign classes.

Dual invariance in question: "$(3.1\ 2.2\ 1.3)\ x\ (3.1\ 2.2\ 1.3)$", x = dual.

1. $(3.1\ 2.2\ 1.3) \Longrightarrow (3.1_3,\ 2.2_{1.2},\ 1.3_3) \Longrightarrow$

$((3.1_3,\ xx,\ 1.3_3),\ (xx,\ 2.1_1,\ xx),\ (xx,\ 2.2_2,\ xx))$.

2. $x(3.1\ 2.2\ 1.3) \Longrightarrow x(3.1_3,\ 2.2_{1.2},\ 1.3_3) \Longrightarrow (3.1_3,\ 2.2_{2.1},\ 1.3_3) \Longrightarrow$

$((3.1_3,\ xx,\ 1.3_3),\ (xx,\ 2.2_2,\ xx),\ (xx,\ 2.1_1,\ xx))$.

For $(3.1\ 2.2\ 1.3) \Longrightarrow (3.1_3,\ 2.2_{1.2},\ 1.3_3)$,

$x(3.1_3,\ 2.2_{1.2},\ 1.3_3) = (3.1_3,\ 2.2_{2.1},\ 1.3_3) \neq (3.1_3,\ 2.2_{1.2},\ 1.3_3)$,

hence $(3.1\ 2.2\ 1.3) \neq x(3.1\ 2.2\ 1.3)$ in the contextural setting.

The special case of a dual invariance, (3.1 2.1 1.3) x(3.1 2.2 1.3), doesn't hold in a *polycontextural* setting because the decomposition shows the inverse order of $(2.2_{1.2})$ in the dual case, i.e.
$(2.2_{1.2})$!= $(2.2_{2.1})$.

Category representation of the *mathematical* and the *polycontextural* approach for $Sem^{(3)}$.

$$\begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & id_1 & \alpha & \beta\alpha \\ 2 & \alpha^\circ & id_2 & \beta \\ 3 & \beta^\circ \alpha^\circ & \beta^\circ & id_3 \end{pmatrix} \text{ versus } \begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & id_{1.3} & \alpha_1 & \alpha_3 \\ 2 & \alpha^\circ{}_1 & id_{1.2} & \alpha_2 \\ 3 & \alpha^\circ{}_3 & \alpha^\circ{}_2 & id_{2.3} \end{pmatrix}$$

1. Categorical:

$(3.1\ 2.2\ 1.3) = (\beta^\circ \alpha^\circ, id_2, \beta\alpha) \Longrightarrow x(3.1\ 2.2\ 1.3) = (\beta^\circ \alpha^\circ, id_2, \beta\alpha)$.

hence $(3.1\ 2.2\ 1.3) = x(3.1\ 2.2\ 1.3)$.

2. Contextural:

$(3.1_3, 2.2_{1.2}, 1.3_3) = (\alpha^\circ{}_3, id_{1.2}, \alpha_3) \Longrightarrow$

$x(3.1_3, 2.2_{2.1}, 1.3_3) = (\alpha^\circ{}_3, id_{2.1}, \alpha_3)$

Hence, $(3.1_3, 2.2_{1.2}, 1.3_3) \neq x(3.1_3, 2.2_{2.1}, 1.3_3)$, esp.

$x(id_{1.2}) = id_{2.1}$ and $id_{1.2} \neq id_{2.1}$.

Polylogical negation example

Similar situation for $m-$ valued and $m-$ contextural logics.

$m = 3$:

logical negation: $p = (1, 2, 3)$, $non_1\ p = (2, 1, 3)$, hence $non_1(3) = 3$.

contextural negation: $p = (T_{1.3}, F_{1.2}, \boldsymbol{F}_{2.3})$, $non_1\ p = (F_{1.2}, T_{1.3}, \boldsymbol{F}_{3.2})$,

hence $non_1(\boldsymbol{F}_{2.3}) = \boldsymbol{F}_{3.2}$ and $\boldsymbol{F}_{2.3} \neq \boldsymbol{F}_{3.2}$.

Again, logical $\neq$ contextural (Kaehr, 1978, 1981).

Duality and identity for $Sem^{(3,1)}$

$dual(id_1, id_2, id_3) = (id_1, id_2, id_3)$ for Bense – Semiotics, i.e.

$dual(id_i) = id_i$, $i = 1, 2, 3$

duality for $Sem^{(3,1)}$
$dual_1(id_1, id_2, id_3) = (\tilde{id}_1, id_3, id_2)$
$dual_2(id_1, id_2, id_3) = (id_3, \tilde{id}_2, id_1)$

Non-Peirce-Bensean self-dual sign classes[4]

Duality, identity and reflectionality.[5]

1.2.6. Diamond semiotics abstract scheme

$$Diamond^{(3,1)}_{Sem} = \left[\begin{matrix} (1.1) \longrightarrow & (1.2) \longrightarrow & (1.3) \\ \downarrow & \downarrow & \downarrow \\ (2.1) \longrightarrow & (2.2) \longrightarrow & (2.3) \\ \downarrow & \downarrow & \downarrow \\ (3.1) \longrightarrow & (3.2) \longrightarrow & (3.3) \end{matrix}\right] \left\| \begin{pmatrix} 1 \longleftarrow 2 \| 2 \longleftarrow 1 \\ 2 \longleftarrow 3 \| 3 \longleftarrow 2 \\ 1 \longleftarrow 3 \| 3 \longleftarrow 1 \end{pmatrix} \right.$$

This kind of presentation (cf. Toth, Ghost, p. 13) is intended to give an idea of the embeddement of semiotics and its diamond environments as a further 'external' embeddement of semiotics. Hence, semiotics in polycontextural diamond constellations are towfold embedded 1. by their *neighbor*-semiotics and 2. by their diamond *environments*. Obviously, semiotics and diamond environments are *equiprimordial* (gleichursprünglich). In a further step of concretization, the construction gets its localization as a 3. embeddemend by its *place-designators* of the kenomic anchors. [6]

1.3. Diamond semiotic interpretations of (M, O,I)

1.3.1. Semiotic interpretation

Independent of the reasonability of the decision to introduce meaningful interpretations to the numeric classification of sign and sub-signs into object, O, medium, M, and interpretation, I, with its differentiations into sub-distinctions, the distinctions are made in the mode of *is-abstractions*. That is, the decision to interpret, e.g. 2.1 as M(I) = sin-sign, is definitive and identical in all possible semiotic contexts. No sin-sign can change in its use to a legi-sign or to a symbol. This fact is further proof of its identity heritage from classical ontology and logic.
Possibilities of thematizations are augmented in the case of embeded semiotics.

Semiotics

$$ZR^{(3,1)} = R(M, I, O)$$

$$M = M(O), M(I), M(M)$$

$$M = \{quali-sign, sin-sign, legi-sign\},$$

$$O = O(O), O(I), O(M)$$

$$O = \{icon, index, symbol\},$$

$$I = I(O), I(I), I(M)$$

$$I = \{rhema, dicent, argument\}.$$

1.3.2. Polysemiotic interpretations

Semiotic interpretation: $Sem^{(4,1)}(M, I, O)$

$$Sem^{(4,1)}(M, I, O) = \begin{bmatrix} M_{1,3,4} & I_{1,3} / M_2 \\ O_{2,3,4} & I_{2,4} / O_1 \end{bmatrix}$$

Polysemiotic interpretations

$$ZR^{(4,1)} = R\left[\left(M_{1,3,4}\right), \left(I_{1,3}/M_2\right), \left(O_1/I_{2,4}\right), \left(O_{2,3,4}\right)\right]$$

$\boldsymbol{M}^{(4)}$:

$$M(1.) = M(M_1),\ M(I_1),\ M(O_1),\ x$$
$$M(2.) = x,\ M(M_2),\ M(I_2),\ M(O_2)$$
$$M(3.) = M(M_3),\ M(I_3),\ x,\ M(O_3)$$
$$M(4.) = M(M_4),\ x,\ M(I_4),\ M(O_4)$$

$\boldsymbol{O}^{(4)}$:

$$O(1.) = O(M_1),\ O(I_1),\ O(O_1),\ x$$
$$O(2.) = x,\ O(M_2),\ O(I_2),\ O(O_2)$$
$$O(3.) = O(M_3),\ O(I_3),\ x,\ O(O_3)$$
$$O(4.) = O(M_4),\ x,\ O(I_4),\ O(O_4)$$

$\boldsymbol{I}^{(4)}$:

$$I(1.) = I(M_1),\ I(I_1),\ I(O_1),\ x$$
$$I(2.) = x,\ I(M_2),\ I(I_2),\ I(O_2)$$
$$I(3.) = I(M_3),\ I(I_3),\ x,\ I(O_3)$$
$$I(4.) = I(M_4),\ x,\ I(I_4),\ I(O_4)$$

1.3.3. Diamond semiotic interpretations

$$Sem_{diam}^{(4,1)}(M, I, O) = \left[\begin{matrix} M_{1,3,4} & I_{1,3}/M_2 \\ O_{2,3,4} & I_{2,4}/O_1 \end{matrix}\right] \Bigg\| \left(i_{1.3} \longleftarrow m_2 \;\|\; i_{2.4} \longleftarrow o_1\right)$$

2. Anchored diamonds

2.1. Anchored semiotics

Semiotics are conceived by Bense/Toth as anchor-free.[7]

$$\left[\begin{matrix}\textbf{sign}\\ \left[\left[\begin{matrix}[1,2,3]\\ \emptyset\\ \langle\emptyset;\emptyset\rangle\end{matrix}\right]\right]\end{matrix}\right], \left[\begin{matrix}\textbf{anchord sign}\\ \left[\left[\begin{matrix}[1,2,3]\\ \emptyset\\ \langle 1;\emptyset\rangle\end{matrix}\right]\right]\end{matrix}\right], \left[\begin{matrix}\textbf{specul env}\\ \left[\left[\begin{matrix}[1,2,3]\\ \emptyset\\ \langle\emptyset;1\rangle\end{matrix}\right]\right]\end{matrix}\right], \left[\begin{matrix}\textbf{specul sign}\\ \left[\left[\begin{matrix}[1,2,3]\\ \emptyset\\ \langle 1;1\rangle\end{matrix}\right]\right]\end{matrix}\right]$$

2.2. Anchored diamonds

$$\left[\begin{matrix}\textbf{sem diamond}\\ \left[\left[\begin{matrix}[1,2,3]\\ 4\\ \langle\emptyset;\emptyset\rangle\end{matrix}\right]\right]\end{matrix}\right], \left[\begin{matrix}\textbf{anch diamond}\\ \left[\left[\begin{matrix}[1,2,3]\\ 4\\ \langle 1;\emptyset\rangle\end{matrix}\right]\right]\end{matrix}\right], \left[\begin{matrix}\textbf{anchord env}\\ \left[\left[\begin{matrix}[1,2,3]\\ 4\\ \langle\emptyset;1\rangle\end{matrix}\right]\right]\end{matrix}\right], \left[\begin{matrix}\textbf{bi - sign}\\ \left[\left[\begin{matrix}[1,2,3]\\ 4\\ \langle 1;1\rangle\end{matrix}\right]\right]\end{matrix}\right].$$

$$\left[\begin{matrix}\textbf{sem - diamond}\\ \left[\left[\begin{matrix}[1,2,3]\\ 4\end{matrix}\right]\atop \langle 1,\ 1,\ 1,\ 1;\emptyset\rangle\right]\end{matrix}\right], \left[\begin{matrix}\textbf{anchord env}\\ \left[\left[\begin{matrix}[1,2,3]\\ 4\end{matrix}\right]\atop \langle\emptyset;1,1,1\rangle\right]\end{matrix}\right], \left[\begin{matrix}\textbf{bi - sign}\\ \left[\left[\begin{matrix}[1,2,3]\\ 4\end{matrix}\right]\atop \langle m=4;\ n=3\rangle\right]\end{matrix}\right]$$

2.3. Dissemination of diamonds

2.3.1. Disseminated semiotics

Diamond – Semiotics $DS^{(1,1)}$

$I.: \quad A \mid a$

$II.: \quad A \longrightarrow B \mid c$

$II.: \quad A \longrightarrow C \mid b_1 \longleftarrow b_2$

$IV.: \quad A \longrightarrow D \mid b_1 \longleftarrow b_2 \parallel c_1 \longleftarrow c_2$

$0.: \quad \emptyset \mid \varnothing$

$$\mathrm{Diss}^{(m,n)}\left(DS^{(1,1)}\right) = \begin{pmatrix} DS^{(1,1)} & DS^{(2,1)} & \ldots & DS^{(m,1)}\\ \ldots & & & \\ DS^{(1,n)} & DS^{(2,n)} & \ldots & DS^{(m,n)}\end{pmatrix}$$

2.3.2. Disseminated diamonds

$$\mathrm{Diss}^{(m,n)}\left(\mathrm{DiamS}^{(1,1)}\right) = \begin{pmatrix} DS^{(1,1)} & DS^{(2,1)} & \ldots & DS^{(m,1)}\\ \ldots & & & \\ DS^{(1,n} & DS^{(2,n)} & \ldots & DS^{(m,n)}\end{pmatrix} \Bigg\| \begin{pmatrix} env^{(1,1)} & env^{(2,1)} & \ldots & env^{(m,1)}\\ \ldots & & & \\ env^{(1,n)} & env^{(2,n)} & \ldots & env^{(m,n)}\end{pmatrix}$$

Notes

1 **Comments**

Peirce triadics or trichotomics is based on his metaphysical *intuition*, nurtured by his studies of Kant and Hegel, and is not a product of a *general generation scheme* with steps from 1 to 3. Such a general generation scheme wouldn' t have a built-in stop function, it could go on to arbitrary magnitudes. Only a reconstructional interest allows to start with 1 and end with 3.

> *"Creation thus means "that Firstness (repertory of 'possible' cases) must be given, so that Secondness (the 'real' case) in the sense of singular, concrete and innovative givenness is selectable in dependency of also given Thirdness (determining law or necessity)" (Bense and Walther 1973, p. 127)."* (Toth, In Transit, p. 49)

From the point of view of the primary trichotomic intuition and its realization, *monadic* and *dyadic* relations occur as reductions of the trichotomic intuition and its realization as a triadic relation. In this diamond theoretical and polyconttextural approach to an embedement of semiotics, nothing is given. The giveness of the semiotic categories, firstness, secondness and thirdness, are a result of a speculative decision for a trichotomic paradigm of thinking and corresponding world model, initiated scientifically bei Peirce.

2 **Classificatory remarks**

As a first result, I would like to point to the scenario, which could be misunderstood. To construct an embeddement of semiotics into diamonds doesn't establish a priority of semiotics over diamonds. Diamond of diamond theory, like diamond semiotics, are not results of trichotomic semiotics, neither of their application. As much as diamond category theory is not part of classical category theory (in all its forms), diamond semiotics is not part of semiotics (and all its forms, say, with retro-semiosis). The same pattern holds for the topics like *"Lambda Calculi in polycontextural situations"*.

The reason is simple. Neither for category theory nor for (mathematical) semiotics (Toth), an interplay between categories and saltatories, categorical composition and saltatorical saltisition (jump-operation), exists. Hence, a reconstruction of diamonds in the framework of semiotics or category theory is, albeit its reductive results, missing the point.

Hence, semiotics (and categories) are defined by objects, morphisms and composition of morphism, and are not including the concepts of separation, hetero-morphism, saltisition and bi-objects and their specific rules.

Neither co-algebras, co-categories or n-categories nor retro-semiosis and similar can be considered as proper concepts and methods for concretizations and realizations in diamond studies.

Again, nevertheless, it might produce interesting results in applying such concepts for reasons of *modeling* and *simulation* in contrast to *realization*.

On the other hand, there is no reason too, to restrict sign theory to the paradigm of Peircean semiotics (as it is represented by the Stuttgarter School). The same holds for category theory. Hence, the term semiotics might be properly used in constellation with diamond theory, polycontexturality and kenogrammatics.

3 **In Transit (Toth, 2008)**

Interestingly, there is a highly sophisticated study accessible by Toth which is on the way to *abolish,* step by step, the arbitrary (and ideological) *restrictions* to semiotics as they had been superimposed by Bense for interpretational reasons and in disharmony with the formal possibility of semiotics as such.

Personally, I just got access to *In Transit* and other free downloadable e-books from UNI Klagenfurt| Reihe KBT: *http://www.uni-klu.ac.at/iff-tewi/inhalt/280.htm*

Toth, Alfred, In Transit. A mathematical-semiotic theory of Decrease of Mind based on polycontextural Diamond Theory. Klagenfurt 2008

It is therefore not yet the place to go into more details to compare mathematical and polycontextural diamond semiotics more properly.

Albeit the fact that Gotthard Gunther fully accepted my approach of a *dissemination of formal systems* as an approach to formalize his polycontextural logic, similar philosophical and anti-formalistic restrictions had been superimposed by Gunther and his followers.

4 **Non-Peirce-Bensean self-dual sign classes**

Additionally to the special self-dual sign class "(3.1 2.2 1.3)" of the 10 accepted sign classes, all other self-dual sign classes, not included in the Peirce-Bensean semiotics, should be accepted. Hence the sign sets, (3.2 2.2 2.3) as a general sign class/set and (2.1 2.2 1.2) as a non-sign class, or (1.2 1.1 1.2) too, are self-dual and structurally, in general, not different from the special case "(3.1 2.2 1.3)" of the privileged 10 sign classes.

In general, all self-dual sign sets (x.y id_i y.x), i=1,2,3 seems to be special sign classes.

A main argument against restrictions in the definition of sign classes might be the fact that accepted operations on signs are able to transform accepted sign classes into non-accepted sign class, then called sign sets.

5 **Duality, identity and reflectionality**

As long as identity relations in formal systems aren't involved into reflectional interactions, the whole system remains classical.

Hence, semiotics of whatever formation, which aren't deconstructing their identity relation have to be classified as classical. Therefore, Toth's transclassical approaches to semiotics are doomed to fail their intentions to overcome classical restrictions.

Reflectionality of identity is possible only if the thematized identity is embedded in a polycontextural environment with neighboring identity systems from whom it can be differentiated. Identity in classical systems is neighbor-less and hence not able to reflect itself.

6 http://www.thinkartlab.com/pkl/lola/Xanadu-textemes/Xanadu-textemes.pdf

7 **Anchors**

This free-floating illusion of anchor-free semiotics gets a concretization in the *anchor*-concept as it is constitutive by definition for diamonds.
Anchors are realized in a *kenomic* grid. This happens at first as a *proto*-numbering of anchors. Anchors of diamonds, and as a consequence of semiotics too, are not part of diamonds or semiotics. That is, anchors are not represented by diamond' s firstness, secondness, thirdness and fourthness. Because anchors are realized in a kenomic grid, there numeric representation level shall be 0, hence Zeroness, also understood as Emptiness or Voidness. It presents the fifth category of anchored diamonds.

Toth introduced *zeroness* in his new design of transclassical semiotics. A comparision of both approaches is forthcoming.

Triadic Diamonds

Robertson's algebra of triadic relations, Gunther's founding relations and diamond triads

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Abstract

Some further thematizations and formalizations of diamond topics, especially triads, are presented. Triads, and founded triads, are presented in the context of Gunther's epistemology, Toth's semiotics with the help of Robertson's "Algebra for triadic relations". It is proposed that until now founding relations had been thematized externally only. An implementation of founding strategies into the system to be founded by the diamond approach is realizing the simultaneity of construction (model) and verification (foundation) of the triad. This approach is open for arbitrary n-ads.

1. Beyond binary relations?

1.1. Triads, trilogs, triplets

Representamen

"My definition of a representamen is as follows:
A REPRESENTAMEN is a subject of a triadic relation TO a second, called its OBJECT, FOR a third, called its INTERPRETANT, this triadic relation being such that the REPRESENTAMEN determines its interpretant to stand in the same triadic relation to the same object for some interpretant." (Peirce)

Further Towards a Triadic Calculus

Christofer R. Longyear's reconstruction of Warren McCullochs hobbyhorse with Peircean triads.

Part 1-3:
http://www.vordenker.de/ggphilosophy/longyear-part_1.pdf

Triplets

```
<rdf:RDF
  xmlns:rdf="http://www.w3.org/1999/02/22-rdf-syntax-ns#"
  xmlns:terms="http://purl.org/dc/terms/">
    <rdf:Description rdf:about="urn:x-states:New%20York">
          <terms:alternative>NY</terms:alternative>
    </rdf:Description>
</rdf:RDF>
```

http://www.instructionaldesign.com.au/Academic/TechnicalTheme1.htm

Trilog

"Unary relations are obviously insufficient and quadratic (4-airy) relations provide only minimally more capacity than triadic relations. Hence the choice is between two and three. Tarski studied binary relations extensively, but relation names played a significant, distinct metadata role. Binary relations are sufficient to represent information in a fixed schema, *but the names of these relations are inaccessible* from the relation contents. Both a benefit and a disadvantage of binary relations is that they are inherently *closed* in an algebra of unary and binary operators."

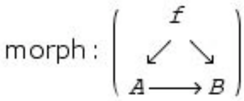
"Trilog is of course equivalent to the use of a fragment of *first order logic* to define ternary predicates, a fragment which has less convenient syntax and safety rules."
Edward L. Robertson, An Algebra for Triadic Relations, 2005
http://www.cs.indiana.edu/~edrbtsn/

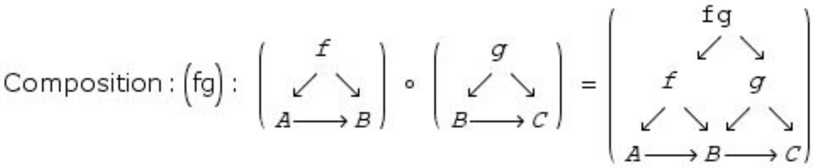
1.2. Morphisms as triads

1.2.1. Category theory

Binary:
morph= f(A, B), morph f: A-->B
morph: $A \xrightarrow{f} B$
composition: (fg): $A \xrightarrow{f} B$ o $B \xrightarrow{g} C$ ==> $A \xrightarrow{fg} C$

Ternary:
morph = (A, f, B), (A, f, B)⊆Morph

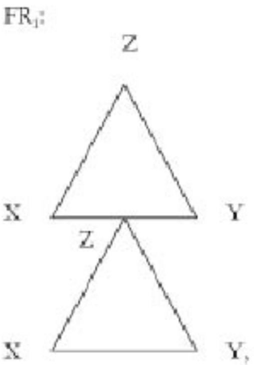




1.2.2. Semiotic foundation relation

Toth: "4. Starting with the geometric model of a sign class or reality thematic as an (equilateral) triangle, we notice that the semiotic foundational relations (FR) are *orthogonal* relations between the categories and the sign relations:
http://www.mathematical-semiotics.com/pdf/FoundRel.pdf

FR_1 := I <--> (M --> O) ≡ (.3.) <--> ((.1.) --> (.2.))
FR_2 := M <--> (O --> I) ≡ (.1.) <--> ((.2.) --> (.3.))
FR_3 := O <--> (M --> I) ≡ (.2.) <--> ((.1.) --> (.3.))



where X, Y, Z ∈ {.1., .2., .3.} and X, Y, Z are pairwise different, which means that for Z any
of the three prime-signs can be chosen, so that for FR_1 the following 6 relations are possible:

(3.a 2.b 1.c)	(3.a 2.b 1.c)	(3.a 2.b 1.c)	(3.a 2.b 1.c)
1.c	2.b	2.b	1.c
3.a	3.a	1.c	3.a
2.b	1.c	3.a	2.b

(3.a 2.b .... 1.c)
1.c
2.b
3.a

(3.a 2.b .... 1.c)
1.c
2.b
3.a

2. Diamond triads

2.1. Robertson’s Trilog

[Obviously, the following presentation is not more than a *wee hint* to a promising direction. Especially, there is no need to over-interprete the triadicity of the triadic approach(es). All the restrictions here to triadicity are for ‘*didactical*’ reasons only.]

Lower case letters (a, b, c, . . ., x, y, z) are used as variables over D.
The basic structures are sets of *triples* over D. We refer to these as *triadic* relations.
Triangular form of notation: (x, y, z) =: $\begin{smallmatrix} & y & \\ x & & z \end{smallmatrix}$ or $\left(\begin{smallmatrix} & y & \\ x & & z \end{smallmatrix}\right)$
”...the triple (x, *l*, z) indicates that the binary *l*(x, z) relationship holds.”

3.1 Definition:
Let R, S, and T be triadic relations. The triadic *join* of R, S, and T, is defined

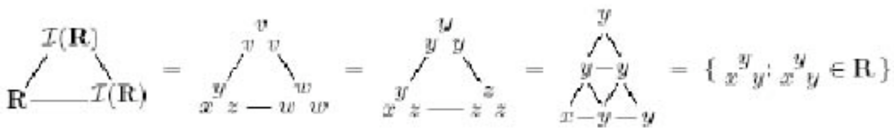
$$\mathit{trijoin}(R, S, T) \overset{\text{def}}{=} \left\{ \begin{smallmatrix} & b & \\ a & & c \end{smallmatrix} : \exists x, y, z \left[\begin{smallmatrix} & x & \\ a & & z \end{smallmatrix} \in \mathbf{R} \;\&\; \begin{smallmatrix} & b & \\ x & & y \end{smallmatrix} \in \mathbf{S} \;\&\; \begin{smallmatrix} & y & \\ z & & c \end{smallmatrix} \in \mathbf{T} \right] \right\}$$

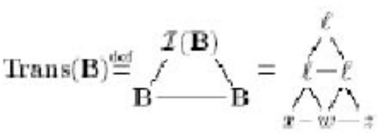
An equivalent diagrammatic notation for **trijoin**(R, S, T) is $\begin{smallmatrix} & S & \\ R & & T \end{smallmatrix}$.

3.2 Definition:

$I(R)$ def $= \left\{ \begin{smallmatrix} & x & \\ x & & x \end{smallmatrix} : x \text{ occurs in the active domain of } R \right\}$

$D^3(R)$ def $= \left\{ \begin{smallmatrix} & y & \\ x & & z \end{smallmatrix} : x, y, \text{ and } z \text{ occur in the active domain of } R \right\}$





Rotation

Definition: The (clockwise) rotation operator ρ is defined over triadic relations in the expected way:

$$\rho(R) \overset{\text{def}}{=} \left\{ \begin{smallmatrix} & a & \\ c & & b \end{smallmatrix} : \begin{smallmatrix} & b & \\ a & & c \end{smallmatrix} \in R \right\}$$

Robertson, An Algebra for Triadic Relations, 2005

2.2. Triadic diamonds

Lower case letters (a, b, c, ..., x, y, z) are used as variables over D.
Lower case letters ($\overline{a, b, c}$, ..., $\overline{x, y, z}$) are used as variables over $\overline{D}$.
The basic structures are bi-sets of diamond-triples over (D, $\overline{D}$).
We refer to these as *triadic diamond* relations.
Acceptional *triadic* relations are called **R**, **S**, **T**.
The complementary rejectional *dyadic* relations are called **r**, **s**, **t**.

Note:
The triadic rejectional relations (**r, s, t**) occur as *complementary* relations to ternary acceptional relations (**R, S, T**). Complementarity in diamond theory is based on an abstraction of the *compositions* of morphisms and is not to confuse with a categorical dualization of *morphisms*. Complementarily, categorical composition of morphism is possible only iff the criteria of saltatorial saltisition is realized.

This is not in conflict with the fact that category theory exists easily without any saltatory theory. Simply because saltatorical conditions of categories are implicitly used in the presupositions and not yet set from the *'mind to the blackboard'* (B. Brecht).

$$\textit{trijoin}_{\text{diam}}((R, r), (S, s), (T, t)) \text{ def} =$$

$$\left\{ \left(\begin{smallmatrix} & b & \\ a & & c \end{smallmatrix} \middle\| \begin{smallmatrix} d_1 \\ d_2 \end{smallmatrix} \right) : \frac{\exists\, x, y, z}{\overline{\exists\, x, y, z}} \left[\left(\begin{smallmatrix} & x & \\ a & & z \end{smallmatrix} \middle\| \begin{smallmatrix} a_1 \\ a_2 \end{smallmatrix} \right) \in \in (\mathbf{R}, \mathbf{r}) \ \&\& \left(\begin{smallmatrix} & b & \\ x & & y \end{smallmatrix} \middle\| \begin{smallmatrix} b_1 \\ b_2 \end{smallmatrix} \right) \in \in (\mathbf{S}, \mathbf{s}) \ \&\& \left(\begin{smallmatrix} & y & \\ z & & c \end{smallmatrix} \middle\| \begin{smallmatrix} c_1 \\ c_2 \end{smallmatrix} \right) \in \in (\mathbf{T}, \mathbf{t}) \right] \right\}$$

An equivalent diamond diagrammatic notation for

$\textit{trijoin}_{\text{diam}}((R, r), (S, s), (T, t))$ is $\left(\begin{smallmatrix} & (\mathbf{S}, \mathbf{s}) & \\ (\mathbf{R}, \mathbf{r}) & & (\mathbf{T}, \mathbf{t}) \end{smallmatrix} \right)$, with distributivity: $\left(\begin{smallmatrix} & \boldsymbol{S} & \\ \boldsymbol{R} & & \boldsymbol{T} \end{smallmatrix} \right) \middle\| \left(\begin{smallmatrix} & \boldsymbol{s} & \\ \boldsymbol{r} & & \boldsymbol{t} \end{smallmatrix} \right)$, hence

$$\textit{trijoin}_{\text{diam}}((R, r), (S, s), (T, t)) = \textit{trijoin}_{\text{diam}}\left[\left(\begin{smallmatrix} & \boldsymbol{S} & \\ \boldsymbol{R} & & \boldsymbol{T} \end{smallmatrix} \right) \middle\| \left(\begin{smallmatrix} & \boldsymbol{s} & \\ \boldsymbol{r} & & \boldsymbol{t} \end{smallmatrix} \right) \right].$$

$$\textit{trijoin}_{\text{diam}}\left(\left(\begin{smallmatrix} & \boldsymbol{S} & \\ \boldsymbol{R} & & \boldsymbol{T} \end{smallmatrix} \right) \middle\| \left(\begin{smallmatrix} & \boldsymbol{s} & \\ \boldsymbol{r} & & \boldsymbol{t} \end{smallmatrix} \right) \right) =$$

$$\left[\begin{matrix} \left(\begin{smallmatrix} & b & \\ a & & c \end{smallmatrix} : \exists\, x, y, z \right) : \\ \left[\begin{smallmatrix} & x & \\ a & & z \end{smallmatrix} \in \mathbf{R} \ \& \begin{smallmatrix} & b & \\ x & & y \end{smallmatrix} \in \mathbf{S} \ \& \begin{smallmatrix} & y & \\ z & & c \end{smallmatrix} \in \mathbf{T} \right] \Big\} \end{matrix} \middle\| \begin{matrix} \left(\begin{smallmatrix} & b & \\ a & & c \end{smallmatrix} : \overline{\exists\, x, y, z} \right) : \\ \left[\begin{smallmatrix} & \bar{x} & \\ \bar{a} & & \end{smallmatrix} \in \mathbf{r} \ \& \begin{smallmatrix} & \bar{b} & \\ \bar{x} & & \end{smallmatrix} \in \boldsymbol{s} \ \& \begin{smallmatrix} & \bar{y} & \\ \bar{c} & & \end{smallmatrix} \in \boldsymbol{t} \right] \Big\} \end{matrix} \right]$$

$$trijoin_{\text{diam}}\left(\left(\begin{matrix} \mathbf{S} \\ \mathbf{R}\ \mathbf{T}\end{matrix}\right) \Big\| \left(\begin{matrix} \mathbf{s} \\ \mathbf{r}\ \mathbf{t}\end{matrix}\right)\right) =$$

$$\left[\begin{matrix} \left(\begin{matrix} b \\ a\ c\end{matrix} : \exists\, x, y, z\right) \\ \left[\begin{matrix} x \\ a\ z\end{matrix} \in \mathbf{R}\right] \\ \left[\begin{matrix} b \\ x\ y\end{matrix} \in \mathbf{S}\right] \\ \left[\begin{matrix} y \\ z\ c\end{matrix} \in \mathbf{T}\right] \end{matrix}\right] \Bigg\| \left[\begin{matrix} \left(\begin{matrix} b \\ a\ c\end{matrix} : \exists\, \overline{x, y, z}\right) \\ \left[\begin{matrix} \bar{x} \\ \bar{a}\end{matrix} \in \mathbf{r}\right] \\ \left[\begin{matrix} \bar{b} \\ \bar{x}\end{matrix} \in \mathbf{s}\right] \\ \left[\begin{matrix} \bar{y} \\ \bar{c}\end{matrix} \in \mathbf{t}\right] \end{matrix}\right]$$

$x, y, z \in D,\ x, y, z \in \bar{D},\ \text{trijoin} \subseteq (D \amalg \bar{D})$

$$trijoin_{\text{diam}}\left(\left(\begin{matrix} \mathbf{S} \\ \mathbf{R}\ \mathbf{T}\end{matrix}\right) \Big\| \left(\begin{matrix} \mathbf{s} \\ \mathbf{r}\ \mathbf{t}\end{matrix}\right)\right) =$$

$$\left[\begin{matrix} \left(\begin{matrix} b \\ a\ c\end{matrix} : \exists\, x, y, z;\ \exists\, \overline{x, y, z}\right) : \\ \left[\begin{matrix} x \\ a\ z\end{matrix} \in \mathbf{R}\right] \Big\| \left[\begin{matrix} \bar{x} \\ \bar{a}\end{matrix} \in \mathbf{r}\right] \\ \left[\begin{matrix} b \\ x\ y\end{matrix} \in \mathbf{S}\right] \Big\| \left[\begin{matrix} \bar{b} \\ \bar{x}\end{matrix} \in \mathbf{s}\right] \\ \left[\begin{matrix} y \\ z\ c\end{matrix} \in \mathbf{T}\right] \Big\| \left[\begin{matrix} \bar{y} \\ \bar{c}\end{matrix} \in \mathbf{t}\right] \end{matrix}\right]$$

$x, y, z \in D,\ x, y, z \in \bar{D},\ \text{trijoin} \subseteq (D \amalg \bar{D})$

2.2.1. Diamond triad rotation

Diamond triad rotation

$$(\rho \amalg \bar{\rho})(R \amalg r) \overset{\text{def}}{=} \rho R \amalg \bar{\rho} r \overset{\text{def}}{=} \left\{ \begin{matrix} a \\ c\ b\end{matrix} \Big\| \begin{matrix} x \\ y\end{matrix} : \begin{matrix} b \\ a\ c\end{matrix} \in R \Big\| \begin{matrix} y \\ x\end{matrix} \in r \right\}$$

2.2.2. From triads to n-ads

2.3. Founded triads modeled by triadic diamonds

2.3.1. Definition of founded triads

Founding relations of (S^s, S^o, O)

$$\left(\begin{matrix} \square & O & \square \\ \square & \swarrow\ \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{matrix}\right) \Longrightarrow \left(\begin{matrix} S^S\, r^F \left(O \longrightarrow S^O\right) \\ O\, r^F \left(S^O \longleftrightarrow S^S\right) \\ S^O\, r^F \left(S^S \longrightarrow O\right) \end{matrix}\right)$$

Founding relations constists of a relation, r^F, between monadic instances, S^S, S^O, O and dyadic relations, $(O \longrightarrow S^O)$, $(S^O \longleftrightarrow S^S)$ and $(S^S \longrightarrow O)$.

Table of founding relations

$S^S\, r^F (O \longrightarrow S^O)$	$O\, r^F (S^O \longleftrightarrow S^S)$	$S^O\, r^F (S^S \longrightarrow O)$
$\left(\begin{matrix} \square & S^S & \square \\ \square & \downarrow & \square \\ O & \longrightarrow & S^o \end{matrix}\right)$	$\left(\begin{matrix} \square & O & \square \\ \square & \downarrow & \square \\ S^o & \longleftrightarrow & S^s \end{matrix}\right)$	$\left(\begin{matrix} \square & S^O & \square \\ \square & \downarrow & \square \\ S^s & \longrightarrow & O \end{matrix}\right)$

Context – valued logic modeling of the founding relation

The unary founding relations r^F are modeled into a contextualized (parametrized) binary function.

$$r_{binary}{}^{F} : (S^S,\ S^o,\ O) \times (S^S,\ S^o,\ O) \longrightarrow \begin{pmatrix} S^S;\ (S^S,\ S^o,\ O) \\ S^o;\ (S^S,\ S^o,\ O) \\ O;\ (S^S,\ S^o,\ O) \end{pmatrix}$$

$$r_{binary}{}^{F}(S^S,\ S^o,\ O) = \begin{pmatrix} r_{S^S}{}^{F} : (S^S,\ S^o,\ O) \times (S^S,\ S^o,\ O) /_{S^S} \longrightarrow (S^S; (S^S,\ S^o,\ O)) \\ r_{S^o}{}^{F} : (S^S,\ S^o,\ O) \times (S^S,\ S^o,\ O) /_{S^o} \longrightarrow (S^o;\ (S^S,\ S^o,\ O)) \\ r_{O}{}^{F} : (S^S,\ S^o,\ O) \times (S^S,\ S^o,\ O) /_{O} \longrightarrow (O;\ (S^S,\ S^o,\ O)) \end{pmatrix}$$

In a ternary functional setting, binary operations, additional to the unary, are easily introduced.

Founding relation in 'trijoin'

Founding relation in epistemic trilog

$$\mathbf{r}^F - trijoin_{epistem}(\boldsymbol{R}, \boldsymbol{S}, \boldsymbol{T}) \stackrel{def}{=}$$

$$\left\{ \begin{matrix} & S^s \\ S^o & O \end{matrix} : \exists\, x, y, z \left[\begin{matrix} & x \\ S^o & z \end{matrix} \Big/ _{(S^0)} \in \mathbf{R}\ \&\ \begin{matrix} & S^s \\ x & y \end{matrix} \Big/ _{(S^s)} \in \mathbf{S}\ \&\ \begin{matrix} & y \\ z & O \end{matrix} \Big/ _{(O)} \in \mathbf{T} \right] \right\}$$

Dyadic diamond foundations of triads

$$\begin{pmatrix} \square & O & \square \\ \square & \swarrow\ \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{pmatrix} \left\| \begin{pmatrix} S^S\, r^F \left(O \longrightarrow S^O\right) \\ S^O\, r^F \left(S^S \longrightarrow O\right) \\ O\, r^F \left(S^O \leftrightarrow S^S\right) \end{pmatrix} \right. \Longrightarrow \begin{pmatrix} \square & O & \square \\ \square & \swarrow\ \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{pmatrix} \left\| \begin{pmatrix} (S^o,\ S^s) \\ (S^o,\ O) \\ (S^s, O) \end{pmatrix} \right.$$

Monadic diamond foundations of triads

$$\begin{pmatrix} \square & O & \square \\ \square & \swarrow\ \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{pmatrix} \left\| \begin{pmatrix} S^S\, r^F \left(O \longrightarrow S^O\right) \\ S^O\, r^F \left(S^S \longrightarrow O\right) \\ O\, r^F \left(S^O \leftrightarrow S^S\right) \end{pmatrix} \right. \Longrightarrow \begin{pmatrix} \square & O & \square \\ \square & \swarrow\ \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{pmatrix} \left\| \begin{pmatrix} (S^s{}_{1.2}) \\ (S^o{}_{1.2}) \\ (O_{1.2}) \end{pmatrix} \right.$$

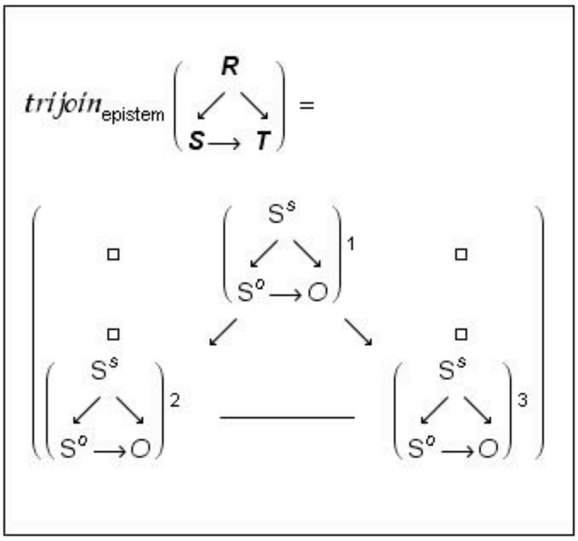
2.3.2. Composition of epistemic triads

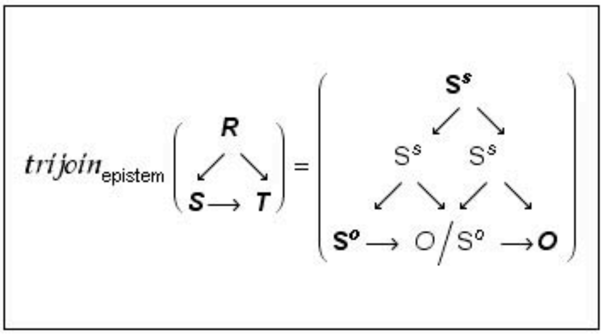
$$trijoin_{epistem}(\boldsymbol{R}, \boldsymbol{S}, \boldsymbol{T}) \stackrel{def}{=} \left\{ \begin{matrix} & S^s \\ S^o & O \end{matrix} : \exists\, x, y, z \left[\begin{matrix} & x \\ S^o & z \end{matrix} \in \mathbf{R}\ \&\ \begin{matrix} & S^s \\ x & y \end{matrix} \in \mathbf{S}\ \&\ \begin{matrix} & y \\ z & O \end{matrix} \in \mathbf{T} \right] \right\}$$

Substitution for $\begin{pmatrix} & b & \\ a & & c \end{pmatrix} \Longrightarrow \begin{pmatrix} & S^s & \\ S^o & & O \end{pmatrix}$, $R,\ S,\ T \subseteq U,\ r,\ s,\ t \subseteq \overline{U}$

$$\boldsymbol{R} = \begin{pmatrix} & S^s & \\ \swarrow & & \searrow \\ S^o & \longrightarrow & O \end{pmatrix}^1,\ \boldsymbol{S} = \begin{pmatrix} & S^s & \\ \swarrow & & \searrow \\ S^o & \longrightarrow & O \end{pmatrix}^2,\ \boldsymbol{T} = \begin{pmatrix} & S^s & \\ \swarrow & & \searrow \\ S^o & \longrightarrow & O \end{pmatrix}^3$$

$$\boldsymbol{r} = \begin{pmatrix} (S^o, S^s) \\ (S^o, O) \\ (S^s, O) \end{pmatrix}^1,\ \boldsymbol{s} = \begin{pmatrix} (S^o, S^s) \\ (S^o, O) \\ (S^s, O) \end{pmatrix}^2,\ \boldsymbol{t} = \begin{pmatrix} (S^o, S^s) \\ (S^o, O) \\ (S^s, O) \end{pmatrix}^3$$





2.3.3. Composition of founded epistemic triads

Monadic foundations of epistemic triads

$$\begin{pmatrix} \square & O & \square \\ \square & \swarrow \; \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{pmatrix} \Bigg\| \begin{pmatrix} (S^s{}_{1.2}) \\ (S^o{}_{1.2}) \\ (O_{1.2}) \end{pmatrix}$$

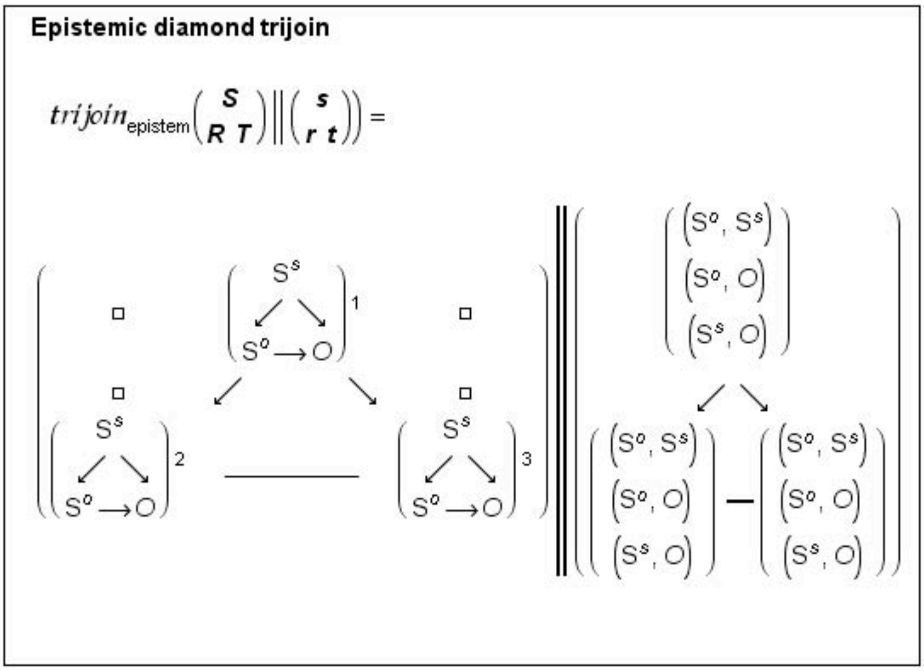
Dyadic foundations of epistemic triads

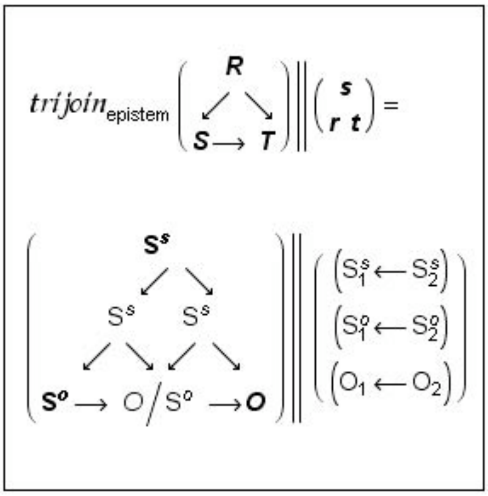
$$\begin{pmatrix} \square & O & \square \\ \square & \swarrow \; \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{pmatrix} \Bigg\| \begin{pmatrix} (S^o, S^s) \\ (S^o, O) \\ (S^s, O) \end{pmatrix}$$

$$trijoin_{\text{diam}}\begin{pmatrix} & \boldsymbol{S} \\ \boldsymbol{R} & \boldsymbol{T} \end{pmatrix} \Big\| \begin{pmatrix} & \boldsymbol{s} \\ \boldsymbol{r} & \boldsymbol{t} \end{pmatrix} =$$

$$\left[\begin{array}{l} \begin{pmatrix} & S^s \\ S^o & O \end{pmatrix} : \exists\, x, y, z \\ \begin{bmatrix} & x \\ S^o & z \end{bmatrix} \in \mathbf{R} \\ \begin{bmatrix} S^s & \\ x & y \end{bmatrix} \in \mathbf{S} \\ \begin{bmatrix} & y \\ z & O \end{bmatrix} \in \mathbf{T} \end{array}\right] \Bigg\| \left[\begin{array}{l} \begin{pmatrix} & S^s \\ S^o & O \end{pmatrix} : \exists\, \overline{x, y, z} \\ \begin{bmatrix} \bar{x} \\ \bar{S^o} \end{bmatrix} \in \mathbf{r} \\ \begin{bmatrix} \bar{S^s} \\ \bar{x} \end{bmatrix} \in \boldsymbol{s} \\ \begin{bmatrix} y \\ \bar{O} \end{bmatrix} \in \boldsymbol{t} \end{array}\right]$$

$x, y, z \in D,\; x, y, z \in \bar{D},\;$ trijoin $\subseteq (D \amalg \bar{D})$





3. Gunther’s founding relation

3.1. The idea of founding relations

Diamonds might be considered as implementations of Gunther’s founding relation for reflectional triads. In other words, Gunther’s founding relation might get a formalization as a special triadic diamond in general diamond theory.

Triads, like (sS, oS, oO), introduced by Gunther as the framework of a new epistemological paradigm, are reflected from each knot of the graph: sS(oS, oO), oS(sS, oO) and oO(sS, oS). Those relations are interpretations, respectively foundations of the triad as a whole from each of its monadic instances. Hence there are two modelings of reflection mapped into one triad: the *triad* and its *foundations*.

The foundations, realized from each standpoint of the triad, are delivering the 3 dyadic relations which are constituting the whole. Those parts, the dyadic relations, might be mirrored “outside” the whole as constitutive parts. Hence, the whole is a triad mirrored, inside-out, by its constitutive *simultaneous* dyads. Furthermore, the dyads of the triad are obtained from the unary elements, hence from monads. The whole construction for triadic reflectionality entails self-reflectionally a triad, consisting of a triad, 3 dyads and 3 monads.

"an *exchange* relation between logical positions
an *ordered* relation between logical positions
a *founding* relation which holds between the member of a relation and a relation itself."

TRIAD =(triad, dyad, monad).

But the wording of the construction suggests a *simultaneity* of the reflectional triad and its foundation by the founding relations of the triadic relation.
That is, Gunther’s trans-classical model of subjectivity is developed in three steps:
1. The stipulation of the *triadic model* as such,
2. The analysis of the triad by the new idea of *founding relations* and
3. The *composition* of the specific founding relations together to the founded triadic model.

This might be interpreted as a diamond construction with:

(oO)==>(sS, oS, oO)||(oS,sS)
(sS)==>(sS, oS, oO)||(oS,oO)
(oS)==>(sS, oS, oO)||(sS,oO)

The unary positions (oO), (sS) and (oS) are all thematizing the corresponding dyads. Hence, the unary monads are invoveld into two aspects based on the binarity of the founded dyads of the triad. Therefore, the monads shall be indexed by the index set={1, 2}.

Gunther’s concept of founding relations found some application in general systems theory (Alfred Locker). The formalism might have its own value from a descriptive viewpoint but is not well prepared for *operative* transformations. One attempt to formalize the epistemic model one step further happend with the application of Gunther’s *Kontextwertlogic* (Contextvalued logic), as opposed to Stellenwertlogik (place-valued logic) (cf. Kaehr 1978, Baldus 1982, Grochowiak 1979).

A new attempt to formalize the idea of founding relations is proposed by the *diamond* approach which takes into account the *simultaneity* of the model and its foundation. It also reflects the fact, that a foundation of an operation is localized on a different level of abstraction. The activity of modeling and the activity of founding are complementary activities demanding different kinds of abstractions. Hence, any applicative iteration of the model on itself is not fulfilling the criteria of foundation.

3.1.1. Chinese Ontology and Diamonds

The idea of in-sourcing *the matching conditions into the definition of diamonds tries to realize the two postulates of* Chinese Ontology*, the permanent* change *of things and the endness or* closeness *of situations. That is, diamonds should be designed as structural explications of the* happenstance *of compositions and not as a succession of events (morphisms). More exactly, diamond are contemplating the interplay of acceptional and rejectional thematizations. Thus, morphisms with their matching conditions and composability are in fact of secondary order for the understanding of diamonds.*

The complementarity of construction *and* verification*, which is happening at once and not in a temporal delay, is a consequence of the finiteness and dynamics postulate of* *polycontextural "ontology". This simultaneous interplay is based on the insight that a delayed verification (or testing in programming) would not necessarily verify the construction in question because, at least, the context will have changed in-between. Delayed verification is possible only in the very special case of frozen dynamics.*

In other words, in a changing open/closed world, the activities of construction and verification (of correctness and relevance) have to happen at once. Otherwise, because the conditions might have changed, the *relevancy* of the construction to be verified would have to be verified itself, again, and this ad nauseam. Obviously, the statement is not about/against the *stability* of the construction (program, system, agreement, contract), this might be rock solid, but about the *relevancy* of the rock solid construction.
(In therapy, even by constructivists, this delayed checks are called “reality check”. Nearly everytime, such a reality to be checked has escaped any relevance.)

In-sourcing the matching conditions
Diamond strategies are offering a fundamentally different approach.
Each step in a diamond world has simultaneously its counter-step. Hence, each operation
has an environment in which a legitimation of it can be stated. The legitimation
is not happening before or after the step is realized but immediately in parallel to it.

Morphisms are representing mappings between objects, seen as domains and codomains of the mapping function.
Hetero-morphisms are representing the conditions of the possibility (Bedingungen der Möglichkeit) of the composition of morphisms. That is, the conditions, expressed by the matching conditions, are reflected at the place of the heteromorphisms.
Hetero-morphisms as reflections of the matching conditions of composition are therefore second-order concepts realized "inside" the diamond system.

Morphisms and their composition are first-order concepts, which have to match the matching conditions defined by the axiomatics of the categorical composition of morphisms. But these matching conditions are not explicit in the composition of morphism but implicit, defined "outside" of the compositional system.
Hence, in diamonds, the matching conditions of categories are explicit, and moved from the "outside" to the inside of the system.
In this sense, the rejectional system of hetero-morphisms is a reflectional system, reflecting the interactions of the compositions of the acceptional system. Heteromorphisms are, thus, the "morphisms" of the matching conditions for morphisms.

3.1.2. Duplicity of reflection

This approach, to model the founding relation in the framework of diamond theory might be achieved with a decomposition of the founding relation into its parts: *monadic* and *dyadic* relations as the rejectional parts of the diamond interplay between model and foundation, i.e. acceptional (categorical) and rejectional (saltatorical) thematizations.
The founding operation itself, r_F, which has unary and binary operands, e.g. $O \in$ unary, $S^S \longleftrightarrow S^O \in$ binary, is not implemented and involved into the definition of the triadic model itself as it is introduced by Gunther.

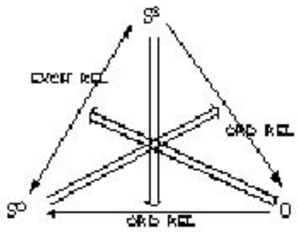
Funding relations and structural relations are complementary and antidromic in their orientation. If there is something like a *"Duplizität des Ich"* (Fichte) (A duplicity of the ego) then such a duplicity is of interest only if this duplicity is a simultaneous duplicity. A successive hierarchy of different levels of epistemic reflection, as it is supposed in Anglo-Saxon philosophy and computational reflection, belongs to a strictly different paradigm of thinking. Similar intricate situations of duplicity of consciousness had been discovered by Edmund Husserl with his distinction of *retention* and *protention* of the temporal structure of reflectional acts.

3.1.3. Subjectivity as a founded triadic model

"What we still have to consider is the relation any of the three terms S^s, O and S^O may assume to the relation which holds between the other two terms. From a purely combinational view point three possibilities exist for a demanded relation ... r^F ... they are:

S^S r^F (O -> S^O)
O r^F (S^O<-> S^S)
S^O r^F (S^S-> O)

Formally speaking it is the relation any of the two realizations of S , namely S^S or S^O, may have toward the connection of the other S and O. We call this the founding relation (r^F) because by it, and only by it, a self reflective subject separates itself from the whole Universe which thus becomes the potential contents of the consciousness of a Self gifted with awareness. In contrast to it the classic relation O r^F(S^O<-> S^S) is still a founding relation but not for consciousness.



But this claim also extracts from the "outside" observer, S^S an interesting admission. He will state that, seen from his vantage point, the inclusive disjunction does not only hold in the case of:

(1) $S^S r^F$(O->S^O) .∨. $S^O r^F$(S^S->O) but also in the other two cases:
(2) $S^O r^F$(S^S->O) .∨. Or^F(S^S<->S^O) (3) SSrF(O->SO) .∨. OrF(SS<->SO) provided, of course, that he uses a two-valued logic.

But in doing so he realizes by self-reflection that he has committed a momentous logical mistake. Since in classic logic only two values are available for the determination of the distinction between subject and object, it is impossible to describe the *triadic* relation between the subjective subject; the objective subject and the object.

This investigation intends only to show that the concept of *Totality* or *Ganzheit* is closely linked to the problem of subjectivity and trans-classic logic and that it is based on three basic structural relations:
an *exchange* relation between logical positions
an *ordered* relation between logical positions
a *founding* relation which holds between the member of a relation and a relation itself.

We are now able to establish the fundamental law that governs the connections between exchange, ordered and founding relation.

Thus we may say: the founding-relation is an exchange relation based on an ordered relation. But since the exchange relations can establish themselves only between ordered relations we might also say: the founding-relation is an ordered relation based

on the succession of exchange-relations. When we stated that the founding relation establishes subjectivity we referred to the fact that a self-reflecting system must always be:
self-reflection of (self- and hetero-reflection)."
(Gunther, Formal Logic, Totality and The Super-additive Principle, 1965) in:
http://www.thinkartlab.com/pkl/archive/Cyberphilosophy.pdf

3.1.4. Semiotic triads

Semiotic triads occur as morphisms between the instances I, O, M and their combinations, called graph theoretic sign models.

1. (I-->O -->M) 4. (O-->M-->I)
2. (M-->O-->I) 5. (I-->M-->O), (M-->I-->O)
3. (I-->M--> O) 6. (O-->I-->M).

"In 1966, Günther showed that the three reflexive categories of a three-valued logic, objective subject (oS), object (O) and subjective subject (sS) correspond (in this order) with the
semiotic categories of medium (firstness), object (secondness) and interpretant (thirdness)."
(cf. Toth 2008, p. 64):
oS <==> M ≡ (.1.)
O <==> O ≡ (.2.)
sS <==> I ≡ (.3.)
http://www.mathematical-semiotics.com/pdf/Obj.andrefl.existence.pdf

3.1.5. Gunther's journeys

Triadic semiotics (Bense, Toth) and triadic epistemology (Gunther). Also Gunther's approach and semiotic triads are fitting, at least at a first glance, well together, Gunther's epistemological triadism shouldn't be taken too seductively, because (t)his obsession lasted only for a short and specific time of Gunther's speculations. In the early 60s, the dialogical concept was replaced by a much more socialist distribution of subjectivity over a mass of 'subject centres' (Chinese Cultural Revolution).

"To sum it up:
A non *Aristotelian* or trans-classical logic is a system of distributed rationality. Our traditional (two valued) logic presents human rationality in a non distributed form. This means: the tradition recognizes only one single universal subject as the carrier of logical operations.

A *non-Aristotelian* logic, however, takes into account the fact that subjectivity is ontologically distributed over a plurality of subject centres. And since each of them is entitled to be the subject of logic human rationality must also be represented in a distributed form. The means to do this is to interpret many valued structures as place-value systems of our two valued logic." (Gunther 1962)
http://www.vordenker.de/ggphilosophy/gg_tradition-of-logic.pdf

In a German paper, 1965, Gunther writes: "Wir sind zum Übergang zu einer vierwertigen Logik genötigt, in der nicht nur Subjekt-überhaupt und Objekt-überhaupt durch logische Werte vertreten sind, sondern in der U sowohl als S_1, S_2, S_3 ontologische Instanzen repräsentieren, von denen jede Vertretung duch einen eigenen Wert beansprucht."
http://www.vordenker.de/ggphilosophy/gg_problem-trans-klass-logik.pdf

Some years later in 1968, Gunther developed a general theory of mediation where anthropological roots had been erased in favor of the history of the world as such - with or without human beings.
http://www.vordenker.de/ggphilosophy/gg_struk-min-theor-obj-geist.pdf

Furthermore , 2 years later, in his *"Die historische Kategorie des Neuen"*, in Moscow in favor for Change, his theory of polycontexturality is strictly neutral to any specific

interpretation. Neither I, Thou, It or other subjectivity constructs are appearing in the historical development of the New.
http://www.vordenker.de/ggphilosophy/gg_category.pdf

The whole conceptual story is well sketched at:
Ditterich, Kaehr: Einübung in eine andere Lektüre
http:///www.vordenker.de/ggphilosophy/kaehr_einuebung.pdf

Nevertheless, it seems that even today it would be a revolution to realize a working 3-contextural scientific paradigm and technology (of computing and social organizations).

Again, the proposed classical theory of triads is fomualted in the framework of a binary and dichotomous First-Order Logic., i.e. n-ary relations are *logically* reducible to binary relations. Hence, it achieves a simulation of trichotomic logic and never a realization. (Cf. Ternary Computers, Moscow)

What do we learn? As Max Bense mentioned correctly, Gunther was a Laborphilosoph (lab philosopher) and not a Kathederphilosoph (lectem philosopher) - this is true, despite the fact that each attempt to his philosophy, albeit only a fragment, was declared with absoluteness. Asked some month later about his theoretical advances and some immanent problems of it, he even didn't remember it. This surely was an exaggeration, but he was, again, some steps further on. And so on.

3.1.6. Toth's founding strategy

Toth's semiotic modeling of Gunther's founding relation is of importance, not only for systematic semiotics alone but for applications in computational semiotics and the triple-approach for semantic implementations in the project of a Semantic Web or Web 3.0.

Nevertheless, both, Gunther and Toth, are stressing on the successive, iterative or orthogonal structure of the idea of founding logic and semiotics. Because the basic triadic structure to be founded remains untouched, the whole approach might be more an *application* then a foundation.

This analysis is confirmed by Toth' s statements:
"In accordance with Günther (1991), these *superizations* [based on categories and relations] are based on semiotic orthogonality." (Toth 2008)

Superization, obviously, is a special application and is immanent of semiotics, not changing anything in its fundamental definition.

"From the logical standpoint, the latter means that the "Thou" founds the order relation
between an "I" and an "It" (OS --> (SS --> O)), that an "I" founds the order relation between an "It" and a "Thou" (SS --> (O --> OS)), and finally, that an "It" founds the exchange relation between an "I" and a "thou" (O --> (SS <-->OS))." (Toth 2008)
http://www.mathematical-semiotics.com/pdf/FoundRel.pdf

All parts of the founding relations are unchanged parts of the logical or epistemic model. The different *roles* of the instances might be used but not represented. That is, e.g. "It" as a founding point of view and "It" as a founded element of the model are technically not distinguished in the semiotic approach. This fact is based on the triviality that the model and its founding relations don' t have the necessary structural complexity. That is, at least two variables would be needed to model the different roles of the elements depending on their context. Context-value logic (Kontextwertlogik) was introduced just for this purpose (Gunther 1968).
There is up to now no equivalent at all to find in Bensean semiotics.

Some more approximations

$$trijoin_{\text{diam}}\big(trijoin_{\text{diam}}((R,r),(S,s),(T,t)),\ trijoin_{\text{diam}}((R,r),(S,s),(T,t)),\ trijoin_{\text{diam}}((R,r),(S,s),(T,t))\big)$$

$$= trijoin_{\text{diam}}\left(trijoin_{\text{diam}}\left[\begin{pmatrix} S \\ R\ T \end{pmatrix} \middle\| \begin{pmatrix} s \\ r\ t \end{pmatrix}\right],\ trijoin_{\text{diam}}\left[\begin{pmatrix} S \\ R\ T \end{pmatrix} \middle\| \begin{pmatrix} s \\ r\ t \end{pmatrix}\right],\ trijoin_{\text{diam}}\left[\begin{pmatrix} S \\ R\ T \end{pmatrix} \middle\| \begin{pmatrix} s \\ r\ t \end{pmatrix}\right]\right)$$

$$= trijoin^{(3,3)}{}_{\text{diam}}\left(\left[\begin{pmatrix} S \\ R\ T \end{pmatrix} \middle\| \begin{pmatrix} s \\ r\ t \end{pmatrix}\right],\left[\begin{pmatrix} S \\ R\ T \end{pmatrix} \middle\| \begin{pmatrix} s \\ r\ t \end{pmatrix}\right],\left[\begin{pmatrix} S \\ R\ T \end{pmatrix} \middle\| \begin{pmatrix} s \\ r\ t \end{pmatrix}\right]\right)$$

$$= trijoin_{\text{iter}}{}^{(3,3)}{}_{\text{diam}}\left[\left[\left(\begin{matrix} \begin{pmatrix} S \\ R\ T \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \end{pmatrix}\begin{pmatrix} S \\ R\ T \end{pmatrix} \end{matrix}\right) \middle\| \left(\begin{matrix} \begin{pmatrix} s \\ r\ t \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \end{pmatrix}\begin{pmatrix} s \\ r\ t \end{pmatrix} \end{matrix}\right)\right]\right].$$

$$trijoin_{\text{iter}}{}^{(3,4)}{}_{\text{diam}}\left(\left[\left[\left(\begin{matrix} \begin{pmatrix} S \\ R\ T \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \end{pmatrix}\begin{pmatrix} S \\ R\ T \end{pmatrix} \end{matrix}\right) \middle\| \left(\begin{matrix} \begin{pmatrix} s \\ r\ t \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \end{pmatrix}\begin{pmatrix} s \\ r\ t \end{pmatrix} \end{matrix}\right)\right]\right]\right) = \left[\left[\left(\begin{matrix} \begin{pmatrix} S \\ R\ T \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \end{pmatrix}\begin{pmatrix} S \\ R\ T \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \end{pmatrix} \end{matrix}\right) \middle\| \left(\begin{matrix} \begin{pmatrix} s \\ r\ t \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \end{pmatrix}\begin{pmatrix} s \\ r\ t \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \end{pmatrix} \end{matrix}\right)\right]\right].$$

$$4 - join_{\text{acc}}{}^{(4,3)}{}_{\text{diam}}\left(\left[\left[\left(\begin{matrix} \begin{pmatrix} S \\ R\ T \\ U \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \\ U \end{pmatrix}\begin{pmatrix} S \\ R\ T \\ U \end{pmatrix} \end{matrix}\right) \middle\| \left(\begin{matrix} \begin{pmatrix} s \\ r\ t \\ u \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \\ u \end{pmatrix}\begin{pmatrix} s \\ r\ t \\ u \end{pmatrix} \end{matrix}\right)\right]\right]\right.$$

$$4 - join_{\text{acc}}{}^{(4,4)}{}_{\text{diam}}\left[\left[\left(\begin{matrix} \begin{pmatrix} S \\ R\ T \\ U \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \\ U \end{pmatrix}\begin{pmatrix} S \\ R\ T \\ U \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \\ U \end{pmatrix} \end{matrix}\right) \middle\| \left(\begin{matrix} \begin{pmatrix} s \\ r\ t \\ u \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \\ u \end{pmatrix}\begin{pmatrix} s \\ r\ t \\ u \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \\ u \end{pmatrix} \end{matrix}\right)\right]\right].$$

4 – join diamond approx

$$\mathbf{4} - join_{\text{diam}}\left(\begin{pmatrix} & \boldsymbol{S} & \\ \boldsymbol{R} & & \boldsymbol{T} \\ & \boldsymbol{U} & \end{pmatrix} \middle\| \begin{pmatrix} & \boldsymbol{s} & \\ \boldsymbol{r} & & \boldsymbol{t} \\ & \boldsymbol{u} & \end{pmatrix}\right) =$$

$$x, y, z, u \in D,\ x, y, z, u \in \bar{D},\ 4 - \text{join} \subseteq \left(D \,\|\, \bar{D}\right)$$

$$\left[\begin{array}{l} \begin{pmatrix} & b & \\ a & & c : \exists\, x, y, z, u \\ & d & \end{pmatrix} \\ \begin{bmatrix} & x & \\ a & & z \\ & u & \end{bmatrix} \in \mathbf{R} \\ \begin{bmatrix} & b & \\ x & & y \\ & u & \end{bmatrix} \in \mathbf{S} \\ \begin{bmatrix} & y & \\ z & & c \\ & u & \end{bmatrix} \in \mathbf{T} \\ \begin{bmatrix} & y & \\ x & & z \\ & d & \end{bmatrix} \in \mathbf{U} \end{array}\right] \Bigg\| \left[\begin{array}{l} \begin{pmatrix} & b & \\ a & & c : \exists\, \overline{x, y, z, u} \\ & d & \end{pmatrix} \\ \begin{bmatrix} & x & \\ a & & z \end{bmatrix} \in \mathbf{r} \,\Big\|\, \begin{bmatrix} \bar{x} \\ \bar{a} \end{bmatrix} \in \mathbf{r} \\ \begin{bmatrix} & b & \\ x & & y \end{bmatrix} \in \mathbf{s} \,\Big\|\, \begin{bmatrix} \bar{b} \\ \bar{x} \end{bmatrix} \in \boldsymbol{s} \\ \begin{bmatrix} & y & \\ z & & c \end{bmatrix} \in \mathbf{t} \,\Big\|\, \begin{bmatrix} \bar{y} \\ \bar{c} \end{bmatrix} \in \boldsymbol{t} \\ \begin{bmatrix} & y & \\ x & & d \end{bmatrix} \in \mathbf{u} \,\Big\|\, \begin{bmatrix} \bar{y} \\ \bar{d} \end{bmatrix} \in \boldsymbol{u} \end{array}\right]$$

Diamond Relations

Sketch of a theory of diamond relations

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Abstract

Because of their concreteness, the complexity of *relations* is more structured and is not always tackled by the axioms or properties of mathematical categories. E.g. the categorical properties of *commutativity* and *transitivity* are not necessarily holding for relations.
As an application, relations and the category of PATH as proposed by Pfalzgraf is presented. Diamond relations and a diamond version of PATH, i.e. JOURN (journey), based on diamond set theory, is sketched.
How to introduce intransitivity (non-commutativity) in category theory? Two approaches are presented: Pfalzgraf's *generalized* morphisms which are re-establishing categorical commutativity on a generalized level of relations and a sketch of polycontextural diamond constructions which are introducing different types of non-commutativity on the level of a generalized (disseminated) paradigm of categoricity.

1. Diamond relations

1.1. Set-theoretical relations

1.1.1. Composition of relations

A relation from set A_0 to *a* set A_1 is *a* triple

(A_0, R, A_1) with $R \subseteq A_0 \times A_1$,

its *composition* with (B_0, R, B_1) is defined iff $A_1 = B_0$.

The composition of two relations $R: A \longrightarrow B$ and $S: B \longrightarrow C$ is given by:

$(a, c) \in S \circ R$ iff for some $b \in B$, $(a, b) \in R$ and $(b, c) \in S$.

That is: $(a, c) \in S \circ R \Longleftrightarrow \exists\, b \in B : (a, b) \in R \wedge (b, c) \in S$.

Obviously, sets like A, B and C with their elements a, b and c belong to the universe of sets U: A, B, C ⊂ U.

1.1.2. Composition of morphisms

"A binary operation o, called *composition* of morphisms, such that for any three objects a, b, and c, we have hom(a, b) × hom(b, c) -> hom(a, c).
The composition of f: a -> b and g: b -> c is written as go f or gf (some authors write fg), governed by two axioms: *Associativity* and *Identity*." (WiKi)

Mostly, the first-orderr logic and set theoretical notions to build category theoretical constructions, are not formalized in the sense of formal logic and set theory.

$A \xrightarrow{f} B$ o $B \xrightarrow{g} C$ => $A \xrightarrow{gf} C$, with cod(f) = dom(g)

Diagram
A-->B
↘ ↓
C commutes.

Category theory as a first-order theory

From: William S. Hatcher (The Foundation of Mathematics, 1968)

(C1) $(D(C(x)) = C(x) \wedge C(D(x)) = D(x)$

(C2) $\forall x \forall y \forall z \forall u (K(x, y, z) \wedge K(y, z, u) \supset z = u)$

(C3) $\forall x \forall y ((Ez : K(x, y, z) \equiv (C(x) = D(y)))$

(C4) $\forall x \forall y \forall z (K(x, y, z) \supset (D(z) = D(x) \wedge C(z) = C(y)))$

(C5) $\forall x (K(D(x), x, x) \wedge K(x, C(x), x))$

(C6) Associativity.

http://www.thinkartlab.com/pkl/lola/graphematische_problem-kae.pdf

1.1.3. Transitivity of relations

$\forall a, b, c : aRb \wedge bRc \Longrightarrow aRc$

"In mathematics, a binary *relation* R over a set X is transitive if whenever an element a is related to an element *b*, and *b* is in turn related to an element *c*, then *a* is also related to *c*." (WiKi)

1.1.4. Intransitivity of relations

$\neg \forall a, b, c : aRb \wedge bRc \Longrightarrow aRc$: Intransitivity

$\forall a, b, c : aRb \wedge bRc \Longrightarrow \neg aRc$: Antitransitivity

1.1.5. Intransitivity for composition

$\forall a, b, c : \text{hom}(a, b) \times \text{hom}(b, c) \longrightarrow \neg \text{hom}(a, c)$

$\neg \forall a, b, c : \text{hom}(a, b) \times \text{hom}(b, c) \longrightarrow \text{hom}(a, c)$

$\forall a, b, c : a \longrightarrow b \circ b \longrightarrow c \Longrightarrow \neg (a \longrightarrow c)$

$\neg \forall a, b, c : a \longrightarrow b \circ b \longrightarrow c \Longrightarrow a \longrightarrow c$

Diagram
A⟶B
¬ ↘ ↓
C does **not** commute.

Does this construction make any sense for category theory? Obviously not. It doesn't accept the main definition of categorical composition of morphism.

1.2. Diamond theory of relations

1.2.1. Diamond relation

Diamond relations are defined over the pluri – verse of acceptional and rejectional sets.

Classical (acceptional) binary relation: $R \subset (X \circ X)$

Non – classical (rejectional) binary relation : $r \subset (x \bullet x)$

Diamond binary relation: $(R, r) \subset \subset ((X, x) \circ\bullet (X, x))$.

Diamond relation

$$(R, r) \subset \subset [(X, x) \diamond (X, x)] \text{ iff } \begin{bmatrix} R \subset (X \circ X) \\ r \subset (x \bullet x) \end{bmatrix}$$

For each *binary* relation $R \subset X \circ X$, with $X \in U$, there is complementary *unary* relation $r \subset (x, x)$ with element $x \in \overline{U}$.

For each *ternary* relation $R \subset X \times X \times X$, with $X \in U$ there is a complementary *binary* relation $r \subset (x, x)$ with $x \in \overline{U}$.

Diamond relation DiamRel:

$R \in \text{Cat}, \ r \in \text{Sat}$

$$(R, r)^{(m)} \Longleftrightarrow \text{Rel}^{(m)} \,\|\, \text{rel}^{(m-1)}$$

1.2.2. Diamond composition of relations

Diamond Composition of Relations

A relation from tupel (A_o^1, A_o^2) to *a* tupel (A_1^1, A_1^2) is *a* 2 – triple $((A_o^1, A_o^2), R^{1.2}, (A_1^1, A_1^2))$ with $R^{1.2} \subseteq (A_o^1, A_o^2) \times\times (A_1^1, A_1^2)$,

its composition with $((B_o^1, B_o^2) R^{1.2} (B_1^1, B_1^2))$

is diamond – defined iff
$$\begin{bmatrix} \begin{bmatrix} \delta(A_1^2) \doteq \delta(B_1^2) \\ \delta(\alpha_2) \doteq \alpha_4 \\ \delta(\omega_1) \doteq \omega_4 \end{bmatrix}_{DIAM} \\ \begin{bmatrix} \omega_1 \simeq \alpha_2 \\ A_1^1 \equiv B_o^1 \end{bmatrix}_{REL} \end{bmatrix}$$

$\left(\left(A_o^1, \alpha_1\right), R^{1.2}, \left(A_1^1, \omega_1^1\right)\right)$ with $R^{1.2} \subseteq \left(A_o^1, \alpha_2\right) \times\times \left(A_1^1, \omega_2\right)$,

is a composition with $\begin{bmatrix} \left(\left(B_o^1, \alpha_3\right), R^{1.2}, \left(B_1^1, \omega_3\right)\right) \\ \left(\left(B_o^1, \alpha_4\right), \overline{R^{1.2}}, \left(A_1^1, \omega_4\right)\right) \end{bmatrix}$

Diamond – Relation composition :

$\left(a, b\right) \in S \Diamond R \Longleftrightarrow \left(\frac{S \circ R}{\overline{S \circ R}}\right) =$

$S \circ R: \exists b = \left(b_1^1 \equiv b_0^2\right) \in B : \left(a_0^1, b_1^1\right) \in R^1 \wedge \left(b_0^2, c_1^2\right) \in S^2$

$\_: \left(a^4, b^4\right) \in \overline{S \circ R} \Longleftrightarrow \exists b_1^1 \in B \wedge \exists b_0^2 \in B: \text{diff}\left(b_1^1, b_0^2\right) \in R^4$

Short:

Binary diamond composition

$\forall A \forall B \in U, \forall D \in \overline{U}, U \frown \overline{U} = \phi,$

$\left(a, b\right) \in S \Diamond R \Longleftrightarrow \exists b \in B \| \exists d \in D:$

$\left(a, b\right) \in R \wedge \left(b, c\right) \in S \| \left(d\right) \in \overline{S \circ R}$

$\left(a, b\right) \in S \Diamond R \Longleftrightarrow \exists b \in B:$

$\left(a, b\right) \in R \wedge \left(b, c\right) \in S \| \exists d \in D: \left(d\right) \in \overline{S \circ R}$

1. A. A. Mullin: *Properties of mutants.*

Let $(A, *)$ denote a nonempty set A together with a closed binary composition law "$*$" defined on A. By a *mutant* of $(A, *)$ is meant a subset M of A that satisfies the condition that $M * M \subseteq \overline{M}$, where $M * M = \{a * b: a \in M \text{ and } b \in M\}$ and $\overline{M}$ is the set of all of the elements of A not in M. If all of the elements of A are idempotent with respect to "$*$" let the empty set be the only mutant of $(A, *)$.

2. Category PATH

2.1. Motivation

Diamond motivation

From the point-view of diamonds, the question about *non*-identitive, *non*-commutative and *non*-transitive compositions for diamonds might arise. Non-transitivity is a well known property for so called “real life” situations. Again, a first observation to remind, is the fact, that in contrast to classical logic, the operator “non” has many well-defined appearances in polycontextural logics.
Non-transitivity in diamond theories, thus, is not simply a total negation or rejection of transitivity but the acceptance of a plurality of different kinds of transitivity, enabling many kind of specific non-transitive relations.
Nontransitivity appears naturally for relations. Categories are by definition transitive (commutative). Hence, intransitivity for categories can be introduced only as a secondary concept. On the other hand, intransitivity for relations might be transformed to transitivity by a kind of a generalization or an abstraction to generalized relations, i.e. *“a more general type of morphism”* based on the difference of *direct* and *indirect* arrows (Pfalzgraf).

It is based on a very different paradigm to ask: *“How to introduce intransitivity on the epistemological level of the definition of categories as such?”*

It shall be shown, say sketched, that such a basic interplay of transitivity and different forms of non-transitivity is accessible in the framework of a polycontextural diamond category theory.

Road Map Metaphor

"Let us consider, for illustration, a simple practical example of *real* life: Looking at general relational structures is quite natural since transitivity and even reflexivity are not always existent in *applications*.
As a practical example let us look at a *road map* where the nodes (objects) are towns and the arcs (arrows) are road connections, then not every pair of towns has a *direct* connection (arrow), in general. Therefore, generally, starting from a point we have to follow a path of direct road connections passing several nodes (towns) before we can reach a goal." (Pfalzgraf) [my emph]

Pfalzgraf gives an example about direct connections between towns. The same observation holds for most intensional verbs, like *win*, *love*, *hate*, etc., e.g. *A loves B*, *B loves C.* Does *A loves C* hold necessarily? Obviously not.

Pfalzgraf's strategy to keep transitivity by *generalization* could be paraphrased as:
A loves B, *B loves C*, *A hate C* , then, by generalization from intransitivity to transitivity:
A is-in-emo-relation to B,
B is-in-emo-relation to C, hence,
A is-in- emo-relation to C .

On the other hand, if A is connected with B, and B is connected with C, then A is connected with C, too. At least in a stable world, where the definition of connection is not suddenly transforming itself.
But is this reflection on intransitivity matching the level of categorical abstractions, like morphisms, compositions, matching conditions? Obviously not!

Such examples are localized on the level of *relational* algebra and logic, i.e. based on *set* theoretical and not on *category* theoretical assumptions.

Pfalzgraf gives a relational definition of situations where transitivity and reflexivity doesn't hold. Then he extends his *"CAT modeling approach"* to *"contain arbitrary relations"*. On the base of this relational concept he abstracts the category **PATH**.

Transitivity gets re-established with Pfalzgraf's approach by the more general abstraction of *"consecutive arrows"* in contrast to *"direct arrows"*. In the example below, "x-->z" is a direct arrow (morphism) which is not covered in the relational approach, but the *consecutiveness* of arrows "x -->y -->z" as such is considered as a *generalized* morphism Mor(x,z).

The proposed "*extension*" of categories, based on the distinction "direct" versus "consecutive" arrows (morphisms), towards a category of relations seems to be an *application* of "pure" category theory to the theory of relations and not in itself a categorical extension, but an application. At the end, all gets saved, back home in the category PATH.

Activity Map Metaphor
Instead of connecting towns, i.e. objects, the diamond approach is focusing on connection *activities*, i.e. arrival/departure-activities, of journeys between towns. The set- and object-aspect of towns is secondary to the activities of *coming (arriving)* and *going (departing)*. It is just this focussing on *activities* which is enabling the discovery of the concept of hetero-morphism and the construction of saltatories that are complementary, and not simply dual, to categories.

First- and second-order thematizations.
Now, thinking seems to be an activity too. Hence, the process of thinking might be connected, step by step. Steps are represented by morphisms, morphisms are informational, information is based on first-order observation. The process of composition isn't in focus. In focus are morphisms based on objects. But objects are moved into the background in favor of morphisms. Therefore it is strict forward to think of non-transitive connections of morphisms as a morphism. This is a kind of an abstraction towards a "more general" concept of morphism. This abstraction is abstracting in two directions, one from the objects and one from the composition of the less general, i.e. ordinary morphisms.

"We point out here that in our definition of agents human agents are included." (Pfalzgraf, 07, p.33)

On this level of abstraction it is not only generous but necessary to include human actors together with any other actors into the general approach of MAS. As far as we reduce human actions to non-reflectional roles, like using a mobile phone, e-Business, this reduction is appropriate. But as usual, the claims are much higher.

If thinking the process of thinking is understood as something different to the mere process of thinking (of something) then, such a second-order thinking is not properly modeled by morphisms but by a reflection on the conditions of morphisms which are, in this scenario, compositions, the operation to compose morphisms. The alter of morphisms are compositions. But that is demanding for a radically new abstraction. An abstraction which is not objectivizing compositions to morphisms of ordinary or general abstraction, but is keeping the processuality of the process in its own right and domain.

The argumentative figure to this difference is this: classic thinking is thinking of something. Therefore, for classical thinking, second-order thinking is thinking of thinking of something, i.e. thinking of something and this something is a “thinking of something”, hence, still accessible to thinking as thinking of something - and nothing else. As a result, classical thinking is inevitable connected with the problem of infinite regress of its meta-levels (type theory, meta-circularity).

Transclassical thinking insists on the structural difference between thinking of *something* and thinking of *thinking* as the process of thinking (of something). It is the processuality which isn’t caught by the ontology of something, even if this something is not stubbornly thought as an identical object but as a temporal object or process (Whitehead).
If there will be, one day, something like a Semantic Web, it will not be based on morphisms or on generalized morphisms.

Second-order thinking or, as it is called more properly, transcendental logic (Kant, Husserl, Gunther), is not formalized by category theory or relational algebra and logic.

The only account I’m aware of is the approach I proposed myself as diamond category theory. Diamond theory takes thinking a step further than polycontextural logic as it was introduced by Gunther as a first step towards a formalization of dialectics and transcendental logic and as it was further developed in the last decades by my own work.

As the transcendental logical tradition (Kant, Fichte, Schelling) pointed out, thinking is doubled. Fichte calls it the *“Duplizität des Ich”*. The process of thinking, which is thinking of thinking, is parallactic and antidromic. This exactly is what diamond theory discovered. The crucial difference is that diamond theory is formal and operative, albeit in a new sense of the terms but nevertheless well connect to the tradition of mathematical thinking, and not speculative descriptive like the transcendental logic forerunner.

Diamond non-transitivity
What could be a reasonable extension of category theoretical definitions *per se* introduced from a diamond approach? The strategy, that has to be *excluded* first, is to go back to application, concretization and other methods, like fiberization, which are, despite their productivity, leaving the level of category theoretical abstractions.
The only chances I see at the time for a structural enhancement of categorical notions, beyond fibered and n-categories, seems to be sketched by the diamond approach to compositionality of morphisms, i.e. the complementarity of categories and saltatories.

Categorical motivation

Following Jochen Pfalzgraf

"For practical reasons - in order to reach a large area of applications - we extend our CAT modeling approach to *arbitrary relations* (X,R). In such a case, we are not able to associate directly a category to the relation as we did it before since transitivity, reflexivity do not hold, in general.

"But from the categorical perspective again we interpret a relational structure as a certain diagram of arrows “visualizing” the given relations between the objects which form the “nodes” of the diagram. It turns out that we can always “embed” such a diagram in an associated PATH-category having comparable behavior as the category associated to a reflexive, transitive relation, although being a little “bigger” concerning the morphism structure.

"We point out: The introduction of the associated category **PATH** allows to use and apply all the modeling principles and constructions provided by **CAT** in the corresponding situations."

2.2. Formal description

"For the technical definitions, again let $R \subset X \times X$ or (X,R) denote a (general) relation. We associate to it the following category denoted by PATH(X,R) or just PATH for short, if no confusion can arise.

"The objects are the elements $x \in X$ and arrows (morphisms) are defined by sequences (paths) of adjacent arrows. That means, there is always a morphism x -> y, if $(x, y) \in R$ (or xRy), but if we have arrows (morphisms) x -> y and y -> z, then, in general we do not have a "direct arrow" x -> z, since the relation need not to be transitive. But what we can always do is forming a sequence (path) of consecutive arrows, like x -> y -> z in the previous case. This is then a morphism of a more general type between x and z.

"More generally, we can have (finite) sequences denoted for example by x_0 -> x_1 -> . . . -> x_n which is a morphism in Mor(x_0, x_n) in the new sense of our definition, but can also be interpreted as the composition of other morphisms which will be represented by adjacent parts of that whole path.

"In a category normally we need the identity arrow id_x for each object x.
We can add this as a requirement if there is really a necessity from a theoretical viewpoint; in practice this may be irrelevant.

"In PATH it can be the case that there exists more than one path between two nodes a and b, therefore in PATH we can have for the sets of morphisms |Mor(a, b)| > 1, in general, in contrast to the category which is associated to a reflexive, transitive relation as considered before.
Based on these considerations we can see that **PATH** becomes a category." (Pfalzgraf)

2.2.1. Formal description of PATH

"Let $R \subset X \times X$ denote a general relation. We associate with it the category denoted by PATH(X,R), PATH(X) or just PATH.
Objects: Elements $x \in X$.
Arrows, *Morphisms*: Sequences (paths) of consecutive arrows.

"This defines *composition* of arrows, in a natural way (concatenation of consecutive arrows) and this composition is associative. The identity arrow id_x, with respect to each object $x \in X$, will be assumed to exist ("tacitly") by definition.

"There is a morphism x -> y, iff xRy.
In general, for arrows x -> y and y -> z, we do not have a "direct arrow" x -> z (the relation can be not transitive) - this causes no problem.

"We can always form a sequence (path) of *consecutive* arrows, like x -> y -> z.
This is a morphism of a more *general* type between x and z. More generally, we can have (finite) sequences, for example x_0 -> x_1 -> . . . -> x_n (a path), this is a morphism in Mor(x_0, x_n) in the new sense of our definition. It can also be interpreted as the composition of other morphisms being represented by adjacent parts of the long sequence. Thus, **PATH** is a category." (Pfalzgraf)

3. Diamond of JOURNs

3.1. The journey map metaphor

Paths in diamonds are not exactly path in the sense of the category PATH but journeys. Thus, diamond paths are building diamond JOURNs. Hence, **JOURN** is not a category but a diamond.

"To practice the complementarity of the movement is not as simple as it sounds. You have to have one eye in the driving mirror and the other eye directed to the front window and, surely, you have to mediate, i.e., to understand together, what you are perceiving: leaving and approaching at once. And the place you are thinking these two counter-movements which happens at once is neither the forward nor the backward direction of your journey. It's your awareness of both. Both together at once and, at the same time, neither the one nor the other. It is your arena where you are playing the play of leaving and arriving."

"This complementarity of movements is just one part of the metaphor.

Because life is complex, it has to be composed by parts. Or it has to be de-composed into parts. We may drive from Dublin to Glasgow and then from Glasgow to London to realize our trip from Dublin to London. This, of course, is again something extremely simple to think and even to realize.
But again, there is a difference to discover which may change the way we are thinking for ever." (Kaehr, The Book of Diamonds, Intro)
http://rudys-diamond-strategies.blogspot.com

JOURN's catalogue of journeys
There are structurally different kinds of journeys on offer.
1. PATH is a very special type of journey. It is an intra-contextural journey in a single contexture without structural environment. Hence, properly formalized as a category.
2. This situation might be distributed. Journeys in different but mediated contextures are possible. Still isolated and each thus intra-contextural.
3. A new kind appears with possible switches (permutation) and transjunctional splitting (bifurcation) simultaneously into paths of different contextures. Still without complementary environment in the sense of diamond theory.
3. Now, each contexture, even an isolated mono-contexture, might be involved into itself and its environment. This happens for diamonds, which are containing antidromically oriented path in categorical and saltatorial systems. Such journeys ar group-journeys with running into opposite directions.
3. Here, a new and risky journey is offered by the travel agency by inviting to use the bridging rules between complementary acceptional and rejectional domains of categories and saltatories of a diamonds. All that happens intra-contexturally, i.e. diamonds are defined as the complementarity of an elementary contexture.
4. Obviously, diamond journeys might be organized for advanced travellers into polycontextural constellations. Hence, there are transcontextural transitions between diamonds to risk. Interestingly, such journeys might be involved into metamorphic changes between acceptional and rejectional domains of different contextures of the polycontextural scenario.

Further Metaphors
As a metaphor, the idea of *colored* contextures, each containing a full PATH-system, involved in *interactions* between neighboring contextures, might inspire the understanding of journeys in pluri-labyrinths of JOURN.
Such journeys are not safely connected in the spirit of secured transitivity but are challenging by jumps, salti and bridging and transjunctional bifurcations and transcontectural transitions.
This metaphor of *colored* categories, logics, arithmetic and set theories gets a scientific implementation with real world systems containing incommensurable and incompatible but interacting domains, like for bio- and social systems.

***"Observation**: An arbitrary binary relation R on X induces a corresponding arrow diagram D, "visualizing" the given relations between objects by corresponding arrows. Vice versa, a given arrow diagram D induces (or defines) a corresponding binary relation R on the set of elements (nodes) of D in the obvious, natural way, i.e. a specific arrow x $\longrightarrow$ y in D defines xRy. In these situations there is always an associated Cat **PATH**."* (Pfalzgraf, ACCAT-TutorialSKRIPT, p. 32, 2004)

3.2. Formal description of JOURN

Let $R^{1.2} \subseteq (A_o^1, A_o^2)$ xx (A_1^1, A_1^2), denote a general bi-relation. We associate with it the *diamond* denoted by JOURN((X,x), $R^{1.2}$), JOURN(X,x) or just JOURN.

Bi-objects: Bi-Elements (X,x) ∈∈ **(X, x)**.

Morphisms: Sequences (paths) of consecutive arrows,
*Hetero-morphisms:*counter-sequences of antidromic arrows.

Complementarity: Category/Saltatory
JOURN is not a product of **PATH**, i.e. JOURN != PATH x PATH but a *complementary* (and not a dual!) interplay between PATH and co-PATH:

JOURN = compl(PATH, $\overline{\text{PATH}}$)

There is a *morphism* X -> Y, iff XRY∈ Cat .
There is a *hetero*-morphism x -> y, iff xry ∈ Salt.
There is a *diamond* if [Cat; Salt].

$$R^{1.2} \subseteq \left(A_o^1, A_o^2\right) \text{xx} \left(A_1^1, A_1^2\right)$$
$$\left(\text{Rr}\right) \subseteq \subseteq \left(A_o^1, a_o^2\right) \text{xx} \left(A_1^1, a_1^2\right)$$

3.2.1. Monocontextural diamond journeys

Diamond journeys might realize intransitivity by changing between categories and saltatories of a diamond. Also transitivity is established, taken in isolation, for both, the categorical composition of morphisms in a category and for the jump-operation (saltisition) of hetero-morphism in saltatories, journeys might follow nontraditional paths between categories and saltatories by exploiting the possibilities opened up by the *bridging* operations.

"Jumping operations are the main operations for hetero-morphisms. A new abstraction, additionally to composition and saltisition, is introduced for the *bridging* of categories and saltatories. Bridging has two faces: *bridge* and *bridging*."

Collecting terms
Category: composition based on matching conditions (coincidence)
Saltatory: saltisition based on jumping conditions
Interactionality: bridge, bridging, transversality, parallelity based on bridging conditions (difference).

Possible chain of operators
composition (o) produced by morphisms, matching condition, domain, codomain,
saltisition (||) produced by complementation (difference) of composition,
bridge (^) produced by composition and difference from category and saltatory,
bridging (•) produced by difference from bridge.

As a consequence, the *composition* (f o g) and the *saltisition* (k || l) are mixed to (l || k) o g).
Bridging vs. jumping shows clearly that not only *what* is achieved matters but *how* it is achieved, i.e. by bridging or by jumping. Each jump in a saltatory has an inverse morphism as a bridge in a category.

Properties of bridging

Bridge and Bridging Conditions BC

1. $\forall k, l, n \in \text{HET}, \forall f, g, h \in \text{MORPH}$:

a. **composition**

$g \circ f,\ g \circ h,$

$(h \circ g) \circ f,\ h \circ (g \circ f) \in \text{MC},$

b. **saltisition**

$l \parallel k,\ n \parallel l,$

$n \parallel (l \parallel k),\ (n \parallel l) \parallel k \in \overline{\text{MC}},$

c. **bridge**

$g \perp k,\ l \perp g,$

$(l \perp g) \perp k,\ l \perp (g \perp k)$ are in $\widehat{\text{BC}}$.

d. **bridging**

$g \bullet k,\ l \bullet g,$

$(l \bullet g) \bullet k,\ l \bullet (g \bullet k)$ are in BC.

2. $(g \bullet k) \in \text{BC}$ iff $\text{dom}(k) = \text{diff}(\text{dom}(g))$,

$(l \bullet g) \in \text{BC}$ iif $\text{cod}(l) = \text{diff}(\text{cod}(g))$,

$(l \bullet g \bullet k) \in \text{BC}$ iff $(g \bullet k),\ (l \bullet g) \in \text{BC}$.

3 . $(g \perp k) \in \widehat{\text{BC}}$ iff $\text{diff}(\text{dom}(k)) = \text{dom}(g)$,

$(l \perp g) \in \widehat{\text{BC}}$ iff $\text{diff}(\text{cod}(l)) = \text{cod}(g)$,

$(l \perp g \perp k) \in \widehat{\text{BC}}$ iff $(g \perp k),\ (l \perp g) \in \widehat{\text{BC}}$.

Bridging

Associativity :

If $k, g, l \in BC$, then $(k \bullet g) \bullet l = k \bullet (g \bullet l)$,

Bridging :

$\text{bridging}_{(g, l, k)}: het(\omega_4, \alpha_4) \bullet hom(\alpha_2, \omega_2) \bullet het(\omega_8, \alpha_8) \longrightarrow het(\omega_9, \alpha_9)$.

Bridge

Associativity :

If $k, g, l \in \widehat{BC}$, then $(k \perp g) \perp l = k \perp (g \perp l)$,

Bridge :

$\text{bridge}_{(g, l, k)}: \text{het}(\omega_4, \alpha_4) \perp \text{hom}(\alpha_2, \omega_2) \perp \text{het}(\omega_8, \alpha_8) \longrightarrow \text{het}(\omega_9, \alpha_9)$.

Bridges vs. Bridging vs. Jumping

$(l \perp g \perp k) \cong (l \bullet g \bullet k) \cong (l \amalg k)$,

$(l \perp g \bullet k) \cong (l \bullet g \perp k) \cong (l \amalg k)$,

$(l \bullet g \perp k) \cong (l \perp g \bullet k) \cong (l \amalg k)$.

$\text{diff}(\perp) = (\bullet)$, $(\perp) = \text{diff}(\bullet)$.

3.2.2. Polycontextural journeys

Polycontextural category theory is enabling journeys between different contextures covering autonomous categories. This might happen in parallel or in cycles. Multiple parallel and cyclic journeys might running concurrently in the polycontextural matrix of disseminated categories.
For each category of a contexture an intra-contextural *path* might be realized and ruled by PATH.

An interplay between *local* and *global* thematization of the scenario, enabling distancing and zooming in, is describing intra- and poly-contexturality of JOURN.

$\text{JOURN}^{(3)}{}_{(\text{cascade})} (\text{Cat}^{(3)}):$

$(\forall \forall \forall)\ A^{(3)}, B^{(3)}, C^{(3)}:$

$$\frac{\begin{bmatrix} \text{Cat}^1: A \longrightarrow B \circ B \longrightarrow C \Longrightarrow A \longrightarrow C \\ \text{Cat}^2: A \longrightarrow B \circ B \longrightarrow C \Longrightarrow A \longrightarrow C \\ \text{Cat}^3: A \longrightarrow B \circ B \longrightarrow C \Longrightarrow A \longrightarrow C \end{bmatrix}}{(A \longrightarrow B \in \text{Cat}^1) \circ\circ\circ (B \longrightarrow C \in \text{Cat}^2) \Longrightarrow (A \longrightarrow C \in \text{Cat}^3).}$$

Modeled in 3 - contextural predicate logic

$(C3) : (1 - \text{contextural linear})$

$\forall x \forall y \left(\left(Ez : K(x, y, z) \equiv \left(C(x) = D(y) \right) \right) \right)$

$(C3a) : (3 - \text{contextural parallel})$

$\forall^{(3)} x \forall^{(3)} y :$

$\left(\left(E^{(3)} z : K^{(3)} (x, y, z) \equiv \left(C^{(3)} (x) = D^{(3)} (y) \right) \right) \right)$

$(C3b) : (3 - \text{contextural cascade})$

$\forall^{(3)} x \forall^{(3)} y :$

$\left(\left(E^1 z : K^{(3)} (x, y, z) \equiv \left(C^1 (x) = D^2 (y) \right) \right) \right),$

$\left(\left(E^2 z : K^{(3)} (x, y, z) \equiv \left(C^2 (x) = D^3 (y) \right) \right) \right).$

That is, for all 3-contextural parallel path-constellations (C3a) there is a cascade-journey (C3b) in (C3a).

$JOURN^{(3)}_{(cycle)} \left(Cat^{(3)} \right) :$

$(\forall \forall \forall)\, A^{(3)}, B^{(3)}, C^{(3)} :$

$$\left[\begin{array}{l} Cat^{1} : A \longrightarrow B \circ B \longrightarrow C \Longrightarrow A \longrightarrow C \\ Cat^{2} : A \longrightarrow B \circ B \longrightarrow C \Longrightarrow A \longrightarrow C \\ Cat^{3} : A \longrightarrow B \circ B \longrightarrow C \Longrightarrow A \longleftarrow C \end{array} \right]$$

$\left(A \longrightarrow B \in Cat^{1} \right) \circ \circ \circ \left(B \longrightarrow C \in Cat^{2} \right)$

$\Longrightarrow$

$\left(A \longleftarrow C \in Cat^{3} \right).$

$(C3c) : (3 - \text{contextural cycle})$

$\forall^{(3)} x \forall^{(3)} y :$

$\left(\left(E^1 z : K^{(3)} (x, y, z) \equiv \left(C^1 (x) = D^2 (y) \right) \right) \right),$

$\left(\left(E^2 z : K^{(3)} (x, y, z) \equiv \left(C^2 (x) = C^3 (y) \right) \right) \right)$

$\left(\left(E^3 z : K^{(3)} (x, y, z) \equiv \left(C^{2 \cdot 3} (x) = D^3 (y) \right) \right) \right)$

That is, for all 3-contextural parallel path-constellations (C3a) there is a (cascade) cyclic-journey (C3c) in (C3a).

3.2.3. Polycontextural diamond journeys

$$\mathrm{JOURN}^{(3)}_{(cycle)}\left(\mathrm{Diam}^{(3)}\right):$$

$$U^{(3)} \frown \overline{U^{(3)}} = \emptyset$$

$$(\forall\forall\forall)\ A^{(3)}, B^{(3)}, C^{(3)} \in U^{(3)} \,\Big\|\, \forall^{4}\ d^{(3)} \in \overline{U^{(3)}}:$$

$$\left[\begin{array}{l} \mathrm{Cat}^{1}: A \longrightarrow B \circ B \longrightarrow C \Longrightarrow A \longrightarrow C \\ \mathrm{Cat}^{2}: A \longrightarrow B \circ B \longrightarrow C \Longrightarrow A \longrightarrow C \\ \mathrm{Cat}^{3}: A \longrightarrow B \circ B \longrightarrow C \Longrightarrow A \longleftarrow C \end{array}\right] \Bigg\| \left[\begin{array}{l} d_1 \longleftarrow d_2 : \mathrm{Salt}^1 \\ d_1 \longleftarrow d_2 : \mathrm{Salt}^2 \\ d_1 \longleftarrow d_2 : \mathrm{Salt}^3 \end{array}\right]$$

$$\left[\begin{array}{cccccc} (A \longrightarrow B) & - & - & - & - & \\ & \circ & (B \longrightarrow C) & - & - & \\ - & - & - & \circ & A \longleftarrow C & \end{array}\right] \Bigg\| \ d_1 \longleftarrow d_2$$

Short :

$$\mathrm{Cat}^{(3)}: \left(A \longrightarrow B \longrightarrow C \Longrightarrow A \longleftarrow C\right) \Big\| \left(d_1 \longleftarrow d_2\right): \mathrm{Salt}^{(3)}.$$

$$(\mathrm{C3c}): (3 - \text{contextural diamond cycle})$$

$$\forall^{(3)}\, x\, \forall^{(3)}\, y \in U^{(3)} \,\Big\|\, \forall^{4}\, v^{(3)} \in \overline{U^{(3)}}:$$

$$\left(\left(E^1 z : K^{(3)}(x, y, z) \equiv \left(C^1(x) = D^2(y)\right)\right)\right) \Big\| \left(d_1 \longleftarrow d_2\right)$$

$$\left(\left(E^2 z : K^{(3)}(x, y, z) \equiv \left(C^2(x) = C^3(y)\right)\right)\right) \Big\| \left(d_1 \longleftarrow d_2\right)$$

$$\left(\left(E^3 z : K^{(3)}(x, y, z) \equiv \left(C^{2.3}(x) = D^3(y)\right)\right)\right) \Big\| \left(d_1 \longrightarrow d_2\right)$$

$$\text{with } \left(d_1 \longleftarrow d_2\right)^{i},\ i = 1, 2, 3:$$

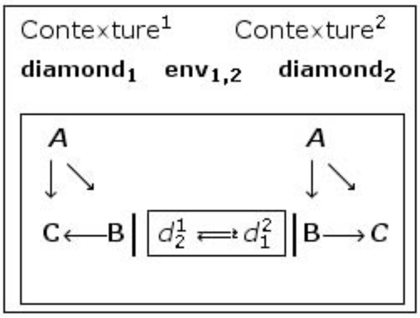
$$\Big\| E^4 v : \mathrm{compl}\left(E^1 z : K^{(3)}(x, y, z)\right) \equiv$$

$$\mathrm{diff}^{i}\left(C(x)\right) = \mathrm{diff}^{i}\left(D(y)\right) \equiv$$

$$\left(d_1 \longleftarrow d_2\right)^{i}.$$

Is the diamond construction working for non-commutative compositions?
Diamond category theory was up to now based on the commutativity of composition of morphisms *f* and *g*, i.e. the coincidence relations for the composite *fg* was restricted to transitivity. Diamond structures are abstractions from compositions and are not depending on special properties of compositions. Hence, diamondization of relational concepts is well founded in the abstraction from composition in general.

3.2.4. Interplay between polycontextural diamond journeys



4. Diamond set theory

The concept of relation and its relational algebra is based on *set theory*. This wasn't always the case (Peirce). But today, set-theory based relation theory is well established.

Diamond relation theory, therefore, is based on diamond set theory. Diamond set theory is only in its very beginnings.

5. Architectonics of disseminated categories

5.1. Rhizomatics

The question behind the proposed constructions for non-commutative (diamond) category is unmasking a deeper structure of category theory not yet mentioned explicitly. That is: What is the general architectonics of categorical constructions? Or: On which architectonical decisions are categories based?
There are first answers to learn in the papers "*ConTeXtures*" and "*PolyLogics*".

5.1.1. Category theory

The leading metaphor of category theory can be found in the use of linerarily ordered compositions of arrows.
"A category can be regarded as a directed graph with structure."
They are symbolizing an information flow from arrow to arrow.
"*...the unifying idea is that of 'information flow'*.
Hence, "*Ordinary category theory uses 1-dimensional arrows - -->.*
Higher-dimensional category theory uses higher-dimensional arrows."(Leinster, 2003)

5.1.2. n-Category theory

In contrast to category theory the leading metaphors of n-category is based in topology. With that, new fundamental or basic topologies or architectonics for categories are naturally motivated and constructed.
"The natural geometry of these higher-dimensional arrows is what makes higher-dimensional category theory an inherently topological subject." (Leinster, 2003)
Some further ideas are developed in the paper: "*Categories and Contextures*"
http://www.thinkartlab.com/pkl/lola/Categories-Contextures.pdf

5.1.3. Disseminatorics

Dissemination of categories over a *kenomic* matrix is not only topological but takes into account the mediation mechanism of proemiality instead of the quite vague matching conditions of category theory.

It could be stated as a theorem, that all possible relational constellations of combinatorial tree-structures can be represented as architectonics of disseminated categories.

Hence, all relational constellations, with commutative and diverse non-commutative properties, might serve as equal architectonics for disseminated, i.e. distributed and mediated, categories.
There is no prime structure.
A Birkhoff arithmetics of such skeletal structures might be considered.

Obviously, the game starts with the number 4. All other cases are structurally equivalent.
Especially for the number 3, there are no formal criteria to distinguish the skeletal architectures of a line from the architectures of a star, both are coinciding.

"Only for the elementary case of m=3, star *and* line *structures are coinciding. This simple structural coincidence may be the hidden reason of profound epistemological controversies in philosophy and sociology."* (cf. Kaehr, Contextures, 2005, Materialien 1978)

With the number 4, the skeletal difference of line- and star-structure is accessible. As the combinatorial table, below, for skeletal structures shows, an interesting coincidence of complexity and structures is given for the number 6. For complexity m>=7, the number of structures is increasing , structures(m)>m. E.g., m=10, structure(10) = 106.

The whole strategy could be called a *unification of relational and categorical structures* on a higher level of abstraction than relational and categorical notions. Instead of a lower level of “generalized morphisms” (Pfalzgraf) or the topological structures of n-categories.

Again, modern strategies of defending paradigmatic fundamentalism
As developed and argued at length at many places, polycontextural notions, theories and formalisms are not *fibered* theories, logics and semiotics, etc. (Pfalzgraf 1988), *"Polycontextural systems are topological fiberings of monocontextural systems” (*Toth 2009*)*, simply because fibering, in all its forms, is based on non-fibered category theory and predicate logic of a single universe.
Toth, Light, 2009:

It might sound more trendy to use fiber-terminology and techniques instead of clumsy many-sorted logical theories with identity (Goguen 1981), both are nevertheless fundamentally rooted in the uniqueness of a universe (of discourse, objects, elements, individuals, signs, marks, etc.) and there is no paradigmatic insight or strategy to abandon this kind of fundamentalism in favor of pluri-verses disseminated over kenomic matrixes in the sense of polycontextural and kenogrammatic endeavors and risks.
With more fun, read: SUSHI'S LOGICS, 2004.

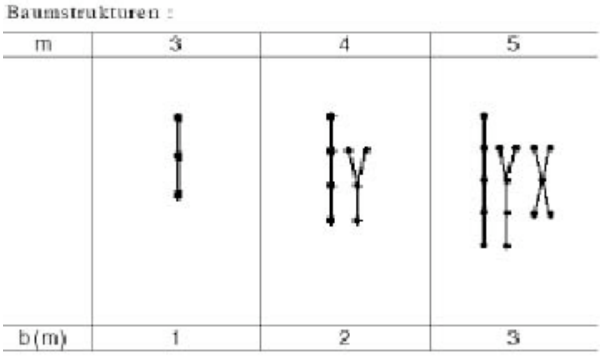
If there is something like a polycontextural category theory - with diamond structure or not - then there exists equally a polycontextural distribution of fibered and many-sorted theories as there exists, since at least 1978, a polycontextural dissemination of multivalued logics.

5.2. Dissemination between lines, stars, circles and flags

5.2.1. Skeletal structures

Arboreal patterns, linear and star-patterns (without root) are skeletal structures for the architectonics of dissemination.
A tabular and matrix approach to disseminated categories opens up a quite natural semantical interpretation of non-transitive mediated categorical systems.
Table of skeletal structures

Gerhard G. Thomas, "On Permatography", Proceeding of the 10th Winter School on Abstract Analysis (Srni 1982), Reyndiconti del Civcolo Matematico di Palermo.



Anzahlen :

m	b(m)
1	1
2	1
3	1
4	2
5	3
6	6
7	11
8	23
9	47
10	106
11	235
12	551
13	1301

A valuation of *skeletal* structures with the key distinctions of categories, like domain and codomain of morphisms, is enabling a distribution of categories over all possible combinatorial constellations. It is proposed that the skeletal structure of classical category theory is given by the skeletal *line*-structure with complexity m=3.

Skeletal structures are not rooted. For the purpose of modeling line- and star-structures for distributed categories the *directionality* of rooted graphs might be first omitted. That is, only one direction for *arrows* as interpreted graphs is chosen. For cyclic graphs, right and left orientations of graphs have to be involved. Questions of *knots* are not yet treated properly.

Hence, from the possibilities of the graph(3) = (•-•-•) and its arrow interpretation and its domain/codomain valuation, only the standard linear structure is considered as an architectonic base for category theory:

$$val_{arrow}(\bullet\text{-}\bullet\text{-}\bullet)=\begin{pmatrix}\bullet\longrightarrow\bullet\longrightarrow\bullet\\ \bullet\longrightarrow\bullet\longleftarrow\bullet\\ \bullet\longleftarrow\bullet\longrightarrow\bullet\\ \bullet\longleftarrow\bullet\longleftarrow\bullet\end{pmatrix} \text{ and } val_{dom/cod}\begin{pmatrix}\bullet\longrightarrow\bullet\longrightarrow\bullet\\ \bullet\longrightarrow\bullet\longleftarrow\bullet\\ \bullet\longleftarrow\bullet\longrightarrow\bullet\\ \bullet\longleftarrow\bullet\longleftarrow\bullet\end{pmatrix}.$$

That is, from $val_{dom/cod}(\bullet\longrightarrow\bullet\longrightarrow\bullet) = \begin{pmatrix}\square & -\\ \square & \square\\ - & \square\end{pmatrix}$ only the linear order

$$\begin{pmatrix}dom_1 & -\\ cod_1 & dom_2\\ - & cod_2\end{pmatrix} \text{ with commutativity } \begin{pmatrix}dom_1 & - & dom_3\\ cod_1 & dom_2 & -\\ - & cod_2 & cod_3\end{pmatrix} \text{ is considered.}$$

Both, left- and right-ordered linear structures, $(\bullet\longrightarrow\bullet\longrightarrow\bullet)$ and $(\bullet\longleftarrow\bullet\longleftarrow\bullet)$, are considered as isomorphic.
Also, left- and right-ordered star structures, $(\bullet\longrightarrow\bullet\longleftarrow\bullet)$ and $(\bullet\longleftarrow\bullet\longrightarrow\bullet)$, are considered as isomorphic.
Furthermore, linear and star structures coincide for m=3.

General procedure

$val(graph) = distr[matrix]$

architect: $\{dom_i,\ cod_i,\ i \in N\} \longrightarrow [distribution-matrix]$

Example

$$\text{val}\left(\longrightarrow \longrightarrow\right) = \begin{pmatrix} \square & - \\ \square & \square \\ - & \square \end{pmatrix}$$

$$\text{architect}\begin{pmatrix} \square & - \\ \square & \square \\ - & \square \end{pmatrix} = \begin{pmatrix} \text{dom}_1 & - \\ \text{cod}_1 & \text{dom}_2 \\ - & \text{cod}_2 \end{pmatrix}.$$

5.2.2. Line, star, cycles and flags

Line$^{(3)}$:

$$\text{architect}_{\text{line}}^{(3)} = \begin{pmatrix} \text{dom}_1 & - \\ \text{cod}_1 & \text{dom}_2 \\ - & \text{cod}_2 \end{pmatrix}$$

$$\text{MC}_{\text{line}}^{(3)} = \left\{\text{cod}_1 \cong \text{dom}_2\right\}$$

$$\text{Hence, } \text{Cat}_{\text{line}}^{(3)} = \begin{pmatrix} \text{dom}_1 & - & \text{dom}_3 \\ \text{cod}_1 & \text{dom}_2 & - \\ - & \text{cod}_2 & \text{cod}_3 \end{pmatrix}$$

$$\text{Cat}_{\text{line}}^{(3)} : (\text{hom}) : \text{hom}(x, y) \circ \text{hom}(y, z) \Longrightarrow \text{hom}(x, z).$$

Star$^{(3)}$:

$$\text{architect}_{\text{star}}^{(3)} = \begin{pmatrix} \text{dom}_1 & - \\ \text{cod}_1 & \text{cod}_2 \\ - & \text{dom}_2 \end{pmatrix}$$

$$\text{MC}_{\text{star}}^{(3)} = \left\{\text{cod}_1 \cong \text{cod}_2\right\}$$

$$\text{Cat}_{\text{star}}^{(3)} = \begin{pmatrix} \text{dom}_1 & - & \text{dom}_3 \\ \text{cod}_1 & \text{cod}_2 & - \\ - & \text{dom}_2 & \text{cod}_3 \end{pmatrix} \text{or} \begin{pmatrix} \text{dom}_1 & - & \text{cod}_3 \\ \text{cod}_1 & \text{cod}_2 & - \\ - & \text{dom}_2 & \text{dom}_3 \end{pmatrix}$$

$$\text{Cat}_{\text{star}}^{(3)} : (\text{hom}) : \text{hom}(x, y) \circ \text{hom}(x, z) \Longrightarrow \text{hom}(x, z) \vee \text{hom}(z, x).$$

$$\Longrightarrow \text{Cat}_{\text{line}}^{(3)} \cong \text{Cat}_{\text{star}}^{(3)}.$$

Star$^{(4)}$:

$$\text{architect}_{\text{star}}^{(4)} = \begin{pmatrix} \text{dom}_1 & - & - \\ \text{cod}_1 & \text{dom}_2 & \text{dom}_3 \\ - & \text{cod}_2 & - \\ - & - & \text{cod}_3 \end{pmatrix}$$

$$\text{MC}_{\text{star}}^{(4)} = \{\text{cod}_1 \cong \text{dom}_2 = \text{dom}_3\}$$

$$\text{Hence, Cat}_{\text{star}}^{(4)} = \begin{pmatrix} \text{dom}_1 & - & - & - & \text{dom}_5 & \text{dom}_6 \\ \text{cod}_1 & \text{dom}_2 & \text{dom}_3 & - & - & - \\ - & \text{cod}_2 & - & \text{dom}_4 & \text{cod}_5 & - \\ - & - & \text{cod}_3 & \text{cod}_4 & - & \text{cod}_6 \end{pmatrix}$$

$$\text{Cat}_{\text{star}}^{(4)}(\text{hom}) = \begin{pmatrix} \text{hom}(x, y) \circ \text{hom}(y, z) \Longrightarrow \text{hom}(x, z) \\ \text{hom}(x, y) \circ \text{hom}(y, v) \Longrightarrow \text{hom}(x, v) \\ \text{hom}(y, z) \circ \text{hom}(z, v) \Longrightarrow \text{hom}(y, v) \end{pmatrix}$$

Line$^{(4)}$:

$$\text{architect}_{\text{line}}^{(4)} = \begin{pmatrix} \text{dom}_1 & - & - \\ \text{cod}_1 & \text{dom}_2 & - \\ - & \text{cod}_2 & \text{dom}_3 \\ - & - & \text{cod}_3 \end{pmatrix}$$

$$\text{MC}_{\text{line}}^{(4)} = \{\text{cod}_1 \cong \text{dom}_2, \text{cod}_2 \cong \text{dom}_3\}$$

$$\text{Cat}_{\text{line}}^{(4)} = \begin{pmatrix} \text{dom}_1 & - & - & \text{dom}_4 & \square & \text{dom}_6 \\ \text{cod}_1 & \text{dom}_2 & - & \square & \text{dom}_5 & \square \\ - & \text{cod}_2 & \text{dom}_3 & \text{cod}_4 & \square & \square \\ - & - & \text{cod}_3 & \square & \text{cod}_5 & \text{cod}_6 \end{pmatrix}$$

$$\text{Cat}_{\text{line}}^{(4)}(\text{hom}) = (\text{hom}(x, y) \circ \text{hom}(y, z) \circ \text{hom}(z, v) \Longrightarrow \text{hom}(x, v))$$

$$\begin{pmatrix} \text{hom}(x, y) \circ \text{hom}(y, z) \Longrightarrow \text{hom}(x, z) \\ \text{hom}(y, z) \circ \text{hom}(z, v) \Longrightarrow \text{hom}(y, v) \end{pmatrix} \Longrightarrow \text{hom}(x, v)$$

$$\Longrightarrow \text{Cat}_{\text{line}}^{(4)} \neq \text{Cat}_{\text{star}}^{(4)}$$

Star – line$^{(5)}$:

$$\text{architect}^{(5)}_{\text{star-line}} = \begin{pmatrix} dom_1 & - & - & - \\ cod_1 & dom_2 & dom_3 & - \\ - & cod_2 & - & dom_4 \\ - & - & cod_3 & - \\ - & - & - & cod_4 \end{pmatrix}$$

$$MC^{(5)}_{\text{star-line}} = \{cod_1 \cong dom_2 \cong dom_3,\ cod_2 \cong dom_4\}$$

Null

$$Cat^{(5)}_{\text{star-line}} = \begin{pmatrix} dom_1 & - & - & - & dom_5 & - & - & dom_8 & - & dom_{10} \\ cod_1 & dom_2 & dom_3 & - & - & - & dom_7 & - & dom_9 & - \\ - & cod_2 & - & dom_4 & cod_5 & - & - & - & - & - \\ - & - & cod_3 & - & - & dom_6 & cod_7 & cod_8 & - & - \\ - & - & - & cod_4 & - & cod_6 & - & - & cod_9 & cod_{10} \end{pmatrix}$$

Cycle$^{(4)}$

$$\text{architect}^{(4)}_{\text{left-cycle}} = \begin{pmatrix} dom_1 & cod_2 & - & - \\ cod_1 & - & - & dom_4 \\ - & - & dom_3 & cod_4 \\ - & dom_2 & cod_3 & - \end{pmatrix}$$

$$\text{architect}^{(4)}_{\text{right-cycle}} = \begin{pmatrix} dom_1 & - & - & cod_4 \\ cod_1 & dom_2 & - & - \\ - & cod_2 & dom_3 & - \\ - & - & cod_3 & dom_4 \end{pmatrix}$$

$$\boxed{\text{architect}^{(4)}_{\text{left-cycle}} \neq \text{architect}^{(4)}_{\text{right-cycle}}}$$

Flag$^{(4)}$

$$architect^{(5)}_{right-cycle-flag} = \begin{pmatrix} dom_1 & - & - & cod_4 & - \\ cod_1 & dom_2 & - & - & - \\ - & cod_2 & dom_3 & - & - \\ - & - & cod_3 & dom_4 & dom_5 \\ - & - & - & - & cod_5 \end{pmatrix}$$

$$architect^{(5)}_{left-cycle-flag} = \begin{pmatrix} dom_1 & cod_2 & - & - & - \\ cod_1 & - & - & dom_4 & - \\ - & - & dom_3 & cod_4 & - \\ - & dom_2 & cod_3 & - & dom_5 \\ - & - & - & - & cod_5 \end{pmatrix}$$

right – cycle – flag

$$\begin{array}{ccccccccc} & & & & & & E & & \\ & & & & & & \uparrow & & \\ A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & D & \longrightarrow & A \end{array}$$

$$architect^{(5)}_{right-cycle-flag} \neq architect^{(5)}_{left-cycle-flag}$$

Diamond – Star$^{(4)}$

$$architect^{(4)}_{star} = \begin{pmatrix} dom_1 & - & - \\ cod_1 & dom_2 & dom_3 \\ - & cod_2 & - \\ - & - & cod_3 \end{pmatrix} \Longrightarrow \left(A \longrightarrow B \circ \begin{pmatrix} B \longrightarrow C \\ B \longrightarrow D \end{pmatrix} \right)$$

Diamond – Star – Rule

$$\frac{A \longrightarrow B \diamond \begin{pmatrix} B \longrightarrow C \\ B \longrightarrow D \end{pmatrix}}{A \longrightarrow B \circ \begin{pmatrix} B \longrightarrow C \\ B \longrightarrow D \end{pmatrix} \Bigg\| \begin{pmatrix} b_1^1 \longleftarrow b_2^1 \\ b_1^2 \longleftarrow b_2^2 \end{pmatrix}}$$

Star$^{(4)}$ – rules

$$\frac{A \longrightarrow B \circ \begin{pmatrix} B \longrightarrow C \\ B \longrightarrow D \end{pmatrix}}{A \longrightarrow C \;\Big|\; A \longrightarrow D \;\Big|\; C \longrightarrow D}$$

Birkhoff arithmetics

Example

$\text{architect}^{(5)}_{\text{star-line}} = \text{architect}^{(4)}_{\text{star}} \oplus \text{architect}^{(2)}_{\text{line}}$:

$$\begin{pmatrix} dom_1 & - & - \\ cod_1 & dom_2 & dom_3 \\ - & cod_2 & - \\ - & - & cod_3 \end{pmatrix} \oplus \begin{pmatrix} dom_1 \\ cod_1 \end{pmatrix} = \begin{pmatrix} dom_1 & - & - & - \\ cod_1 & dom_2 & dom_3 & - \\ - & cod_2 & - & dom_4 \\ - & - & cod_3 & - \\ - & - & - & cod_4 \end{pmatrix}$$

Elements of Diamond Set Theory

Some more parts of the mosaic towards semiotics, logic, arithmetic and category theory

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Abstract

Further elements are sketched towards an interplay of polycontextural logic, semiotics, arithmetic and set theory. Basics for junctional and transjunctional quantification in polycontextural logic are presented. Hints to metamorphic changes between sets, classes and conglomerates in pluri-verses are given.

1. Diamond set theory

1.1. Sets, universes, conglomerations

It is said that category theory is a departure from set theory, other are more radical and insists that category theory has nothing to do with set theory at all.
From a foundational point of view, Herrlich makes it clear that a proper mathematical formalization of categories needs different sorts of collections of different generality. He distinguishes *sets*, *classes* and *conglomerates* as the *collections* of a universe appropriate to deal with categories.
Nevertheless, there is no special conflict necessary between set theory and category theory. Both are based on different, even complementary, thematizations of formal thinking. And as such, both are using mutually methods from each other. And both are, logically and semiotically, if blindness is not dominating, based on common grounds.

Collections of the universe U = [sets, classes, conglomerates].

The objects of category theory belong to these collections. Obviously, categorical objects are not simply sets but, e.g., categories of categories, hence surpassing all reasonable, i.e., contradiction-free notions of set theory. Hence, *"One universe as a foundation of category theory",* (Mac Lane, 1969)

Diamond theory is in no way less general than category theory, but the objects of diamonds are not only collections of different degrees of abstractions, but are *bi-objects* from their very beginning. Bi-objects are complementary objects constructed as an interplay between acceptional and rejectional aspects of diamond theory.

Hence the objects of diamonds are not simply belonging to the universe U of conglomerates with its classes and sets, but to the 2-verse (di-verse) as a complementarity of the universe of acceptional and the "universe" of rejectional objects.

Category theory happens in a universe, polycontexturality in a pluri-verse and diamond theory in a di-verse 2-U of complementarity.
Thus, 2-U = [collections || collections].
Hence, 2-U = [(set||set), (class||class), (conglomerate||conglomerate)].

A di-verse conception of collections opens up the possibility of *metamorphic* chiasms between their constituents [set, class, conglomerate].

This happens in a similar way like in *polycontexturally* disseminated categories. That is, a set in one contexture can be seen as a class in another contexture, etc. This happens on the base of as-abstractions. In category theory "a set is a set, a class is a class and a conglomerate is a conglomerate"; and nothing else happens. The hierarchy is strict and well defined. The notions, set, class, conglomerate, are defined by the is-abstraction.

This is different for polycontextural systems but also for diamond theory. For both, collections are still well defined and placed in their hierarchy. But because of the multitude of universes, interactions are possible between different kinds of collections. These interactions are strictly defined, too. They are ruled by the mechanism of *chiastic metamorphosis*.

Obviously, to describe the rules of sets, classes and conglomerates in di-verses we need some knowledge from diamond theory, which is based then just on such rules. That is, the whole idea of a di-verse is based on conceptions of diamond theory.
In diamond theory, conglomerates are not covering the situations of bi-objects. Bi-objects are polycontextural, thus they are members of disseminated conglomerates.

Contexture(Conglomerate(Class(Set)))

On the base of other conceptualizations of the diamond way of thematization, a transition from 2-verses to n-verses is not excluded. This should not be confused with the general multi-verses of polycontextural systems.

1.2. Diamond strategies for bi-objects

Bi-objects are strictly divided into a saltatorical and a categorical part. With the interplay and interactivity between categories and saltatories, ruled by the bridging conditions and operations, a new type of object emerges: bi-objects with mixed parts. Hence, diamonds are involved not simply in bi-objects but in bridges, too.
Bridges are composed by difference operation into a combination of categorical and saltatorical parts. In this sense, they are the both-at-once aspect of diamond bi-objects. A change of perspective in favor to the bridging operation as such, abstracting from its bi-objects, the neither-nor structure of bi-objects might be constructed.
Hence, we have to distinguish 4 aspects of diamonds: categorical, saltatorical, interplay (bridging as a mix) and interactionality (bridging as such).

1.3. Elements of a diamond theory of conglomerates

Both approaches, the polycontextural approach to logic and diamond theory as well the approach of mathematical semiotics, is first and mainly considered of abstract cognitive and volitive structures and transformations. Propositions about elements of the semiotic, polycontextural and diamond theoretical domains are not yet proposed in a formal and formalized way. Like classical propositional logic is enhanced by a theory of quantification, which allows to state statements about elements, properties and quantifications with all ($\forall$), some ($\exists$), exactly one ($\exists!$), the same shall be introduced for transclassical approaches to formal thinking.
In classical logic, the logic of predicates, i.e. first-order logic, is defined on the base of a single, uniform or structured, *universe* of individual elements or objects, transclassical logic has to be defined in concert with a plurality of domains, called *pluri-verse*. Diamond theory of pluriverses is reflecting the otherness of any thematized universe of a pluriversal "set" or "domain" theory.
Classical logic and set theory is restricted to structure its single universe into sorts, sub-domains, layers, levels etc. without touching the strict hierarchy between the single universe and its parts or subsystems.
Today, set theory is enlarged, for the reasons of category theoretical aims, to a theory of universes, conglomerates, large and small sets, i.e classes and sets.

Laws for sets
Laws for classes
Laws for conglomerates
Laws for universes

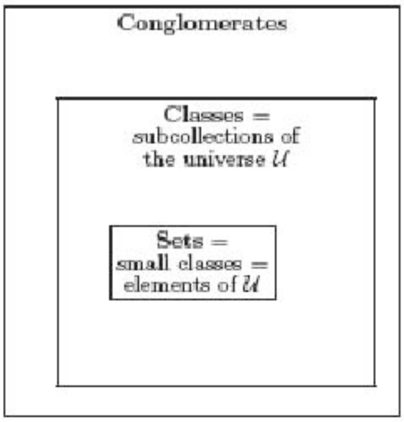
Universes are founded in uniqueness
Laws for chiasms between universes.

Metamorphic interchanges between universes, conglomerates, classes and sets in a polycontextural framework are the fundamental mechanisms of change.
Changes might be *iterative* or *accretive*.
Iterative changes happens in a stable framework, accretive changes are augmenting the complexity of the framework.

This is not the place to enter into the intriguing world of mathematical foundations, its strategies of avoiding paradoxes and extending the fields of mathematical reasoning.

One small hint should be recalled. There is no primary need to avoid paradoxes in polycontextural theories because they are accessible to a paradox-free implementation based on the chiasm between elements and predicates, or sets and elements. It was sketched in (Kaehr 1978), that for each contexture, a contexture specific local paradox might be constructed and that the system as such is not involved, globally, into the well known unavoidable paradoxes of self-referentiality.

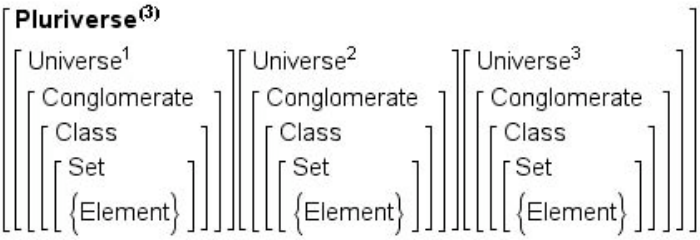
$$\text{Chiasm}_{\text{iter}}\left(\text{Pluriverse}^{(m)}\right) = \text{Pluriverse}^{(m)}$$
$$\text{Chiasm}_{\text{acc}}\left(\text{Pluriverse}^{(m)}\right) = \text{Pluriverse}^{(m+n)}$$

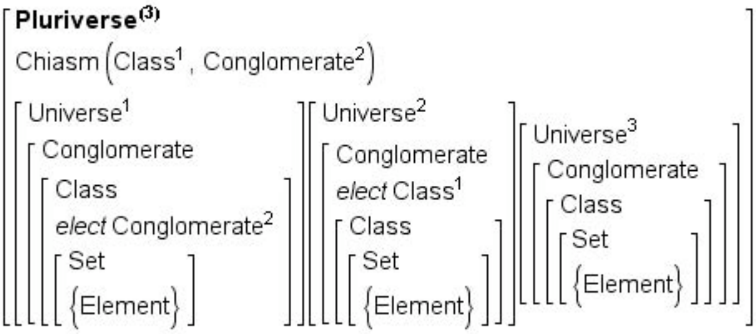


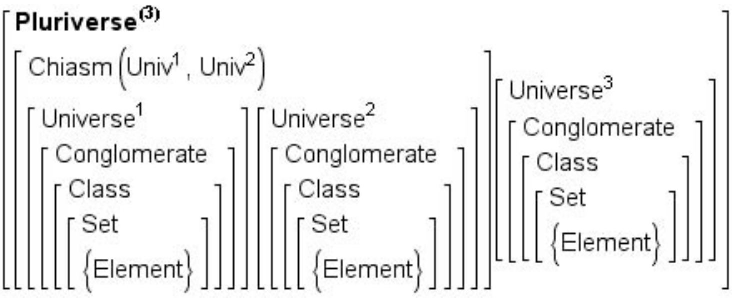
The hierarchy of "collections"

Universe = Conglomerates[Classes[Sets[Elements]]] after Herrlich.

$$\text{Universal Category Theory} = \left[\begin{array}{l}\textbf{Universe}\\ \left[\begin{array}{l}\text{Conglomerate}\\ \left[\begin{array}{l}\text{Class}\\ \left[\begin{array}{l}\text{Set}\\ \{\text{Element}\}\end{array}\right]\end{array}\right]\end{array}\right]\end{array}\right]$$







$$\text{Chiasm}(\text{Set, Element}) = \begin{pmatrix} \text{Set}^1 & \longrightarrow & \text{Element}^1 \\ \updownarrow & x & \updownarrow \\ \text{Element}^2 & \longleftarrow & \text{Set}^2 \end{pmatrix}$$

$$\text{Diamond}(\text{Set, Element}) = \begin{pmatrix} \text{Element}^3 & - & \text{Set}^1 & \longrightarrow & \text{Element}^1 & - & \text{Element}^4 \\ \downarrow & & \updownarrow & x & \updownarrow & \square & \uparrow \\ \text{Set}^3 & - & \text{Element}^2 & \longleftarrow & \text{Set}^2 & - & \text{Set}^4 \end{pmatrix}$$

1.3.1. Ontology

General Set Theory (Boolos)

"GST features a single primitive ontological notion, that of set, and a single ontological assumption, namely that all individuals in the universe of discourse (i.e., all mathematical objects) are sets. There is a single primitive binary relation, set membership; that set a is a member of set b is written a∈b (usually read "a is an element of b")." WiKi

1. *Axion of Extensionality:*
The sets x and y are the same set if they have the same members.
∀x ∀y [∀ z [z∈x <--> z∈ y] --> x = y].

2. Axiom Schema of Specification

Separation or Restricted Comprehension): If z is a set and ϕ is any property which may be satisfied by all, some, or no elements of z, then there exists a subset y of z containing just those elements x in z which satisfy the property ϕ.

∀z ∃y ∀x [x ∈ y <--> (x ∈z ∧ ϕ(x))].

3. Axiom of Adjunction
If x and y are sets, then there exists a set w, the adjunction of x and y, whose members are just y and the members of x.[1]

∀x ∀y ∃w ∀z [z ∈ w<-> [z∈ x ∨ z=y]].

General Diamond Set Theory
GDST$^{(m)}$ features for each contexture a single primitive contextural notion, that of bi-set, and for each a single contextural assumption, namely that all bi-individuals in the pluri-verse of thematizations (i.e., all mathematical objects) are bi-sets of complexity m.

There is *locally* for each contexture of a polycontexturality a single primitive binary relation, bi-set membership; that set (A,a) is a member of set (B,b) is written (A,a)∈∈(B,b) (usually read "a is an element of b").

There is *globally* for each constellation of contextures and uni-verses a chiastic exchange relation between contextures and uni-verses of the pluri-verse.

The logic of GDST$^{(m)}$ is the polycontextural diamond logic PolyLogic$^{(m, n)}$.

GST is derived from GDST by reducing pluri-verses to uni-verse, bi-sets to sets and by omitting chiasm.

GDST	GST
pluri – verse	universe
bi – sets	sets
bi – mebership	membership
diamond relation	binary relation
polylogic	logic
chiasm	––

Between the concept of "Urelement" and the concept of "Contexture" a duality holds.
A Urelement is an element which might be a member of a set but it doesn't contain itself any members.
A contexture isn’t a member of a set but contains all sets of itself.
In a chiastic scenario, a contexture might change its functionality into the functionality of an element of another contexture.

2. Quantification in polycontextural logics

First-order logic quantification is distributed over different contextural domains of polycontextural logic.

As for FOL, quantification in polylogics requires quantifiers which are applied to predicates and functions and their variables. And substitution is required too.

Quantification in polylogics is naturally realized by *universal* and *existential* quantification introduced in analogy to FOL for each contexture. Because of polycontexturality, additional to the separated actions of contextural quantifiers, quantifier for interactions between different contextures have to be introduced, i.e. *transjunctional* quantifiers.
For a 3-contextural logic Log$^{(3)}$ *universal* and *existential* quantifiers are distributed over 3 places. In general, for m there are $\binom{m}{2}$ places to consider.
Hence, the patterns are in correspondence to the distribution of propositional conjunctions and disjunctions.

Typical polycontextural quantifiers are the *transjunctional* quantifiers Q and G. Similar to the duality of logical quantifiers, ∀ and ∃, the transjunctional quantifiers Q and G are interchangeable.

Quantification is in analogy to propositional versions. That is,

$$non_3\left(QQQ\left(x^{(3)}\right) non_1\left(P^{(3)} x^{(3)}\right)\right) \text{ equal } GGG\left(x^{(3)}\left(P^{(3)}\left(x^{(3)}\right)\right)\right).$$

Some " propositional " versions : " □" like Q, " ≀" like G :

$$non_3\left(non_1\, p \wr\wr\wr q\right) \equiv p \,\square\square\square\, q.$$

$$non_3\left(p \wr\wr\wr non_1\, q\right) = non_2\left(p \,\square\square\square\, q\right).$$

$$non_3\left(non_3\, p \wr\wr\wr non_3\, p\right) = p \wr\wr\wr q$$

$$non_3\left(non_3\, p \,\square\square\square\, non_3\, p\right) = p \,\square\square\square\, q.$$

Junctional quantifiers

∀∀∀, ∀∀∃, ∀∃∀, ∀∃∃,
∃∃∃, ∃∃∀, ∃∀∃, ∃∀∀

Transjunctional quantifiers

QQQ,
QQ∀, Q∀Q, ∀QQ,
∀Q∀, ∀∀Q, Q∀∀,
QQ∃, Q∃Q, ∃QQ,
∃Q∃, ∃∃Q, Q∃∃.

GGG,
GG∀, G∀G, ∀GG, GG∃,
G∃G, ∃GG, GG∃, G∃G,
∃GG, G∃∃, ∃G∃, ∃∃G.

Models

$$Q\exists\forall\ x^{(3)} P^{(3)} x^{(3)}\ \text{eq}\ P^{(3)} k_1^{(3)} \blacklozenge \vee \wedge P^{(3)} k_2^{(3)}) \blacklozenge \vee \wedge \ldots \blacklozenge \vee \wedge P^{(3)} k_n^{(3)}$$

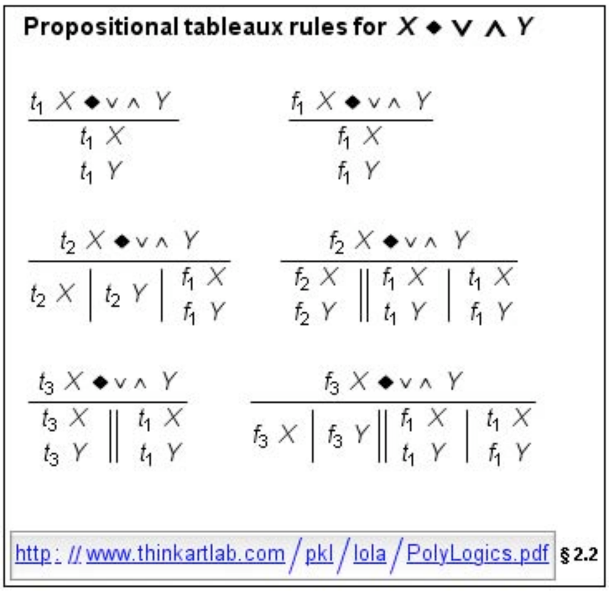
Om $k_i^{(3)} \in U^{(3)}$

Wording for first-Order Logic:

"The *domain* D is a set of "objects" of some kind. Intuitively, a first-order formula is a statement about *objects*; for example, ∃x.P(x) states the existence of an object x such that the predicate P is true where referred to it. The domain is the set of considered objects. As an example, one can take D to be the set of integer numbers." (WiKi)

Because first-order logic objects are obviously characterized by a single domain (universe, contexture), the domain of polycontextural quantification is not a single universal set of objects but, metaphorically, a poly-set of relations.

An interesting *example* for "poly-sets of relations" is given by the system of triadic-trichotomic sign relations. An adequate thematization, modeling and logification of triadic-trichotomic domains, like Peircean semiotics, needs a structurally adequate logical apparatus. First-order logic is reducing such complex structures to mono-contextural predicative objects.



2.1. Tableau rules for polycontextural quantifiers

2.1.1. Syntactic schemes

$$\begin{array}{l}
\left(J^1\, J^2\, J^3\right) x^{(3)} P^{(3)}\ x^{(3)} \\
\hline
\left(J^1\, x^1\right) P^1\, x^1 \\
\left(J^2\, x^2\right) P^2\, x^2 \\
\left(J^3\, x^3\right) P^3\, x^3 \\
\\
J = \{\exists, \forall\}
\end{array}$$

$$\begin{array}{l}
\left(Q\, \exists\, \forall\right) x^{(3)} P^{(3)}\ x^{(3)} \\
\hline
\left(Q^1\left(x_1^1,\ x_2^1\right)\right)\left(\left(P_1^1,\ P_2^1\right)\left(x_1^1,\ x_2^1\right)\right) \\
\left(\exists^2\, x^2\right) P^2\, x^2 \\
\left(\forall^3\, x^3\right) P^3\, x^3
\end{array}
\qquad
\begin{array}{l}
\left(\exists\, \forall\, Q\right) x^{(3)} P^{(3)}\ x^{(3)} \\
\hline
\left(\exists^1\, x^1\right) P^1\, x^1 \\
\left(\forall^2\, x^2\right) P^2\, x^2 \\
\left(Q^3\left(x_1^3,\ x_2^3\right)\right)\left(\left(P_1^3,\ P_2^3\right)\left(x_1^3,\ x_2^3\right)\right)
\end{array}$$

$$\begin{array}{l}
\left(\exists\, Q\, \forall\right) x^{(3)} P^{(3)}\ x^{(3)} \\
\hline
\left(\exists^1\, x^1\right) P^1\, x^1 \\
\left(Q^2\left(x_1^2,\ x_2^2\right)\right)\left(\left(P_1^2,\ P_2^2\right)\left(x_1^2,\ x_2^2\right)\right) \\
\left(\forall^3\, x^3\right) P^3\, x^3
\end{array}
\qquad
\begin{array}{l}
\left(\exists\, Q Q\right) x^{(3)} P^{(3)}\ x^{(3)} \\
\hline
\left(\exists^1\, x^1\right) P^1\, x^1 \\
\left(Q^2\left(x_1^2,\ x_2^2\right)\right)\left(\left(P_1^2,\ P_2^2\right)\left(x_1^2,\ x_2^2\right)\right) \\
\left(Q^3\left(x_1^3,\ x_2^3\right)\right)\left(\left(P_1^3,\ P_2^3\right)\left(x_1^3,\ x_2^3\right)\right)
\end{array}$$

Relation $R = (P_1^1,\ P_2^1)$ is a dual – predicate over the polycontextural tuple $(x_1^1,\ x_2^1)$

2.1.2. Tableaux rules for $(Q\ \exists\ \forall)$

$$\frac{t_1 Q \exists \forall \; x^{(3)} P^{(3)} \; x^{(3)}}{\begin{matrix} t_1 \; P^1 \, {}^{x^1}_{a_1} \\ t_1 \; P^1 \, {}^{x^1}_{b_1} \end{matrix}}$$

$$\frac{f_1 Q \exists \forall \; x^{(3)} P^{(3)} \; x^{(3)}}{\begin{matrix} f_1 \; P^1 \, {}^{x^1}_{a_1} \\ f_1 \; P^1 \, {}^{x^1}_{b_1} \end{matrix}}$$

$$\frac{t_2 Q \exists \forall \; x^{(3)} P^{(3)} \; x^{(3)}}{t_2 \; P^1 \, {}^{x^2}_{a_2} \left\| \begin{matrix} f_1 \; P^1 \, {}^{x^1}_{a_1} \\ f_1 \; P^1 \, {}^{x^1}_{b_1} \end{matrix} \right.}$$

$$\frac{f_2 \; Q \exists \forall \; x^{(3)} P^{(3)} \; x^{(3)}}{f_2 \; P^2 \, {}^{x^2}_{a_2} \left\| \begin{matrix} t_1 \; P^1 \, {}^{x^1}_{a_1} \\ f_1 \; P^1 \, {}^{x}_{b_1} \end{matrix} \right| \begin{matrix} f_1 \; P^1 \, {}^{x^1}_{a_1} \\ t_1 \; P^1 \, {}^{x^1}_{b_1} \end{matrix}}$$

$$\frac{t_3 Q \exists \forall \; x^{(3)} P^{(3)} \; x^{(3)}}{t_3 \; P^1 \, {}^{x^3}_{a_3} \left\| \begin{matrix} f_1 \; P^1 \, {}^{x^1}_{a_1} \\ f_1 \; P^1 \, {}^{x^1}_{b_1} \end{matrix} \right.}$$

$$\frac{f_3 \; Q \exists \forall \; x^{(3)} P^{(3)} \; x^{(3)}}{f_3 \; P^3 \, {}^{x^3}_{a_3} \left\| \begin{matrix} t_1 \; P^1 \, {}^{x^1}_{a_1} \\ f_1 \; P^1 \, {}^{x}_{b_1} \end{matrix} \right| \begin{matrix} f_1 \; P^1 \, {}^{x^1}_{a_1} \\ t_1 \; P^1 \, {}^{x^1}_{b_1} \end{matrix}}$$

With $a_1, \; b_1 \in U^i, \; a_1 \neq b_1$ for $\forall^i, \; \exists^i, \; i = 1, 2, 3$ and $U^{(3)} = \left\{ \left(a_i , \; b_i \right) \middle| \; a, b \; \in U^{(3)} \right\}$ for $Q, \; G$.

For $\left(Q \exists \forall \right): \; U^{(3)} = \left[\left(a_1 \, b_1 \right), \; \left(a_2 \left(a_1 \, b_1 \right) \right), \; \left(a_3 \left(a_1 \, b_1 \right) \right) \right]$.

For all junctional quantificators there exists a quantificational representation with single objects *a*.

For all transjunctional quantificators there exists a quantificational representation with transcontextural tupels *(a, b)*.

Objectivity and Objectionality

The objectivity of an object in First-Order Logic and set theory is characterized by its

ontology of *individual* and *predicate*, i.e. substance and attribute. There is a strict hierarchic order between individuals and predicates. Objectivity is excluding, logically and ontologically, self-referential statements between individuals and predicates, i.e. a predicate can not change into an individual and vice versa without producing in its logical framework a contradiction.
Objectionality of a polycontextural object depends on the complexity of the interplay between entities and characteristics of different contextures. (Kaehr, Materialien 1976, Siemens-Studie, 1985)

Linguistic example
A 3-contextural object $O^{(3)}$ like a (contexturally) complementary object *waveparcel* (Heisenberg) has the property $P^{(3)}$ with locally P^1 for the *wave* and for the *parcel* O^2 the property P^2 and globally the property P^3 for the composed notion of wave and parcel, O^3, *waveparcel*.
Quantification happens intra-contexturally for all contextures by the junctional quantifiers ∀ and ∃.
Quantification between different contextures, focusing different contextures at once, happens with transjunctional quantifiers, e.g. Q and G.

Famous application to Russell' s devilish construction, but escaping paradox

1. $\exists^{(3)} F^{(3)} \forall^{(3)} f^{(3)}: F^{(3)}\left(f^{(3)}\right) \equiv \neg^1 f^{(3)}\left(f^{(3)}\right)$: postulation of constellation
2. $\forall^1 f^1 : F_0\left(f^1\right) \equiv \neg^1 f^1\left(f^1\right)$: specification + contextural selection
3. $f^2 \cong F_0 : \forall^2 f^2 : F_0\left(f^2\right) \equiv \neg^1 f^2\left(f^2\right)$: accretive substitution Q
4. $: F_0\left(F_0\right) \equiv \neg^1 F_0\left(F_0\right)$: comparison : non – contradiction

Strategy : chiasm $\left(\forall, \exists, F, f\right)$

Kaehr, Materialien 1973 – 75, in: Günther, Grundriss, 1978, *p*.57

2.1.3. Mimicking General Set Theory (GST)

GST:

1. $\forall x \forall y\left[\forall z\left[z \in x \longleftrightarrow z \in y\right] \longrightarrow x = y\right]$.
2. $\forall z \exists y \forall x\left[x \in y \longleftrightarrow \left(x \in z \wedge \varphi(x)\right)\right]$.
3. $\forall x \forall y \exists w \forall z\left[z \in w \longleftrightarrow \left[z \in x \vee z = y\right]\right]$.

Axioms of Extensionality for 3 – contextural set theory $GST^{(3)}$

Homogeneous extensionality

1. $\forall^{(3)} x^{(3)} \forall^{(3)} y^{(3)}\left[\forall^{(3)} z^{(3)}\left[z^{(3)} \in^{(3)} x^{(3)} \longleftrightarrow^{(3)} z^{(3)} \in^{(3)} y^{(3)}\right] \longrightarrow^{(3)} x^{(3)} =^{(3)} y^{(3)}\right]$.

Extensionality Axiom, 3 – homogeneous $(\forall\ \forall\ \forall)$

sops = (id, id, id)

$$\left[\begin{matrix}\forall x \forall y\\ \left[\begin{matrix}\forall z\\ \left[\begin{matrix} z \in x \leftrightarrow z \in y\\ [x = y]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\left[\begin{matrix}\forall x \forall y\\ \left[\begin{matrix}\forall z\\ \left[\begin{matrix} z \in x \leftrightarrow z \in y\\ [x = y]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\left[\begin{matrix}\forall x \forall y\\ \left[\begin{matrix}\forall z\\ \left[\begin{matrix} z \in x \leftrightarrow z \in y\\ [x = y]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]$$

Extensionality Axiom, 3 – heterogeneous $(\forall\ \forall\ \exists)$

sops = (id, id, id)

$$\left[\begin{matrix}\forall x \forall y\\ \left[\begin{matrix}\forall z\\ \left[\begin{matrix} z \in x \leftrightarrow z \in y\\ [x = y]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\left[\begin{matrix}\forall x \forall y\\ \left[\begin{matrix}\forall z\\ \left[\begin{matrix} z \in x \leftrightarrow z \in y\\ [x = y]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\left[\begin{matrix}\exists x \exists y\\ \left[\begin{matrix}\exists z\\ \left[\begin{matrix} z \in x \leftrightarrow z \in y\\ [x = y]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]$$

Extensionality Axiom, homogeneous, transjunctional

sops = (bif, perm, perm)

$$\left[\begin{matrix}\text{Contexture}_1\\ \left[\begin{matrix}\text{Contexture}_{1.1}\\ \left[\begin{matrix}\forall x \forall y\\ \left[\begin{matrix}\forall z\\ \left[\begin{matrix} z \in x \leftrightarrow z \in y\\ [x = y]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\left[\begin{matrix}\text{Contexture}_{2.1}\\ \left[\begin{matrix}\forall x \forall y\\ \left[\begin{matrix}\forall z\\ \left[\begin{matrix} z \in x \leftrightarrow z \in y\\ [x = y]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\left[\begin{matrix}\text{Contexture}_2\\ \left[\begin{matrix}\text{Contexture}_{2.3}\\ \left[\begin{matrix}\forall x \forall y\\ \left[\begin{matrix}\forall z\\ \left[\begin{matrix} z \in x \leftrightarrow z \in y\\ [x = y]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\left[\begin{matrix}\text{Contexture}_3\\ \left[\begin{matrix}\text{Contexture}_{3.2}\\ \left[\begin{matrix}\forall x \forall y\\ \left[\begin{matrix}\forall z\\ \left[\begin{matrix} z \in x \leftrightarrow z \in y\\ [x = y]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]\end{matrix}\right]$$

Extensionality Axiom, homogeneous, transjunctional

sops = (bif, id, red)

Contexture$_1$
- Contexture$_{1.1}$: $\forall x \forall y$ [$\forall z$ [$z \in x \leftrightarrow z \in y$] [$x = y$]]
- Contexture$_{2.1}$: $\forall x \forall y$ [$\forall z$ [$z \in x \leftrightarrow z \in y$] [$x = y$]]

Contexture$_2$
- Contexture$_{2.1}$: $\forall x \forall y$ [$\forall z$ [$z \in x \leftrightarrow z \in y$] [$x = y$]]

Contexture$_3$
- Contexture$_{2.3}$: $\forall x \forall y$ [$\forall z$ [$z \in x \leftrightarrow z \in y$] [$x = y$]]

Extensionality Axiom, homogeneous, reflectional

sops = (repl, id, red)

Contexture$_1$
- Contexture$_{1.1}$: $\forall x \forall y$ [$\forall z$ [$z \in x \leftrightarrow z \in y$] [$x = y$]]
- Contexture$_{1.2}$: $\forall x \forall y$ [$\forall z$ [$z \in x \leftrightarrow z \in y$] [$x = y$]]

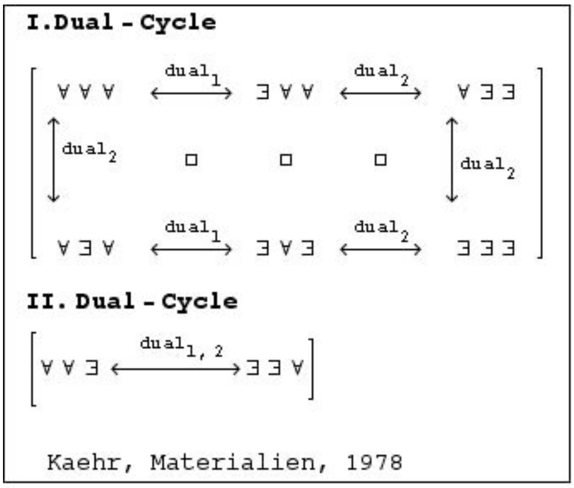
Contexture$_2$
- Contexture$_{2.1}$: $\forall x \forall y$ [$\forall z$ [$z \in x \leftrightarrow z \in y$] [$x = y$]]

Contexture$_3$
- Contexture$_{2.3}$: $\forall x \forall y$ [$\forall z$ [$z \in x \leftrightarrow z \in y$] [$x = y$]]

More **brackets** at:
ConTeXtures. Programming Dynamic Complexity.
http://www.thinkartlab.com/pkl/lola/ConTeXtures.pdf

2.2. Polylogical quantification rules

2.2.1. DeMorgan rules



Dual (Junctional Quantifiers) = Hierarchic ⋃ Heterarchic

First – order logic duality

$$\neg \forall ((x)(\neg P(x)) = (\exists x) P(x)$$

$$\neg \exists ((x)(\neg P(x)) = (\forall x) P(x)$$

Polylogical duality for junctional quantifiers

$$\text{dual}_1 : \neg_1 (\forall_1 \forall_2 \forall_3)((x)(\neg_1 P(x)) = (\exists_1 \forall_3 \ \forall_2)(x) P(x)$$

$$\text{dual}_2 : \neg_2 (\forall_1 \forall_2 \forall_3)((x)(\neg_2 P(x)) = (\forall_3 \exists_2 \forall_1)(x) P(x)$$

Polylogical self – duality for transjunctional quantifiers

$$\text{dual}_1 : \neg_1 (Q_1 \forall_2 \forall_3)((x)(\neg_1 P(x)) = (\mathcal{Q}_1 \forall_3 \ \forall_2)(x) P(x)$$

$$\text{dual}_2 : \neg_2 (\mathcal{Q}_1 \forall_2 \forall_3)((x)(\neg_2 P(x)) = (\forall_3 Q_2 \forall_1)(x) P(x)$$

$$\text{dual}_1 : \neg_1 (G_1 \forall_2 \forall_3)((x)(\neg_1 P(x)) = (\mathcal{G}_1 \forall_3 \ \forall_2)(x) P(x)$$

$$\text{dual}_2 : \neg_2 (G_1 \forall_2 \forall_3)((x)(\neg_2 P(x)) = (\forall_3 G_2 \forall_1)(x) P(x)$$

The quantifiers Q and G are *self – dual*, i.e dual$(Q) = Q$ and dual$(G) = G$.

2.2.2. Distribution rule for transjunctional quantifiers

Propositional distribution rule

$$\frac{(\alpha \text{ simul } \delta) \text{ et } (\beta \text{ simul } \gamma)}{(\alpha \text{ et } \beta) \text{ simul } (\delta \text{ et } \gamma)}$$

Example of a 3 element domain quantificational rule : (ıı = sim = simultaneous)

$$\mathbf{Q \forall \exists \ x^{(3)} P^{(3)} \ x^{(3)}}$$

$$\frac{\left[\left(P^1 k_1^1 \text{ sim } P^2 k_1^2\right) \text{ et } \left(P^1 k_2^1 \text{ sim } P^2 k_2^2\right) \text{ et } \left(P^1 k_3^1 \text{ sim } P^2 k_3^2\right)\right] \| \left[P^2 k_1^2 \text{ et } P^2 k_2^2 \text{ et } P^2 k_3^2\right] \| \left[P^3 k_1^3 \text{ or } P^3 k_2^3 \text{ or } P\right.}{\left[\left(P^1 k_1^1 \text{ et } P^1 k_2^1 \text{ et } P^1 k_3^1\right) \text{ sim } \left(P^2 k_1^2 \text{ et } P^2 k_2^2 \text{ et } P^2 k_3^2\right)\right] \| \left[P^2 k_1^2 \text{ et } P^2 k_2^2 \text{ et } P^2 k_3^2\right] \| \left[P^3 k_1^3 \text{ or } P^3 k_2^3 \text{ or } P^3\right.}$$

$$Q \forall \exists \; x^{(3)} P^{(3)} \; x^{(3)}$$

$$\begin{array}{l|l|l}
\left(P^1 k_1^1 \,\|\, P^2 k_1^2\right) & & \\
\left(P^1 k_2^1 \,\|\, P^2 k_2^2\right) & \begin{array}{l} P^2 k_1^2 \\ P^2 k_2^2 \\ P^2 k_3^2 \end{array} & P^3 k_1^3 \,|\, P^3 k_2^3 \,|\, P^3 k_3^3 \\
\left(P^1 k_3^1 \,\|\, P^2 k_3^2\right) & &
\end{array}$$

$$\begin{array}{l\|l|l|l}
\begin{array}{l} P^1 k_1^1 \\ P^1 k_2^1 \\ P^1 k_3^1 \end{array} & \begin{array}{l} P^2 k_1^2 \\ P^2 k_2^2 \\ P^2 k_3^2 \end{array} & \begin{array}{l} P^2 k_1^2 \\ P^2 k_2^2 \\ P^2 k_3^2 \end{array} & P^3 k_1^3 \,|\, P^3 k_2^3 \; P^3 k_3^3
\end{array}$$

2.3. Quantification for diamond theory

2.3.1. 3-contextural diamond

Scheme for diamond – quantification (reduced)

$$(S_1\; S_2\; S_3\; S_4) = \begin{bmatrix} & S_4 & \\ S_1 & | & S_2 \\ & S_3 & \end{bmatrix} \quad \text{with } S \in \{\forall,\ \exists,\ Q,\ G\}$$

$$(Q_1\; \exists_2\; \forall_3\; \exists_4) \equiv \begin{bmatrix} & \exists_4 & \\ Q_1 & | & \exists_2 \\ & \forall_3 & \end{bmatrix}$$

Example of a diamond – quantificational formula

$$\begin{pmatrix} & \exists_4 & \\ Q_1 & | & \exists_2 \\ & \forall_3 & \end{pmatrix} \left\langle \begin{bmatrix} & x_4 & \\ x_1 & | & x_2 \\ & x_3 & \end{bmatrix} \left\langle \begin{bmatrix} & P_4 & \\ P_1 & | & P_2 \\ & P_3 & \end{bmatrix} \begin{bmatrix} & x_4 & \\ x_1 & | & x_2 \\ & x_3 & \end{bmatrix} \right\rangle \right\rangle$$

quantors | variables | predicates | variables

$$\text{quant}^{(4)} \;|\; \left\langle \left(x^{(4)}\right) \;|\; \left\langle \left(P^{(4)} \quad \left(x^{(4)}\right)\right\rangle \right\rangle\right.$$

Duality for diamond – quantificational formulas

$\text{dual}_1\left(\text{dual}_2\left(\text{dual}_1\left(D^{(3)}\right)\right)\right) =$

$$\begin{bmatrix} & S_4 & \\ S_1 & \mid & S_2 \\ & S_3 & \end{bmatrix} \xrightarrow{\text{dual1}} \begin{bmatrix} & \bar{S}_4 & \\ \bar{S}_1 & \mid & S_3 \\ & S_2 & \end{bmatrix} \xrightarrow{\text{dual2}} \begin{bmatrix} & \bar{S}_4 & \\ \bar{S}_3 & \mid & S_1 \\ & \bar{S}_2. & \end{bmatrix} \xrightarrow{\text{dual1}} \begin{bmatrix} & \bar{S}_4 & \\ \bar{S}_2 & \mid & \bar{S}_1 \\ & \bar{S}_3 & \end{bmatrix}$$

$\text{dual}_2\left(\text{dual}_1\left(\text{dual}_2\left(D^{(3)}\right)\right)\right) =$

$$\begin{bmatrix} & S_4 & \\ S_1 & \mid & S_2 \\ & S_3 & \end{bmatrix} \xrightarrow{\text{dual2}} \begin{bmatrix} & S_4 & \\ S_3 & \mid & \bar{S}_2 \\ & S_1 & \end{bmatrix} \xrightarrow{\text{dual1}} \begin{bmatrix} & \bar{S}_4 & \\ S_2 & \mid & \bar{S}_3 \\ & \bar{S}_1 & \end{bmatrix} \xrightarrow{\text{dual2}} \begin{bmatrix} & \bar{S}_4 & \\ \bar{S}_2 & \mid & \bar{S}_1 \\ & \bar{S}_3 & \end{bmatrix}$$

Hence, cycle $Z^{(4)}$: $\text{dual}_1\left(\text{dual}_2\left(\text{dual}_1\left(D^{(3)}\right)\right)\right) = \text{dual}_2\left(\text{dual}_1\left(\text{dual}_2\left(D^{(3)}\right)\right)\right)$.

Example : Tableaux rules for $\left(Q_1\ \exists_2\ \forall_3\ \exists_4\right)$

$$\left(Q_1\ \exists_2\ \forall_3\ \exists_4\right) \equiv \begin{bmatrix} & \exists_4 & \\ Q_1 & \mid & \exists_2 \\ & \forall_3 & \end{bmatrix}$$

$$\frac{t_4\left(Q\ \exists\ \forall\ \exists\right) x^{(3)} P^{(3)}\ x^{(3)}}{\begin{matrix} t_1\ P^1\ {}^{x^1}_{a_1} \\ t_1\ P^1\ {}^{x^1}_{b_1} \end{matrix}}$$

$$\frac{f_4\left(Q\ \exists\ \forall\ \exists\right) x^{(3)} P^{(3)}\ x^{(3)}}{\begin{matrix} f_1\ P^1\ {}^{x^1}_{a_1} \\ f_1\ P^1\ {}^{x^1}_{b_1} \end{matrix}}$$

2.3.2. 4-contextural diamond

$$\begin{bmatrix} id_9 \\ id_4\ id_8 \\ dual_1\ id_2\ id_5 \\ id_3\ id_6 \\ id_7 \end{bmatrix} : \begin{bmatrix} S_9 \\ S_4 \mid S_8 \\ S_1 \mid S_2 \mid S_5 \\ S_3 \mid S_6 \\ S_7 \end{bmatrix} \xrightarrow{dual_1} \begin{bmatrix} S_9 \\ \bar{S}_4 \mid S_8 \\ \bar{S}_1 \mid S_3 \mid S_5 \\ S_2 \mid S_7 \\ S_6 \end{bmatrix}$$

Example

$$\begin{bmatrix} \forall_9 \\ \forall_4 \mid \forall_8 \\ \forall_1 \mid \forall_2 \mid \forall_5 \\ \forall_3 \mid \forall_6 \\ \forall_7 \end{bmatrix} \xrightarrow{dual_1} \begin{bmatrix} \forall_9 \\ \exists_4 \mid \forall_8 \\ \exists_1 \mid \forall_3 \mid \forall_5 \\ \forall_2 \mid \forall_7 \\ \forall_6 \end{bmatrix}$$

$$\begin{pmatrix} \forall_9 \\ \exists_4 \mid \forall_8 \\ \exists_1 \mid \forall_3 \mid \forall_5 \\ \forall_2 \mid \forall_7 \\ \forall_6 \end{pmatrix} \left\langle \begin{bmatrix} x_9 \\ x_4 \mid x_8 \\ x_1 \mid x_3 \mid x_5 \\ x_2 \mid x_7 \\ x_6 \end{bmatrix} \left\langle \begin{bmatrix} P_9 \\ P_4 \mid P_8 \\ P_1 \mid P_3 \mid P_5 \\ P_2 \mid P_7 \\ P_6 \end{bmatrix} \right] \begin{bmatrix} x_9 \\ x_4 \mid x_8 \\ x_1 \mid x_3 \mid x_5 \\ x_2 \mid x_7 \\ x_6 \end{bmatrix} \right\rangle\right\rangle$$

2.4. Smullyan unification for diamond quantification

2.4.1. Smullyan's unification rules for "propositional" constellations

PolyLogics
Towards a formalization of polycontextural Logics.
http://www.thinkartlab.com/pkl/lola/PolyLogics.pdf

From Dialogues to Polylogues
http://www.thinkartlab.com/pkl/lola/Games-short.pdf

Place-valued logics around Cybernetic Ontology, the BCL and AFOSR
http://www.thinkartlab.com/pkl/lola/AFOSR-Place-Valued-Logic.pdf

2.4.2. Smullyan's unification rules for "quantificational" constellations

3. Interplay of semiotics, logics, set theory and arithmetic

3.1. Strategy

A study of polycontextural *semiotics*, focused on semiotics alone, is not yet guaranteeing its polycontexturality. The logical, arithmetical and set theoretical status of semiotics, mono- and polycontextural, remains undetermined if its corresponding logics, arithmetic and set theory (incl. category theory) are not determined and explicitly developed as polycontextural systems.

On the other hand, what value would have a semiotic system without any chances to proof statements, studying its arithmetical, set and category theoretical properties? Until now, arithmetic, e.g., in semiotics, is not recognizing semiotical complexity but is calculating some combinatorial properties which are independent of the genuine, say triadic-trichotomous structure. Similar mismatches happens with well known inadequate combinatorial studies of morpho- and kenogrammatics.

The same situation has to be recognized for other formal systems. A formalization of polycontextural *logic* is easily reduced to monocontexturality by *arithmetization* (Gödelization) if there is not at the same time a polycontextural arithmetic at hand to defend the strategies of polycontextural logic. And obviously, because there is no *initial* origin, the *carousel* has to go through all stations of logic, arithmetic, semiotic, category and set theory, thematization, meta- and protolanguage, etc. to deliver and interplaying foundation for each other.
Proto- and meta-languagues of formal systems, as *normed* natural languages, are importand to rule the relation between natural and formal languages, especially in the case of the interpretation of formal terms for philosophical or applicative aims. If proto-language-based considerations are limiting the formal possibilities of formal constructions, the reasons for the restrictional decision should be made as explicite as possible. Also should formal possibilities be accepted which haven't yet found an interpretation.

Earlier on, there was a big philosophical topic to fight against the advent of traditional many-valued logic with the argument that the natural meta-language used to motivate and to develop many-valuedness is *a priori* two-valued. Hence, there is no escape from the two-valuedness of human thinking with the help of many-valued logic. Today, not even the question is recognized.

3.2. Sketch

For the purpose of recent introductory sketches of a *descriptive* characterization of the idea of poly-semiotics, it might be sufficient to hint to the decision to use 3-contextural subsystems of 4-contextural logics and arithmetics. Instead of the usual decomposition into elementary contextures.

Hence, from a 4-contextural logic, $Log^{(4)}$, with its six 2-contextures, $Log^{(4,2)}$, its four 3-contextures, $Log^{(4,3)}$, only the four 3-contxtural subsystems are in direct correspondence to the 4-contextural (poly)semiotics, decomposed into its 3-contextural semiotic parts. The same holds in general for the interplay between arithmetic, set theory and semiotics.

$$\mathbf{Graphematics}^{(4,3,2)} = \left(Sem^{(4,2)},\ Log^{(4.2)},\ Arith^{(4,2)},\ Set^{(4,2)}\right)$$

with

$$\mathbf{Sem}^{(4,2)} = \left(Sem^{(3,1)},\ Sem^{(3,2)},\ Sem^{(3,3)},\ Sem^{(3,4)}\right)$$

$$\mathbf{Log}^{(4.2)} = \left(Log^{(3,1)},\ Log^{(3,2)},\ Log^{(3,3)},\ Log^{(3,4)}\right)$$

$$\mathbf{Arith}^{(4,2)} = \left(Arith^{(3,1)},\ Arith^{(3,2)},\ Arith^{(3,3)},\ Arith^{(3,4)}\right)$$

$$\mathbf{Set}^{(4,2)} = \left(Set^{(3,1)},\ Set^{(3,2)},\ Set^{(3,3)},\ Set^{(3,4)}\right)$$

$\mathbf{Sem}^{(4,2)}$ is a 4 – contextural semiotics, which is realizing the paradigmatic and conceptual transformations of the 4-contextural logics $Log^{(4)}$, set theory $Set^{(4)}$ and arithmetic Arith

$^{(4)}$.

Log$^{(4,2)}$ is a 4 – contextural logic, which is realizing the structural and deductional transformations of the 4-contextural semiotics Sem $^{(4)}$, set theory Set$^{(4)}$ and arithmetic Arith$^{(4,2)}$.

Arith$^{(4,2)}$ is a 4 – contextural arithmetic, which is realizing the structural and computational transformations of the 4-contextural semiotics Sem $^{(4)}$, set theory Set$^{(4)}$ and logics Log$^{(4,2)}$.

Set$^{(4,2)}$ is a 4 – contextural set theory, which is realizing the objectional and predicational transformations of the 4-contextural semiotics Sem $^{(4)}$, arithmetic Arith$^{(4)}$ and logics Log$^{(4,2)}$.

Graphematics$^{(4,3,2)}$
is the interplay of semiotics, logics , set and category theory and arithmetic of complexity compl$^{(4,2)}$.

Interactional operators in diamond semiotics

From polylogical transjunctions to polysemiotic interactions and reflections

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Abstract

Comparing polycontextural logics and semiotics, the idea of interactionality is introduced as a further step of interaction in embedded semiotics. To achieve interactionality/reflectionality for semiotics some new concepts had been introduced. For polylogical systems, transjunctional operators are defining interactions between logics. After a sketch of polysemiotics, poly-semiotic formulations of interaction and reflection operators are introduced.

1. Semiotics and polylogics

1.1. Motivation

Transjunction, as important operators of interaction, are well known in polycontextural logics. Semiotics offers a different approach to cognitive/volitive modeling. In this paper, some steps to sketch an interactional approach in semiotics along the experiences, models and formalizations of polycontextural logic, is undertaken.[1]

The semiotic matrix is introduced as the “Cartesian product” of sub-signs (Bense, Toth).

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

"A **sub-sign** is obtained by mapping the three sign relations (.1, .2, .3) into themeselves.”
"The rows are called **triadic values** and the colomns **trichotomic values** of the matrix. In order to build a a **sign class**, one sub-sign has to be taken out of each of the three rows, the rows thus being different."
"Therefore, **sign sets** like *(3.1 3.2 1.3), *(2.1 2.2 1.2), *(1.1 1.3 3.1) are not considered sign classes.” (Toth, Ghost, p.9)

Cartesian *products* as a conceptual point of contact.[2]
The aim of polycontextural semiotics is to design a dynamic sign theory without any fixation on a special or privileged n-ary and m-adic system. Another attempt to augment the structural and architectonic flexibility of semiotics is proposed by Toth’s approach to a 3- and 4-dimensional semiotics resulting in complex topological structures. (Cf. Transit-Korridor, 2009)

1.2. Is there a privileged number for dissemination?

An introduction of the topics of polycontextural formal systems, like polylogics, poly-arithmetic or polysemiotics, has to deal with the question of a *privileged number* of a possible extension of 2-valued logics, semiotics and arithmetic. This has been thematized at different places and can't be exposed *in extenso* in this *Short Study* to Polysemiotics.[3]

1.2.1. Gunther's approach to many-valued logics

In the advent of many-valued logics there was a big run to find a privileged number of truth-values, logical functions and their semantic interpretation.
Gunther's Program. Each single value and each single logical function is entitled to have a logical meaning.
It is absurd to chase for the meaning of logical values and functions for arbitrary many-valued systems. Special value classes of some interest had been studied by logicians for 2, 3, 4, and infinite.
Hence, a method, like the arithmetic position system which is able to determine arbitrary numbers on a finite base system, has to be invented. This was Gunther's approach to many-valued place-value systems (Stellenwertlogik).
Semiotics, today, is still in a pre-decompositional, i.e. conceptionally static state of research, not necessarily in the spirit of Peirce's 'speculations'.

1.2.2. Gunther's criticism of Peirce/Bense's trinitarism

Gunther has taken the opportunity to write down and publish, what was clear at least since the advent of his place-valued logics in the 50s. That the restriction of Peirce and his decade long friend Max Bense is a heritage of Western and Christian thinking, which was conceived by Gunther as dead, at least since Nietzsche and American Cybernetics.

1.2.3. Beyond Gunther's stance on numbers

Gunther repeated the argumentation of Aristotle against a privileged number, say for his m-valued polycontextural logic, but was nevertheless the only one who himself introduced a (Neo)Pythagorean concept and some formalism of transclassic numbers, called "Philosophical numbers" (Gattungszahlen).
In short: In polycontextural logic, no special number is privileged because each number has its own specific characteristics, hence its own privilege. With this paradoxical characterization of 'privileged'/'unprivileged numbers, the whole idea of a privileged number in the traditional sense is obsolete. But this polycontextural magnitude of de-privileged privileges is based on a strategy of a finite structure, the number 4 of '*tetraktomai*', i.e. of *doing* the tetraktys, also called proemial relationship or diamond strategies. Again, this number of the *praxis* of tetraktomai, i.e. diamodization, isn't a member of any arithmetical number system.[4]

1.2.4. Toth's criticism of Bense's triadic-trichotomic semiotics

"Um es kurz zu sagen: Bense hatte - es ist fast nicht zu glauben - *n-äre und n-adische* Logiken verwechselt: Obwohl die Peirce-Bense-Semiotik triadisch ist, bleibt sie dennoch binär, und das, obwohl sie einen zehnfach ausdifferenzierten Realitätsbegriff besitzt." (Toth, Semiotische Strukturen und Prozesse, 2008). This, and other ebooks by Alfred Toth at:

2. Dissemination of semiotics

Interaction between different logical or semiotic systems is depending on the *architectonics* of the framework. In the proposed case, only two cases are presented.

First an architectonics based on a decomposition of the system into (2, 2)-subsystem.
And *second*, an architectonics based on the decomposition of the system into (3, 2)-

and subsystems.

The decomposition into (2, 2)-subsystems of 3-contextural systems corresponds to the usual polycontextural approach as introduced by Gotthard Gunther for his *place-valued* logic. It can be understood as a dissemination of *contextures* towards polycontexturality as the base for polycontextural logics in general.

This strategy of decomposing Peirce/Bense/Toth-semiotics into its dyadic-dichotomic parts opens up the possibility for a *polycontextural* approach to a logic, arithmetic and categorification of semiotics as a mediation of semiotically, logically and categorically independent elementary contextures of a mediated compound. This approach is in strict contrast to a modeling of triadic-trichotomic semiotics with methods of classical relation, set and category theory.

The (3, 3)-subsystem decomposition of 4-contextural systems, albeit it goes back to my early studies of polycontexturality, has been introduced recently for a new formalization of semiotics towards polysemiotics.

Polysemiotics are disseminating, in a first step, classical triadic-trichotomic semiotics, $Sem^{(3,2)}$, over different kenomic places to build more complex configurations.

2.1. Contextural decomposition triadic systems

2.1.1. Unary matrix$^{(3,1)}$

$Sem^{(3,1)} = (Sem^1, Sem^2, Sem^3)$

Valuation (val) of Sem :

$val(Sem^1, Sem^2, Sem^3) = (1_{1.3}, 2_{1.2}, 3_{2.3})$

Hence,

$Sem^1 = Sem_{1.3}$

$Sem^2 = Sem_{1.2}$

$Sem^3 = Sem_{2.3}$.

Matching conditions :

$(1)_1 \cong (1)_3$

$(2)_1 \cong (2)_2$

$(3)_2 \cong (3)_3$.

2.1.2. Binary matrix$^{(3,2)}$ and scheme$^{(3,2)}$

$Sem^{(3,2)} = Sem^{(3,1)} \times Sem^{(3,1)} = (1_{1.3}, 2_{1.2}, 3_{2.3}) \times (1_{1.3}, 2_{1.2}, 3_{2.3})$

$Sem^{(3,2)} = [(Sem^1 \times Sem^1), (Sem^2 \times Sem^2), (Sem^3 \times Sem^3)]$:

$val(Sem^1 \times Sem^1) = (1, 2)_1 \times (1, 2)_1$

$val(Sem^2 \times Sem^2) = (2, 3)_2 \times (2, 3)_2$

$val(Sem^3 \times Sem^3) = (1, 3)_3 \times (1, 3)_3$.

Matching conditions :

$(1, 1)_1 \cong (1, 1)_3$

$(2, 2)_1 \cong (2, 2)_2$

$(3, 3)_2 \cong (3, 3)_3$.

Scheme of Sem$^{(3,2)}$:

$$Semiotics^{(3,2)} = \begin{bmatrix} (1.1)_{1.3} \longrightarrow & (1.2)_1 & \longrightarrow & (1.3)_3 \\ \downarrow \quad \times & \downarrow & \times & \downarrow \\ (2.1)_1 \longrightarrow & (2.2)_{1.2} & \longrightarrow & (2.3)_2 \\ \downarrow \quad \times & \downarrow & \times & \downarrow \\ (3.1)_3 \longrightarrow & (3.2)_2 & \longrightarrow & (3.3)_{2.3} \end{bmatrix}$$

Sub – system decomposition of Sem$^{(3,2)}$:

$$sub-system_1 = \begin{bmatrix} (1.1) \longrightarrow & (1.2) \\ \downarrow \quad \times & \downarrow \\ (2.1) \longrightarrow & (2.2) \end{bmatrix}$$

$$sub-system_2 = \begin{bmatrix} (2.2) \longrightarrow & (2.3) \\ \downarrow \quad \times & \downarrow \\ (3.2) \longrightarrow & (3.3) \end{bmatrix}$$

$$sub-system_3 = \begin{bmatrix} (1.1) \longrightarrow & (1.3) \\ \downarrow \quad \times & \downarrow \\ (3.1) \longrightarrow & (3.3) \end{bmatrix}$$

3 – contextural semiotic matrix

$$Sem^{(3,2)} = \begin{pmatrix} MM & .1_{1.3} & .2_{1.2} & .3_{2.3} \\ 1_{1.3} & \mathbf{1.1_{1.3}} & \mathbf{1.2_1} & \mathbf{1.3_3} \\ 2_{1.2} & \mathbf{2.1_1} & \mathbf{2.2_{1.2}} & \mathbf{2.3_2} \\ 3_{2.3} & \mathbf{3.1_3} & \mathbf{3.2_2} & \mathbf{3.3_{2.3}} \end{pmatrix}$$

The mediation scheme of Semiotics$^{(3,2)}$:

$$mediation\left(Semiotics^{(3,2)}\right) = \begin{bmatrix} (1.1)_1 \longrightarrow & (2.2)_1 & \square \\ \square & \updownarrow & \\ \square & (2.2)_2 \longrightarrow & (3.3)_2 \\ | & & | \\ (1.1)_3 \longrightarrow & \longrightarrow & (3.3)_3 \end{bmatrix}$$

Chiastic structure

$$Order\ relations = \left\{ \begin{array}{l} \square(1.1)_1 \longrightarrow (2.2)_1, \\ (2.2)_2 \longrightarrow (3.3)_2, \\ (1.1)_3 \longrightarrow (3.3)_3 \end{array} \right\},$$

$$Exchange\ relation = \{(2.2)_1 \updownarrow (2.2)_2\},$$

$$Coincidence\ relations = \left\{ \begin{array}{l} (1.1)_1 - (1.1)_3, \\ (3.3)_2 - (3.3)_3 \end{array} \right\}.$$

For systems, $m = 3$, $n = 2$, the matrix$^{(3,2)}$ and scheme$^{(3,2)}$ representation coincide.

Sign classes for classical Semiotics

Sign classes are traditionally defined by :

$ZR = (a, (a \Longrightarrow b), (a \Longrightarrow b \Longrightarrow c))$

for

$a = \{1.1, 1.2, 1.3\}$

$b = \{2.1, 2.2, 2.3\}$

$c = \{3.1, 3.2, 3.3\}$

General sign relation :

$ZR = <3.x, 2.y, 1.z>$ mit $x, y, z \in \{1, 2, 3\}$

with $x \leq y \leq z$.

Resulting in the 10 sign classes :

3.1 2.1 1.1	3.1 2.3 1.3
3.1 2.1 1.2	3.2 2.2 1.2
3.1 2.1 1.3	3.2 2.2 1.3
3.1 2.2 1.2	3.2 2.3 1.3
3.1 2.2 1.3	3.3 2.3 1.3

Classical semiotics is not *mediating* its sub-systems, hence, no matching conditions are required. Therefore, classical semiotics is forced to introduce externally different *restriction* rules to determine the set of accepted sign classes.

Sign classes for $Sem^{(3,1,2)}$

$$\text{decomp}([3.a,\ 2.b,\ 1.c]) = \begin{bmatrix} 3.x, & 2.y, & -- \\ -- & 2.y, & 1.z \\ 3.x, & -, & 1.z \end{bmatrix}$$

Examples

$$\text{decomp}([3.2\ 2.1\ 1.1]) = \begin{bmatrix} 3.2, & 2.1, & -- \\ --, & 2.1, & 1.1 \\ \underline{3.2,} & \underline{--,} & \underline{1.1} \end{bmatrix}$$

$$[3.2\ \ 2.1\ \ 1.1] :: (\text{Sem}^2,\ \text{Sem}^1,\ \text{Sem}^3)$$

$$\text{decomp}([3.1\ 2.2\ 1.2]) = \begin{bmatrix} 3.1, & 2.2, & -- \\ --, & 2.2, & 1.2 \\ \underline{3.1,} & \underline{--,} & \underline{1.2} \end{bmatrix}$$

$$[3.1\ \ 2.2\ \ 1.1] :: (\text{Sem}^2,\ \text{Sem}^1,\ \text{Sem}^3)$$

$$\text{decomp}([3.2\ 2.2\ 1.2]) = \begin{bmatrix} 3.2, & 2.1, & -- \\ --, & 2.1, & 1.2 \\ \underline{3.2,} & \underline{--,} & \underline{1.2} \end{bmatrix}$$

$$[3.2\ \ 2.1\ \ 1.2] :: (\text{Sem}^2,\ \text{Sem}^1,\ \text{Sem}^3)$$

$$\text{decomp}([2.2\ 2.2\ 1.2]) = \begin{bmatrix} 2.2, & 2.1, & -- \\ --, & 2.1, & 1.2 \\ \underline{2.2,} & \underline{--,} & \underline{1.2} \end{bmatrix}$$

$$[[2.2\ \ 2.1\ \ 1.2] :: (\text{Sem}^1,\ \text{Sem}^1,\ \text{Sem}^1)$$

Translation

$a.[3.1\,2.1\,1.1] \Leftarrow; (\mathbf{3.1}_3\ \mathbf{2.1}_1\ \ \mathbf{1.1}_{1.3})$

$\Longrightarrow [(\mathbf{3.1}_3\ \mathbf{xx}\ \mathbf{1.1}_3), (\mathbf{xx}\ \mathbf{2.1}_1\ \mathbf{1.1}_1)]$

$b.[3.2\ 2.1\,1.1] \Leftarrow; (\mathbf{3.2}_2\ \mathbf{2.1}_1\ \ \mathbf{1.1}_{1.3})$

$\Longrightarrow [(\mathbf{3.2}_2\ \mathbf{xx}\ \ \mathbf{xx}), (\mathbf{xx}\ \mathbf{2.1}_1\ \mathbf{1.1}_1), (\mathbf{xx},\ \mathbf{xx},\ \mathbf{1.1}_3)],$

$c.[3.1\,2.1\,1.1] \Leftarrow; (\mathbf{3.1}_3\ \mathbf{2.1}_1\ \ \mathbf{1.1}_{1.3})$

$\Longrightarrow [(\mathbf{3.1}_3\ \mathbf{xx}\ \mathbf{1.1}_3), (\mathbf{xx}\ \mathbf{2.1}_1\ \mathbf{1.1}_1)]$

Super-operators for semiotic mappings

Independent of a specification of sign classes by accepting or abbolishing the *restriction* rules for semiotics (Toth, Ghost, p.9), mappings from sign class to sign class might be classified by the *super-operators* as they are defined in polycontextural logic:

Super – operators for semiotics

$$Sem^{(m,n)} : \left[Sem^{(m,n)}\right]_{refl,\ act} \xrightarrow[sops]{} \left[Sem^{(m,n)}\right]_{refl,\ act}$$

$$id(i, j): \quad \forall\, i, j \in s(m): \quad (Sem^{i.j}) \xrightarrow[id]{} (Sem^{i.j})$$

$$perm(i, j): \forall\, i, j \in s(m): (Sem^{i}, Sem^{j}) \xrightarrow[perm]{} (Sem^{j}, Sem^{i})$$

$$red(i, j): \forall\, i, j \in s(m): (Sem^{i}, Sem^{j}) \xrightarrow[red]{} (Sem^{i}, Sem^{i})$$

$$bif(i, j): \forall\, i, j \in s(m): (Sem^{i}, Sem^{j}) \xrightarrow[bif]{} ((Sem^{i} \,||\, Sem^{j}), Sem^{j})$$

$$repl\,(i, j): \forall\, i, j \in s(m): (Sem^{i}, Sem^{j}) \xrightarrow[repl]{} ((Sem^{i} \,|\, Sem^{i}), Sem^{j})$$

$$sops = \{id, perm, red, bif, repl\}$$

http://www.thinkartlab.com/pkl/lola/ConTeXtures.pdf

http://www.thinkartlab.com/pkl/lola/From Ruby to Rudy.pdf **§ 11.3**

Examples

$$id_{1.2.3}: \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix}$$

$$repl_{1.1.1.1}: \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} S_1 & \square & \square \\ S_1 & S_2 & \square \\ S_{1.1} & \square & S_3 \end{pmatrix}$$

$$perm_{1-2}: \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} S_2 & \square & \square \\ \square & S_1 & \square \\ \square & \square & S_3 \end{pmatrix}$$

$$red_{1-2}: \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} S_1 & \square & \square \\ \square & S_1 & \square \\ \square & \square & S_3 \end{pmatrix}$$

$$bif_{1-2}: \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} S_1 & S_2 & S_2 \\ S_2 & S_2 & \square \\ S_2 & \square & S_3 \end{pmatrix}$$

Considering the 3 principles of semiotic restrictions, i.e. triadic diversity, degenerative triadic order and trichotomic inclusion, *permutation* and *reduction* operations might add some more structure to semiotics without surpassing its general framework. The operation of permutation, which had a case as a *dualisation* (Bense) only, is complemented by Toth's concept of *transpositions*.

2.1.3. Ternary matrix$^{(3,3)}$ and scheme$^{(3,3)}$

$$\text{Sem}^{(3,3)} = \left(\text{Sem}^{(3,1)} \times \text{Sem}^{(3,1)} \times \text{Sem}^{(3,1)}\right) = \begin{bmatrix} \left(\text{Sem}^1 \times \text{Sem}^1 \times \text{Sem}^1\right), \\ \left(\text{Sem}^2 \times \text{Sem}^2 \times \text{Sem}^2\right), \\ \left(\text{Sem}^3 \times \text{Sem}^3 \times \text{Sem}^3\right) \end{bmatrix}$$

$$\text{val}\left(\text{Sem}^{(3,3)}\right) = \begin{bmatrix} \left(1_{1.3}, 2_{1.2}, 3_{2.3}\right) \times \\ \left(1_{1.3}, 2_{1.2}, 3_{2.3}\right) \times \\ \left(1_{1.3}, 2_{1.2}, 3_{2.3}\right) \end{bmatrix}:$$

$$\text{val}\left(\text{Sem}^1 \times \text{Sem}^1 \times \text{Sem}^1\right) = (1, 2)_1 \times (1, 2)_1 \times (1, 2)_1$$
$$\text{val}\left(\text{Sem}^2 \times \text{Sem}^2 \times \text{Sem}^2\right) = (2, 3)_2 \times (2, 3)_2 \times (2, 3)_2$$
$$\text{val}\left(\text{Sem}^3 \times \text{Sem}^3 \times \text{Sem}^3\right) = (1, 3)_3 \times (1, 3)_3 \times (1, 3)_3$$

Matching conditions:

$$(1, 1, 1)_1 \cong (1, 1, 1)_3$$
$$(2, 2, 2)_1 \cong (2, 2, 2)_2$$
$$(3, 3, 3)_2 \cong (3, 3, 3)_3.$$

Sign classes Sem$^{(3,3)}$

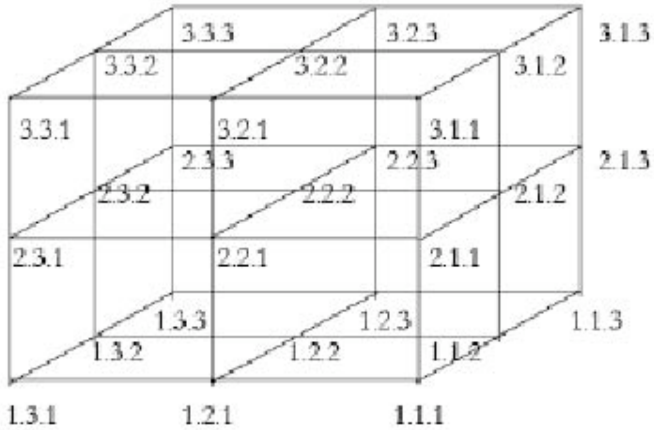
$$(3.1\ 2.2\ 1.3) \in \text{Sem}^{(3,2)}$$
$$(3.1\ .3\ \ 2.1\ .3\ \ 1.3\ .3) \in \text{Sem}^{(3,3)}$$

3 – contextural semiotic matrix $^{(3,3)}$

$$\text{Sem}^{(3,3)} = \begin{bmatrix} \begin{pmatrix} \text{MM} & .1_{1.3} & .2_{1.2} & .3_{2.3} \\ 1_{1.3} & \mathbf{1.1_{1.3}} & \mathbf{1.2_1} & \mathbf{1.3_3} \\ 2_{1.2} & \mathbf{2.1_1} & \mathbf{2.2_{1.2}} & \mathbf{2.3_2} \\ 3_{2.3} & \mathbf{3.1_3} & \mathbf{3.2_2} & \mathbf{3.3_{2.3}} \end{pmatrix} \\ \begin{pmatrix} 1_{1.3} & \mathbf{1.1_{1.3}} & \mathbf{1.2_1} & \mathbf{1.3_3} \\ 2_{1.2} & \mathbf{2.1_1} & \mathbf{2.2_{1.2}} & \mathbf{2.3_2} \\ 3_{2.3} & \mathbf{3.1_3} & \mathbf{3.2_2} & \mathbf{3.3_{2.3}} \end{pmatrix} \\ \begin{pmatrix} 1_{1.3} & \mathbf{1.1_{1.3}} & \mathbf{1.2_1} & \mathbf{1.3_3} \\ 2_{1.2} & \mathbf{2.1_1} & \mathbf{2.2_{1.2}} & \mathbf{2.3_2} \\ 3_{2.3} & \mathbf{3.1_3} & \mathbf{3.2_2} & \mathbf{3.3_{2.3}} \end{pmatrix} \end{bmatrix}$$

$$\sum = 27$$

Different presentation of the matrix$^{(3,3)}$
(Toth, Strukturen + Processe, *p*.36, 2008)



Semiotics scheme for Sem$^{(3,3)}$

Sem$^{(3,3)}$ – scheme

$(1.1.1)_{1.3}$	⟶	(1.1.2)	⟶	(1.2.1)	⟶	(1.2.2)	⟶	(1.1.3)	⟶	(1.3.1)	⟶	(1.3.3)
↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓
(2.1.1)	⟶	(2.1.2)	⟶	(2.2.1)	⟶	$(2.2.2)_{1.2}$	⟶	(2.2.3)	⟶	(2.3.2)	⟶	(2.3.3)
↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓
(3.1.1)	⟶	(3.1.3)	⟶	(3.3.1)	⟶	(3.2.2)	⟶	(3.2.3)	⟶	(3.3.2)	⟶	$(3.3.3)_{2.3}$

$\sum = 21$

Combinatorics

Matrix: 3 x3x3 = 27

Scheme: $(3 \text{x} 3 \text{x} 3)_{/MC} = (3 \text{x} 3 \text{x} 3) - 6 = 21$

$\{1, 2, 3\} = \{(1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)\}$

For $m = 3$, $n = 2$: $|\text{matrix}| = |\text{scheme}|$

In general: $m \geq 2$, $n \geq 3$: $|\text{matrix}| > |\text{scheme}|$.

Sub – system decomposition of Sem$^{(3, 3)}$:

sub – system$_1$ =

$(1, 1) \text{x} (1) = (1.1.1)$

$(1, 1) \text{x} (2) = (1.1.2)$

$(1, 2) \text{x} (1) = (1.2.1)$

$(1, 2) \text{x} (2) = (1.2.2)$

$(2, 1) \text{x} (1) = (2.1.1)$

$(2, 1) \text{x} (2) = (2.1.2)$

$(2, 2) \text{x} (1) = (2.2.1)$

$(2, 2) \text{x} (2) = (2.2.2)$.

The same for sub – system$_2$ and sub – system$_3$.

sub – systems Sem$^{(3, 3)}$:

$$\text{sub-system}_1 = \begin{bmatrix} (1.1.1) \longrightarrow & (1.1.2) & \longrightarrow & (1.2.1) & \longrightarrow & (1.2.2 \\ \downarrow \quad x & \downarrow & x & \downarrow & x & \downarrow \\ (2.1.1) \longrightarrow & (2.1.2) & \longrightarrow & (2.2.1) & \longrightarrow & (2.2.2) \end{bmatrix}$$

$$\text{sub-system}_2 = \begin{bmatrix} (2.2.2) \longrightarrow & (2.2.3) & \longrightarrow & (2.3.2) & \longrightarrow & (2.3.3 \\ \downarrow \quad x & \downarrow & x & \downarrow & x & \downarrow \\ (3.2.2) \longrightarrow & (3.2.3) & \longrightarrow & (3.3.2) & \longrightarrow & (3.3.3) \end{bmatrix}$$

$$\text{sub-system}_3 = \begin{bmatrix} (1.1.1) \longrightarrow & (1.1.3) & \longrightarrow & (1.3.1) & \longrightarrow & (1.3.3 \\ \downarrow \quad x & \downarrow & x & \downarrow & x & \downarrow \\ (3.1.1) \longrightarrow & (3.1.3) & \longrightarrow & (3.3.1) & \longrightarrow & (3.3.3) \end{bmatrix}$$

Possible constellation of the *matrix* of Sem$^{(3, 3)}$, like (1, 2, 3), (1, 3, 2), that is, constellations with (i, j, k), i!=j!=k, for the ternary function AxBxC of the matrix are not decomposable into Sem$^{(2, 2)}$ – subsystems of the semiotic *scheme*.

For systems m, n>=3, well known combinatorial problems of decomposition into

sub-systems have to be solved (Kaehr, Mahler, § 9, 1993). http://www.thinkartlab.com/pkl/media/mg-book.pdf
As a consequence of the matching conditions of decomposition, the semiotic system $Sem^{(3,3)}$ is not delivering $3^9 = 19683$ different decomposable semiotic functions as demanded by (Toth, Gost, p. 9, 2008).

$Sem^{(4,3)}$ – scheme

(1.1.1)	→	(1.1.2)	→	(1.2.1)	→	(1.2.2)	→	(1.1.3)	→	(1.3.1)	→	(1.3.3)	→	(1.1.4)	→	(1.4.1)	→	(1.4
↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓
(2.1.1)	→	(2.1.2)	→	(2.2.1)	→	(2.2.2)	→	(2.2.3)	→	(2.3.2)	→	(2.3.3)	→	(2.2.4)	→	(2.4.2)	→	(2.4
↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓
(3.1.1)	→	(3.1.3)	→	(3.3.1)	→	(3.2.2)	→	(3.2.3)	→	(3.3.2)	→	(3.3.3)	→	(3.3.4)	→	(3.4.3)	→	(3.4
↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓	□	↓
(4.1.1)	→	(4.1.4)	□	(4.4.1)	→	(4.2.2)	→	(4.2.4)	→	(4.4.2)	→	(4.4.3)	→	(4.3.4)	→	(4.4.3)	→	(4.4

$\sum$ scheme = 40, matrix = 4 x4x4 = 64

2.2. Semiotic decomposition of tetradic systems

2.2.1. Unary tetradic matrix

$$Sem^{(4,1,3)} = \left(Sem^1, Sem^2, Sem^3, Sem^4\right)$$

$$val\left(Sem^{(4,1,3)}\right) = \begin{array}{|l|} 1_1 \longrightarrow 2_1 \longrightarrow 3_1 \longrightarrow x \\ x \longrightarrow 2_2 \longrightarrow 3_2 \longrightarrow 4_2 \\ 1_3 \longrightarrow 2_3 \longrightarrow x \longrightarrow 4_3 \\ 1_4 \longrightarrow x \longrightarrow 3_4 \longrightarrow 4_4 \end{array}$$

$$val\left(Sem^{(4,1,3)}\right) = \left(1_{1.3.4}, 2_{1.2.3}, 3_{1.2.4}, 4_{2.3.4}\right).$$

2.2.2. Binary tetradic matrix

$Sem^{(4,2,3)} = Sem^{(4,1,3)} \times Sem^{(4,1,3)} =$
$\left[\left(Sem^1 \times Sem^1\right), \left(Sem^2 \times Sem^2\right), \left(Sem^3 \times Sem^3\right), \left(Sem^4 \times Sem^4\right)\right].$

$val\left(Sem^{(4,1,3)} \times Sem^{(4,1,3)}\right) =$

$\left(1_{1.3.4}, 2_{1.2.3}, 3_{1.2.4}, 4_{2.3.4}\right) \times \left(1_{1.3.4}, 2_{1.2.3}, 3_{1.2.4}, 4_{2.3.4}\right)$

with :

$val\left(Sem^1 \times Sem^1\right) = (1, 2, 3)_1 \times (1, 2, 3)_1$

$val\left(Sem^2 \times Sem^2\right) = (2, 3, 4)_2 \times (2, 3, 4)_2$

$val\left(Sem^3 \times Sem^3\right) = (1, 2, 4)_3 \times (1, 2, 4)_3$

$val\left(Sem^4 \times Sem^4\right) = (1, 3, 4)_4 \times (1, 3, 4)_4.$

As presented at " Semiotics in Diamonds " *a* coloring of the subsystems might emphazise

http : // www.thinkartlab.com / pkl / lola / Semiotics – in – Diamonds / Semiotics – in – Diamonds.html

$$val\left(Sem^{(4,1,3)} \times Sem^{(4,1,3)}\right) =$$

$$sem^1 \times sem^1 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1.3.4} & 1.2_{1} & 1.3_{1} & 1.4 \\ 2 & 2.1_{1} & 2.2_{1.2.3} & 2.3_{1} & 2.4 \\ 3 & 3.1_{1} & 3.2_{1} & 3.3_{1.2.4} & 3.4 \\ 4 & 4.1 & 4.2 & 4.3 & 4.4 \end{pmatrix}$$

$$sem^2 \times sem^2 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1} & 1.2_{1} & 1.3_{1} & 1.4 \\ 2 & 2.1_{1} & 2.2_{1.2} & 2.3_{1.2} & 2.4_{2} \\ 3 & 3.1_{1} & 3.2_{1.2} & 3.3_{1.2} & 3.4_{2} \\ 4 & 4.1 & 4.2_{2} & 4.3_{2} & 4.4_{2} \end{pmatrix}$$

$$sem^3 \times sem^3 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1.3} & 1.2_{1.3} & 1.3_{1} & 1.4_{3} \\ 2 & 2.1_{1.3} & 2.2_{1.2.3} & 2.3_{1.2} & 2.4_{2.3} \\ 3 & 3.1_{1} & 3.2_{1.2} & 3.3_{1.2} & 3.4_{2} \\ 4 & 4.1_{3} & 4.2_{3.2} & 4.3_{2} & 4.4_{3.2} \end{pmatrix}$$

$$sem^4 \times sem^4 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1.3.4} & 1.2_{1.3} & 1.3_{1.4} & 1.4_{3.4} \\ 2 & 2.1_{1.3} & 2.2_{1.2.3} & 2.3_{1.2} & 2.4_{2.3} \\ 3 & 3.1_{1.4} & 3.2_{1.2} & 3.3_{1.2.4} & 3.4_{2.4} \\ 4 & 4.1_{3.4} & 4.2_{3.2} & 4.3_{2.4} & 4.4_{2.3.4} \end{pmatrix}$$

4 – contextural semiotic matrix

$$Sem^{(4,2,3)} = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1.3.4} & 1.2_{1} & 1.3_{1.4} & 1.4_{3.4} \\ 2 & 2.1_{1} & 2.2_{1.2.3} & 2.3_{2} & 2.4_{2.3} \\ 3 & 3.1_{1.4} & 3.2_{2} & 3.3_{1.2.4} & 3.4_{2.4} \\ 4 & 4.1_{3.4} & 4.2_{3.2} & 4.3_{2.4} & 4.4_{2.3.4} \end{pmatrix}$$

Sign classes for $Sem^{(4,1,2)}$

$ZR^{(3,2)} = <3.x,\ 2.y,\ 1.z>$ with $x, y, z \in \{1, 2, 3\}$

$ZR^{(4,2)} = (ZR^1, ZR^2, ZR^3, ZR^4)$:

$ZR^1 = <3.x,\ 2.y,\ 1.z,\ -->$

$ZR^2 = <--, 3.x,\ 2.y,\ 1.z>$

$ZR^3 = <3.x,\ 2.y,\ --,\ 1.z>$

$ZR^4 = <3.x,\ --,\ 2.y,\ 1.z>$

$$\text{decomp}(ZR^{(4,2)}) = \begin{bmatrix} 3.x, & 2.y, & 1.z, & -- \\ --, & 3.x, & 2.y, & 1.z \\ 3.x, & 2.y, & --, & 1.z \\ 3.x, & --, & 2.y, & 1.z \end{bmatrix}$$

$$\text{comp}\left(\begin{bmatrix} 3.x, & 2.y, & 1.z, & -- \\ --, & 3.x, & 2.y, & 1.z \\ 3.x, & 2.y, & --, & 1.z \\ 3.x, & --, & 2.y, & 1.z \end{bmatrix}\right) = [4.a,\ 3.b,\ 2.c,\ 1.d]$$

Matching conditions

$(3.x)_1 \cong (3.x)_3 \cong (3.x)_4$

$(2.y)_1 \cong (3.x)_2 \cong (2.y)_3$

$(1.z)_1 \cong (2.y)_2 \cong (2.y)_4$

$(1.z)_2 \cong (1.z)_3 \cong (1.z)_4$.

Each sign class of $Sem^{(4,2)}$ is decomposable into its 4 $Sem^{(3,2)}$ sign classes.

Example

$[4.3\ 3.2\ 2.1\ 1.1] \in Sem^{(4,2)}$

$$\text{decomp}([4.3\ 3.2\ 2.1\ 1.1]) = \begin{bmatrix} (4.3, 3.2, 2.1, --) \in Sem^1 \\ (--, 3.2, 2.1, 1.1) \in Sem^2 \\ (4.3, 3.2, --, 1.1) \in Sem^3 \\ (4.3, --, 2.1, 1.1) \in Sem^4 \end{bmatrix}$$

$$[4.3\ 3.2\ 2.1\ 1.1] \in Sem^{(4,2)}$$

2.2.3. (Some) Sign classes for $Sem^{(4,2)}$

Class A = (4.1)

```
4.1 3.1 2.1 1.1
4.1 3.1 2.1 1.2
4.1 3.1 2.1 1.3
4.1 3.1 2.1 1.4
4.1 3.1 2.2 1.2
4.1 3.1 2.2 1.3
4.1 3.1 2.2 1.4

4.1 3.2 2.2 1.2
4.1 3.2 2.2 1.3
4.1 3.2 2.2 1.4

4.1 3.2 2.2 1.2
4.1 3.2 2.2 1.3
4.1 3.2 2.2 1.4

4.1 3.2 2.3 1.3
4.1 3.2 2.3 1.4
4.1 3.3 2.3 1.4

4.1 3.2 2.4 1.4
4.1 3.3 2.4 1.4
4.1 3.4 2.4 1.4
```

Class B = (4.2)

```
4.2 3.1 2.1 1.1
4.2 3.1 2.1 1.2
4.2  3.1 2.1 1.3
4.1  3.1 2.1 1.4
4.2 3.1 2.2 1.2
4.2 3.1 2.2 1.3
4.2 3.1 2.2 1.4

4.2 3.2 2.2 1.2
4.2 3.2 2.2 1.3
4.2 3.2 2.2 1.4

4.2 3.2 2.2 1.2
4.2 3.2 2.2 1.3
4.2 3.2 2.2 1.4

4.2 3.2 2.3 1.3
4.2 3.2 2.3 1.4
```

Class C = (4.3)

```
4.3 3.3 2.3 1.3
4.3 3.3 2.3 1.4
4.3 3.3 2.4 1.4
4.3 3.4 2.4 1.4
```

Class D = (4.4)

```
4.4 3.4 2.4 1.4
```

Combinatorics

$|\text{ClassA}| = 5 \times 3 = 15$

$|\text{Class B}| = 6 \times 3 + 1 = 19$

$|\text{Class C}| = 4$

$|\text{Class D}| = 1$

Total = 1 + 15 + 19 = 35

2.3. Interplay of semiotics, logics and arithmetic

A study of polycontextural semiotics, focused on semiotics alone, is not yet guaranteeing its polycontexturality. The logical and arithmetical status of semiotics, mono- and polycontextural, remains undetermined if its corresponding logics are not determined.
There are many ways open to formalize, logically and arithmetically, semiotics and polysemiotics. Good candidates are the logics from the modal logic pool. Nevertheless, they have all to be classified as mono-contextural.

For the purpose of this introductory sketch of a *descriptive* characterization of the idea of poly-semiotics, it might be sufficient to hint to the decision to use 3-contextural subsystems of 4-contextural logics and arithmetics. Instead of the usual decomposition into elementary contextures.

As a consequence, it turns out that the apparatus of classical category theory is not adequate to formalize semiotics and polysemiotics.

Hence, from a 4-contextural logic, $Log^{(4)}$, with its six 2-contextures, $Log^{(4,2)}$, its four 3-contextures, $Log^{(4,3)}$, only the four 3-contxtural subsystems are in direct correspondence to the 4-contextural (poly)semiotics, decomposed into its 3-contextural semiotic parts.

$$\mathbf{Graphematics}^{(4,3,2)} = \left(Sem^{(4,2)},\ Log^{(4.2)},\ Arith^{(4,2)}\right)$$

with

$$\mathbf{Sem}^{(4,2)} = \left(Sem^{(3,1)},\ Sem^{(3,2)},\ Sem^{(3,3)},\ Sem^{(3,4)}\right)$$

$$\mathbf{Log}^{(4.2)} = \left(Log^{(3,1)},\ Log^{(3,2)},\ Log^{(3,3)},\ Log^{(3,4)}\right)$$

$$\mathbf{Arith}^{(4,2)} = \left(Arith^{(3,1)},\ Arith^{(3,2)},\ Arith^{(3,3)},\ Arith^{(3,4)}\right)$$

$Sem^{(4,2)}$ is a 4 – contextural semiotics, which is realizing the paradigmatic and conceptual transformations of the 4-contextural logics $Log^{(4)}$ and arithmetic $Arith^{(4)}$.

$Log^{(4,2)}$ is a 4 – contextural logic, which is realizing the structural and deductional transformations of the 4-contextural semiotics $Sem^{(4)}$ and arithmetic $Arith^{(4,2)}$.

$Arith^{(4,2)}$ is a 4 – contextural arithmetic, which is realizing the structural and computational transformations of the 4-contextural semiotics $Sem^{(4)}$ and logics $Log^{(4,2)}$.

$Graphematics^{(4,3,2)}$ is the interplay of semiotics, logics and arithmetic of complexity $compl^{(4,2)}$.

2.4. Multi-dimensional and polycontextural semiotics

2.4.1. Toth's multi-dimensional semiotics

Scheme of *a* 3 – dimensional semiotics

Toth introduced in (Transit – Korridor, 2009) *a* 3 – dimensional sign relation

$$3 - ZKL = ((a.3.b)(c.2.d)(e.1.f))$$

with its 27 variations.

Examples

(1.3.3 1.2.2 1.1.1) (1.3.1 1.2.2 1.1.3)

(2.3.3 2.2.2 2.1.1) (2.3.1 2.2.2 2.1.3)

(3.3.3 3.2.2 3.1.1) (3.3.1 3.2.2 3.1.3).

Scheme of a Transit – Korridor :

$TK = \{<a.3.3.b \;\; c.2.2.d \;\; e.1.1.f>\}$

with $a, c, e \in \{1, 2, 3\}$ and $b, d, f \in \{1, 2, 3, 4\}$.

In general (I guess) :

$m - ZKL =$

$((a.3.3 \ldots 3 b_1 b_2 \ldots b_m)(c.2.2 \ldots 2 d_1 d_2 \ldots d_m)(e.1.1 \ldots 1 f_1 f_2 \ldots f_m))$

with $a, c, e \in \{1, 2, 3\}$ and $b, d, f \in \{1, 2, \ldots, m\}$.

2.4.2. Polycontextural (uni-dimensional) semiotics

$Sem^{(4,2)} - scheme = [4.a, 3.b, 2.c, 1.d]$

with $a, b, c, d \in \{1, 2, 3, 4\}$.

Sign classes for $Sem^{(4)}$ are defined by :

$ZR = (a, (a \Longrightarrow b), (a \Longrightarrow b \Longrightarrow c), (a \Longrightarrow b \Longrightarrow c \Longrightarrow d))$

for

$a = \{1.1, 1.2, 1.3, 1.4\}$

$b = \{2.1, 2.2, 2.3, 2.4\}$

$c = \{3.1, 3.2, 3.3, 3.4\}$

$d = \{4.1, 4.2, 4.3, 4.4\}$

General sign relation for $ZR^{(4)}$:

$ZR^{(4)} = <4.u, 3.x, 2.y, 1.z>$ with $u, x, y, z \in \{1, 2, 3, 4\}$

and $u \leq x \leq y \leq z$

In general :

$Sem^{(m,2)} = [m.a_m, m-1.a_{m-1}, \ldots, 2.a_2, 1.a_1]$

2.4.3. Comparisons

$2 - ZKL = <3.x, 2.y, 1.z>$

with $x, y, z \in \{1, 2, 3\}$ and $x \leq y \leq z. \Longrightarrow$ 10 sign classes

$3 - ZKL = ((a.3.b)(c.2.d)(e.1.f)) \Longrightarrow$ 27 sign classes

$Sem^{(4,2)} - scheme = [4.a, 3.b, 2.c, 1.d] \Longrightarrow 35 (??)$

3. Interactivity in poly-semiotics

3.1. Interactions between (2,2)-subsystems of $Sem^{(3,2)}$

Interactional semiotic functions

Interactions, in the form of transjunctions are of great importance in polycontextural logic
In fact, from a combinatorial point of view, most polylogical functions are transjunctional.
Therefore, they should deserve a prominent place in a polycontextural semiotics.
At this place, not more than a short hint can be given.

(transjunction, junction, junction) $_{Polylogic}$
$\Longrightarrow$
(interaction, action, action) $_{Polysemiotics}$

$$Sem^{(3,2)}_{(inter,\ act,\ act)} = \left(Sem^{(3,1)}\ x\ Sem^{(3,1)}\right)_{(inter,\ act,\ act)} =$$

$$\left[\left(\left(Sem^{1}\ x\ Sem^{\ 1}\right) \| \left(Sem^{2.3}\ x\ Sem^{2.3}\right)\right), \left(Sem^{2}\ x\ Sem^{\ 2}\right), \left(Sem^{3}\ x\ Sem^{\ 3}\right)\right]:$$

$$val\left(inter_{1}\left(Sem^{1}\ x\ Sem^{\ 1}\right)\right) = (1,\ 1)_{1}\ x\ (2,\ 2)_{1} \| (2,\ 3)_{2.3}\ x\ (3,\ 2)_{2.3},$$

$$val\left(act_{2}\left(Sem^{2}\ x\ Sem^{\ 2}\right)\right) = (2,\ 3)_{2}\ x\ (2,\ 3)_{2},$$

$$val\left(act_{3}\left(Sem^{3}\ x\ Sem^{\ 3}\right)\right) = (1,\ 3)_{3}\ x\ (1,\ 3)_{3},$$

$$\text{with } (2,\ 3)_{2.3}\ x\ (3,\ 2)_{2.3} = (2_{2},\ 3_{2.3})\ x\ (3_{2.3},\ 2_{2})$$

Operational notation

$$Op_{(inter,\ act,\ act)} : \left[\left(Sem^{1}\ x\ Sem^{\ 1}\right), \left(Sem^{2}\ x\ Sem^{\ 2}\right), \left(Sem^{3}\ x\ Sem^{\ 3}\right)\right]$$

$$\Longrightarrow$$

$$\left[\left(\left(Sem^{1}\ x\ Sem^{\ 1}\right) \| \left(Sem^{2.3}\ x\ Sem^{2.3}\right)\right), \left(Sem^{2}\ x\ Sem^{\ 2}\right), \left(Sem^{3}\ x\ Sem^{\ 3}\right)\right]$$

or short :

$$Op_{(inter,\ act,\ act)} : Sem^{(3,2)} \Longrightarrow bif_{1.2.3}\left(id_{2.3}\left(Sem^{(3,2)}\right)\right)$$

$$[\textbf{inter, act, act}] \equiv [\blacklozenge,\ \circ,\ \circ]$$

$Sem^{(3,2,2)}_{(inter,\ act,\ act)} =$

[◆, ∘, ∘]	1	2	3
1	$1.1_{1.3}$	$2.3_{2.3}$	1.3_{3}
2	$3.2_{2.3}$	$2.2_{1.2}$	2.3_{2}
3	3.1_{3}	3.2_{2}	$3.3_{2.3}$

Different modi of interaction with Sem^1 :

$[\blacklozenge_1, \circ, \circ]$	1	2	3
1	$id_{1.3}$	$\alpha_{2.3}$	α_3
2	$\alpha^\circ_{2.3}$	$id_{1.2}$	α_2
3	α°_3	α°_2	$id_{2.3}$

$[\blacklozenge_2, \circ, \circ]$	1	2	3
1	$id_{1.3}$	$\alpha^\circ_{2.3}$	α_3
2	$\alpha_{2.3}$	$id_{1.2}$	α_2
3	α°_3	α°_2	$id_{2.3}$

$[\blacklozenge_3, \circ, \circ]$	1	2	3
1	$id_{1.3}$	$\alpha^\circ_{2.3}$	α_3
2	$\alpha^\circ_{2.3}$	$id_{1.2}$	α_2
3	α°_3	α°_2	$id_{2.3}$

$[\blacklozenge_4, \circ, \circ]$	1	2	3
1	$id_{1.3}$	$\alpha_{2.3}$	α_3
2	$\alpha_{2.3}$	$id_{1.2}$	α_2
3	α°_3	α°_2	$id_{2.3}$

General distribution tables for [inter, act, act]

$[\blacklozenge, \circ, \circ]$	O_1	O_2	O_3
M_1	sem_1	x	x
M_2	$trans_2$	sem_2	x
M_3	$trans_3$	x	sem_3

3.2. Interactions between (3, 3)-subsystems of $Sem^{(4,2,3)}$

$$Sem^{(4,2,3)}_{(inter,act,act,act,act,inter)} =$$

$$\left[\left((Sem^1 \ x\, Sem^1) \| (Sem^{2.3} \ x\, Sem^{2.3})\right),\right.$$

$$(Sem^2 \ x\, Sem^2),\ (Sem^3 \ x\, Sem^3),$$

$$\left.\left((Sem^4 \ x\, Sem^4) \| (Sem^{2.3} \ x\, Sem^{2.3})\right)\right].$$

$$val\left(Sem^{(4,1,3)} \ x\, Sem^{(4,1,3)}\right) =$$

$$(1_{1.3.4},\ 2_{1.2.3},\ 3_{1.2.4},\ 4_{2.3.4})\, x\, (1_{1.3.4},\ 2_{1.2.3},\ 3_{1.2.4},\ 4_{2.3.4})$$

with:

$$val(Sem^1 \ x\, Sem^1) = (1, 2, 3)_1 \ x\, (1, 2, 3)_1 \| \left((2, 3, 4)_2 \ x\, (2, 3, 4)_2,\ (1, 2, 4)_3 \ x\, (1, 2, 4)_3\right)$$

$$val(Sem^2 \ x\, Sem^2) = (2, 3, 4)_2 \ x\, (2, 3, 4)_2$$

$$val(Sem^3 \ x\, Sem^3) = (1, 2, 4)_3 \ x\, (1, 2, 4)_3$$

$$val(Sem^4 \ x\, Sem^4) = (1, 3, 4)_4 \ x\, (1, 3, 4)_4 \| \left((2, 3, 4)_2 \ x\, (2, 3, 4)_2,\ (1, 2, 4)_3 \ x\, (1, 2, 4)_3\right)$$

$$Op_{(inter,act,act,act,act,inter)} : Sem^{(4,2)} \Longrightarrow bif_{1.2.3}\left(id_{2.3.4.5}\left(bif_{6.3.2}\left(Sem^{(4,2)}\right)\right)\right)$$

4 – contextural semiotic matrix

$Sem^{(4,2,3)} =$

MM	1	2	3	4
1	$1.1_{1.3.4}$	$\mathbf{2.3_{2.3}}$	$1.3_{1.4}$	$1.4_{3.4}$
2	$\mathbf{3.2_{2.3}}$	$2.2_{1.2.3}$	2.3_{2}	$2.4_{2.3}$
3	$3.1_{1.4}$	3.2_{2}	$3.3_{1.2.4}$	$\mathbf{2.3_{2.3}}$
4	$4.1_{3.4}$	$4.2_{3.2}$	$\mathbf{3.2_{2.3}}$	$4.4_{2.3.4}$

[◆, ○, ○, ○, ○, ◆]	O_1	O_2	O_3	O_4
M_1	sem_1	x	x	x
M_2	$trans_2$	sem_2	x	$trans_2$
M_3	$trans_3$	x	sem_3	$trans_3$
M_4	x	x	x	sem_4

3.3. Interaction and reflectionality

Following the concepts and methods developed in *"ConTeXtures. Programming Dynamic Complexity"* (Kaehr, 2005), short hints of their application to disseminated semiotics are given. Both, the bracket and the table notation are emphazing the architectonic structure of reflection and interaction.

Reflections in $Sem^{(3,2)}$

$$\left[\begin{array}{l} \left[\begin{array}{l} O_1 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ \quad (Sem_{110}) \end{pmatrix} \end{array}\right] \\ \left[\begin{array}{l} O_2 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ \quad (Sem_{222}) \end{pmatrix} \end{array}\right] \\ \left[\begin{array}{l} O_3 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ \quad (Sem_{033}) \end{pmatrix} \end{array}\right] \end{array}\right]$$

[reflection]	O_1	O_2	O_3
M_1	sem_1	sem_2	x
M_2	sem_1	sem_2	sem_3
M_3	x	sem_2	sem_3

Iterative reflection in $Sem^{(3,2)}$

$$\left[\begin{array}{l} \left[\begin{array}{l} O_1 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ (Sem_{110}) \begin{pmatrix} M_1\ M_2\ M_3 \\ (Sem_{110}) \begin{pmatrix} M_1\ M_2\ M_3 \\ (Sem_{110}) \begin{pmatrix} M_1\ M_2\ M_3 \\ (Sem_{110}) \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{array}\right] \\ \left[\begin{array}{l} O_2 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ \quad (Sem_{222}) \end{pmatrix} \end{array}\right] \\ \left[\begin{array}{l} O_3 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ \quad (Sem_{033}) \end{pmatrix} \end{array}\right] \end{array}\right]$$

[reflections]	O_1	O_2	O_3
M_1	$sem_{1.1.1.1}$	sem_2	x
M_2	$sem_{1.1.1.1}$	sem_2	sem_3
M_3	x	sem_2	sem_3

Interplay between interactionality and reflectionality in $Sem^{(3,2)}$

$$\begin{bmatrix} \begin{bmatrix} O_1 \\ \begin{pmatrix} M_1 \; M_2 \; M_3 \\ \begin{pmatrix} Sem_{111} \end{pmatrix} \end{pmatrix} \end{bmatrix} \\ \begin{bmatrix} O_2 \\ \begin{pmatrix} M_1 \begin{pmatrix} \begin{pmatrix} Sem_{100} \end{pmatrix} \\ \begin{pmatrix} M_2 \begin{pmatrix} M_3 \\ \begin{pmatrix} Sem_{003} \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{pmatrix} \\ \begin{pmatrix} Sem_{222} \end{pmatrix} \end{pmatrix} \end{bmatrix} \\ \begin{bmatrix} O_3 \\ \begin{pmatrix} M_1 \; M_2 \; M_3 \\ \begin{pmatrix} Sem_{033} \end{pmatrix} \end{pmatrix} \end{bmatrix} \end{bmatrix}$$

[interact, reflect]	O_1	O_2	O_3
M_1	sem_1	$sem_{2.1}$	x
M_2	sem_1	$sem_{2.0}$	sem_3
M_3	sem_1	$sem_{2.3}$	sem_3

Reflections in $Sem^{(4,2)}$

[reflection]	O_1	O_2	O_3	O_4
M_1	sem_1	x	x	x
M_2	sem_1	sem_2	x	sem_4
M_3	sem_1	x	sem_3	sem_4
M_4	x	x	x	sem_4

Interactions and reflections in $Sem^{(4,2)}$

[inter, refl]	O_1	O_2	O_3	O_4
M_1	sem_1	x	sem_3	x
M_2	$trans_2$	sem_2	sem_3	$trans_2$
M_3	$trans_3$	sem_2	sem_3	$trans_3$
M_4	x	x	x	sem_4

4. Logification of semiotics

On the base of the introduced concepts for semiotic interactions interesting operations, rules and transformations (deductions) might be studied. Much of the work in semiotics and pre-semiotics is mainly *descriptive*, introducing its concepts and demonstrating some transformations. But there is nearly no work done for a kind of a deductive treatment in the semiotic field. This goes back mainly to the fact that semiotics in general has not yet accepted the concept of a polycontextural deductional system. On the other hand, a logical and deductive treatment of a genuine triadic-trichotomic semiotics by a *classical* logical approach goes hand in hand with a *reduction* procedure of the triadic-trichotomic complexity of classical semiotics to a dyadic-dichotomic model.

Logification of semiotics becomes relevant if we want to study semiotic operations in poly-semiotic systems. Like for logical systems, we can ask for

a specific state of the system in transformation.
Transformations might produce conflicting results, similar to contradictions in logic. Such irregularities can be easily detected by the *tableaux* method for decomposed semiotic constellations. Therefore, deductive aspects, semiotic model theory ('semantics'), proof theory, etc. of semiotic systems are accessible to be studied for their specific characteristics.

$$Sem^{(3,2,2)}_{(inter,\, act,\, act)} = \begin{pmatrix} [\blacklozenge, \circ, \circ] & 1 & 2 & 3 \\ 1 & \mathbf{1.1}_{1.3} & \mathbf{2.3}_{2.3} & \mathbf{1.3}_{3} \\ 2 & \mathbf{3.2}_{2.3} & \mathbf{2.2}_{1.2} & \mathbf{2.3}_{2} \\ 3 & \mathbf{3.1}_{3} & \mathbf{3.2}_{2} & \mathbf{3.3}_{2.3} \end{pmatrix}$$

$$\xrightarrow{\text{logification}} \begin{pmatrix} [\blacklozenge, \vee, \wedge] & T_{1.3} & F_{1.2} & \mathbf{F}_{2.3} \\ T_{1.3} & T_{1.3} & \mathbf{F}_{2.3} & \mathbf{F}_{3} \\ F_{1.2} & \mathbf{F}_{2.3} & F_{\mathbf{1.2}} & F_{2} \\ \mathbf{F}_{2.3} & \mathbf{F}_{3} & F_{2} & \mathbf{F}_{2.3} \end{pmatrix}$$

$$\log\left(Sem^{(3,2,2)}_{[\blacklozenge, \vee, \wedge]}\right)$$

with:

$T_{1.3} \equiv 1.1_{1.3} \equiv t_1, t_3$

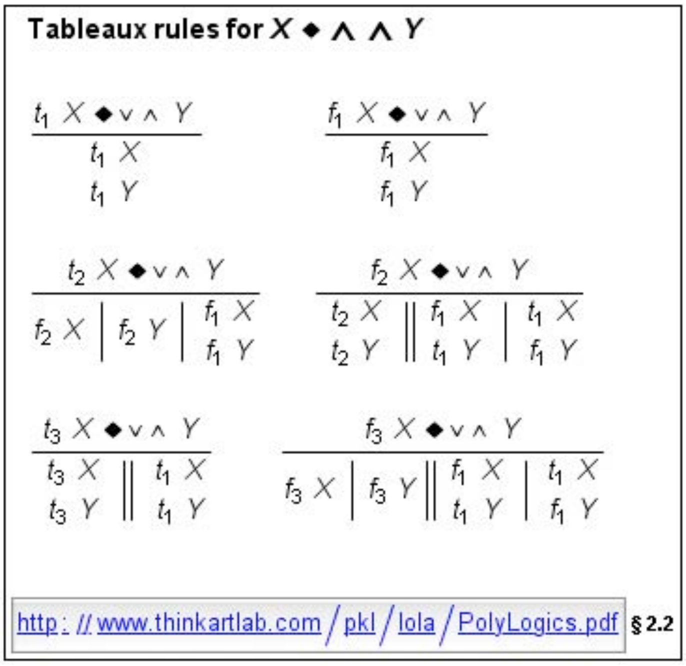
$F_{1.2} \equiv 2.2_{1.2} \equiv f_1, t_2$

$\mathbf{F}_{2.3} \equiv 3.3_{2.3} \equiv f_2, f_3$

Example

$$\frac{F_2\left(Sem^{(3,2,2)}_{(inter,\, act,\, act)}\right)}{F_2\, X \;\middle|\; F_2\, Y} \qquad \frac{\mathbf{F}_2\left(Sem^{(3,2,2)}_{(inter,\, act,\, act)}\right)}{\begin{matrix}\mathbf{F}_2\, X \\ \mathbf{F}_2\, Y\end{matrix}}$$

$$\frac{t_2\left(Sem^{(3,2,2)}_{[\blacklozenge, \vee, \wedge]}\right)}{t_2\, X \;\middle|\; t_2\, Y} \qquad \frac{f_2\left(Sem^{(3,2,2)}_{[\blacklozenge, \vee, \wedge]}\right)}{\begin{matrix}f_2\, X \\ f_2\, Y\end{matrix}}$$



With such a mapping of semiotics onto logics, the whole machinery of combinatorics as studied earlier, might be directly applied (Kaehr, Mahler, 1993).

5. Interactions in diamonds

5.1. Interactions in diamonds

The new distinctions for diamonds between semiotic *systems* and their *environments* are allowing new kinds of interactions. Additionally, *anchored* semiotics and diamonds might be involved into even more radical interactions , like interventions and metamorphosis.

In general, it seems not to be realistic to deal with multi-leveled autonomous systems, say polysemiotics, in their isolation, without considering their complex interactions, e.g. *interpenetrations* (Luhmann), between heterarchically distributed sub-systems.

Interpenetration
"First, interpenetration is not a general relation between system and environment but an intersystem relation between systems that are environments for each other. In the domain of intersystem relations, the concept of interpenetration indicates a very specific situation, which must be distinguished above all from input/output relations (performances). We speak of "penetration" if a system makes its own complexity (and with it indeterminancy, contingency, and the pressure to select) available for constructing another system." (Niklas Luhmann)

Mediation scheme for semiotic diamond$^{(3,2)}$

$$\text{Diamond}^{(3,2)} = \begin{bmatrix} \square & \square & (2.2)_4 & \longleftarrow & (2.2)_4 & \square & \square \\ \square & \square & \updownarrow & \square & \updownarrow & \square & \square \\ (1.1)_1 & \longrightarrow & (2.2)_1 & \diamond & (2.2)_2 & \longrightarrow & (3.3)_2 \\ | & \square & & \square & \square & \square & | \\ (1.1)_3 & \longrightarrow & - & - & - & \longrightarrow & (3.3)_3 \end{bmatrix} \equiv \begin{pmatrix} \square & S_4 & \square \\ S_1 & \square & S_2 \\ \square & S_3 & \square \end{pmatrix}$$

Correspondences for the diamond semiotics Sem_4 : $\delta(2.2)_2 \equiv (2.2)_4$, $\delta(2.2)_1 \equiv (2.2)_4$.

Semiotic diamond scheme for interaction

$$\text{Diamond}^{(3,2,2)}_{(inter,\, act,\, act,\, act)} = \left[\begin{bmatrix} [\blacklozenge, \circ, \circ] & 1 & 2 & 3 \\ 1 & \mathbf{1.1}_{1.3} & \mathbf{2.3}_{2.3} & \mathbf{1.3}_{3} \\ 2 & \mathbf{3.2}_{2.3} & \mathbf{2.2}_{1.2} & \mathbf{2.3}_{2} \\ 3 & \mathbf{3.1}_{3} & \mathbf{3.2}_{2} & \mathbf{3.3}_{2.3} \end{bmatrix} \right] \left\| \left[(2.2)_{(4.4)} \right] \right.$$

A *polylogical* modeling of a semiotic diamond, $\text{Diamond}^{(4,2)}_{(act,inter,act,act)}$, as $\text{Diamond}^{(4,2)}_{(\vee\blacklozenge\vee\wedge)}$ with interaction, transjunction, in sub-system$_2$ and its *interference* in the environmental sub-system$_4$, conjunction, gives some insight into the internal structure of a diamond with a *weak interaction* with sub-system$_4$.

Weak interaction in a diamond with sub – system$_4$

$$\mathrm{val}\left(X^{(4)}\begin{pmatrix} \wedge \\ \vee \ \blacklozenge \\ \vee \end{pmatrix}Y^{4)}\right) \equiv \mathrm{val}(X \vee \blacklozenge \vee \wedge Y)$$

$$\frac{t_1\ X \vee \blacklozenge \vee \wedge Y}{t_1\ X \;\big|\; t_1\ Y \;\Big\|\; \begin{matrix} f_2\ X \\ t_2\ Y \end{matrix} \;\Big|\; \begin{matrix} t_2\ X \\ f_2\ Y \end{matrix}} \qquad \frac{f_1\ X \vee \blacklozenge \vee \wedge Y}{\begin{matrix} f_1\ X \\ f_1\ Y \end{matrix} \;\Big\|\; \begin{matrix} f_4\ X \\ f_4\ Y \end{matrix} \;\Big\|\; \begin{matrix} t_2\ X \\ t_2\ Y \end{matrix}}$$

$$\frac{t_2\ X \vee \blacklozenge \vee \wedge Y}{\begin{matrix} t_2\ X \\ t_2\ X \end{matrix} \;\Big|\; \begin{matrix} t_4\ X \\ t_4\ X \end{matrix}} \qquad \frac{f_2\ X \vee \blacklozenge \vee \wedge Y}{\begin{matrix} f_2\ X \\ f_2\ Y \end{matrix}}$$

$$\frac{t_3\ X \vee \blacklozenge \vee \wedge Y}{t_3\ X \;\big|\; t_3\ Y \;\Big\|\; \begin{matrix} f_2\ X \\ t_2\ Y \end{matrix} \;\Big|\; \begin{matrix} t_2\ X \\ f_2\ Y \end{matrix}} \qquad \frac{f_3\ X \vee \blacklozenge \vee \wedge Y}{\begin{matrix} f_3\ X \\ f_3\ Y \end{matrix} \;\Big\|\; \begin{matrix} f_2\ X \\ f_2\ Y \end{matrix}}$$

$$\frac{t_4\ X \vee \blacklozenge \vee \wedge Y}{\begin{matrix} t_2\ X \\ t_2\ Y \end{matrix} \;\Big\|\; \begin{matrix} t_4\ X \\ t_4\ Y \end{matrix}} \qquad \frac{f_4\ X \vee \blacklozenge \vee \wedge Y}{\begin{matrix} f_1\ X \\ f_1\ Y \end{matrix} \;\Big\|\; f_4\ X \;\big|\; f_4\ Y}$$

5.2. Interactions between diamonds

As introduced in *Diamond Text Theory*, special interactions between diamonds are building networks of textemes. In this case, interaction between semiotic systems happens mediated by their neighboring environments.

Notes

1 *Computational* semiotics is interested in modeling interactions in computational scenarios. As much as there is no proper logic of interaction there is even much less development in computational semiotics. There is not even an awareness about the conceptual lack of interactivity constructs in theoretical semiotics. Despite the many *applicative* approaches to semiotic interactions, e.g. in human-computer interface research, it seems, that theoretical and foundational research for a semiotic theory of interaction and reflection is not supported.

Christopher R. Longyear, Further Towards a Triadic Calculus (Part 1, 2, 3)
http://www.vordenker.de/ggphilosophy/longyear-part_1.pdf

2 Independent of later steps of *abolishing* restrictions in the traditional definition of sign classes by Toth's studies, the concept of a Cartesian product remains a fundamental construction to build up a semiotic system.

This fact allows to study the semiotic matrix under a different angle: the *polycontextural* approach of dissemination, i.e. distribution and mediation, of sub-systems as a mechanism to construct and to deconstruct the semiotic matrix. In this sense, an extension of the semiotic matrix for complex sign systems, called polysemiotics, is introduced.

To use Cartesian products doesn't mean that they will remain stable in the development of a general theory of polylogics and polysemiotics. As shown at other places, what was a good starting point, became the main obstacle for further developments. Here again, the abstract mathematical frame (set and category theory) is not always adequate for the project of formalizing transclassical approaches.

This disseminative approach to the semiotics matrix allows to introduce a comparison of semiotic and logical constructions. As main operators of logical interaction, the polylogical *transjunctions* had been studied *in extenso.(Kaehr, 1978, 2005)*

In analogy and translation or transposition from the polycontextural to the semiotic topics, semiotic interactions between semiotic sub-systems shall be introduced. Semiotic sub-systems are a result of a decomposition of the semiotic matrix into its sub-systems. Such a decomposition is dynamic, depending on the complexity of the semiotic matrix. In this paper, only two cases are introduced. The decomposition into (2, 2)-subsystems, with $S_1 = \{1, 2\}$, $S_2 = \{2, 3\}$, $S_3 = \{1, 3\}$. And the decomposition into (3, 3)-subsystems of a polysemiotic system $Sem^{(4,2)}$.

3 Nevertheless, a specific redundancy has to be repeated because of its established and deep-rooted sheepishness and stultifying ignorance. The more or less only answer or ‘feed-back’ I got, when I was emphasizing the importance of a number, e.g. 4, was, *“Why an extension to 4 and not to 7 or 13 or 5112?”* Nobody ever questioned the fact that their response is based on the number 2 (TWO). And surely I never privileged a single natural number of the established number system.

A criticism of such an idea of a privilege of a single natural number was perfectly done long before by Aristotle with his refutation of Pythagorean number theory.
It seems to be better to live and to die with the number TWO than to question it.
As far, it was an important scientific step by Peirce to introduce his triadic-trichotomic semiotics and first sketches to a trichotomic mathematics.

4 "Die systematische Auszeichnung der 4 mag willkürlich erscheinen; warum nicht die 3 oder die 11 und warum eine und nicht mehrere oder gar alle Zahlen?
Die Kritik Aristoteles' an der pythagoräischen Auszeichnung der 4 bzw. der 10 setzt die Linearität der natürlichen Zahlen und das Prinzip der potentiellen Realisierbarkeit voraus. Erst dann entstehtein Konflikt zwischen der Reihe der natürlichen Zahlen, d.h. einer beliebigen Zahl und der Auszeichnung einer Zahl dieser Reihe als Gattungszahl der Reihe selbst.
Wird jedoch unter der 4 die '*Gattungszahl*' der 4 Schrifttypen der Graphematik verstanden, also das Geviert der geschlossenen Proemialität, dann entsteht kein Widerspruch zwischen Auszeichnung einer Zahl und der Zahlenreihe selbst. Die 4 eröffnet die Vielfalt der Zahlensysteme der Polykontexturalität, liegt jedoch als solche nicht in der Reihe der natürlichen Zahlen einer beliebigen Kontextur. Aristoteles lehnt die Auszeichnung der 4 (und mit ihr die der 10) ab, ist aber selbst gezwungen, die 1 auszuzeichnen. Denn die Uni-Linearität der Reihe der natürlichen Zahlen setzt die 1 als Maß der Zahlen und als unum der Unizität der Reihe voraus. Die Auszeichnung der 4 unter der Voraussetzung der Uni-Linearität heißt, daß die vertikale Sprachachse der Graphematik auf die horizontale Linie der natürlichen Zahlen projiziert wird.
Der Widerspruch zwischen 'Gattungszahl' und 'Reihenzahl' ist somit das Produkt einer Verdeckung, einer Koinzidenz der beiden 'Zahlenachsen'. Dabei wird auch stillschweigend vorausgesetzt, daß die Zahlziffern selbst eindeutig und nicht einer Überdetermination ausgesetzt sind. Aristoteles' Kritik verfängt auch dann nicht, wenn sich die 4 vertikalen Sprachschichten nicht legitimieren lassen und ihre Anzahl vergrößert oder verkleinert werden muß.
Die Kritik an der Auszeichnung einer bestimmten Zahl vor der anderen durch die transklassische Arithmetik, kann sich jedoch nicht auf Aristoteles berufen, denn seine Kritik umfaßt generell die Mehrlinigkeit der platonischen Zahlen und diese wiederum ist ein wesentlicher Charakter der transklassischen Zahlentheorie.
So argumentiert Günther: *„Aristoteles ist im Recht. Es ist notwendig, konsequent zu sein. Entweder sehen wir uns gezwungen, nicht nur der Monas, der Dyas, der Triade usw., kurz jeder pythgagoräischen n-Zahl den Rang einer ontologischen Idealität zuzubilligen oder aber die ganze Problemsicht ist verfehlt und keine Zahl hat die Würde einer Idee-außer vielleicht die Einheit und die aoristos duas, die man aber beide nicht als Zahlen zu betrachten braucht. Daß die zweite Auffassung nicht haltbar ist, lehrt die Geistesgeschichte vergangener Epochen."*
Günther insistiert also auf der Auszeichnung jeder Zahl und nicht nur der pythagoräischen Tetraktys. D.h. jede Zahl hat die Würde einer Idee und erhält somit eine logisch-strukturelle Relevanz in der Polykontexturalitätstheorie. Dort entspricht jeder natürlichen Zahl m eine bestimmte irreduzible m-kontexturale Qualität.
Damit geht aber die Idee der Auszeichnung, des Abschlusses und die Dialektik von offenem und geschlossenem System, wie sie sonst in der Kenogrammatik von Relevanz ist, verloren. Läßt sich keine Zahl auszeichnen, sondern müssen umgekehrt alle Zahlen einer Auszeichnung würdig sein, so führt sich die Idee der Auszeichnung ad absurdum. Daß alle natürlichen Zahlen logisch-strukturell ausgezeichnet werden können, ist aber das Resultat einer vollständigen Dekonstruktion der Konzeption der uni-linearen aristotelischen Arithmetik wie sie in der Kenogrammatik und der Polykontexturalitätstheorie vollzogen wurde. Mit der isolierten Thematisierung der Iterierbarkeit der

m-kontexturalen Zahlensysteme wird das wenig dialektische Moment der schlechten Unendlichkeit zugelassen.” (Kaehr, Einschreiben in Zukunft, § 6,1981)
http://www.thinkartlab.com/pkl/media/DISSEM-final.pdf

Polycontextural and diamond dynamics

Sketches and exercises for dynamics and metamorphosis for formal systems

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Abstract

The Ancient Chinese idea of a permanently changing world in which stable formulations, i.e. axioms in logic, are obsolete is thematized by the polycontextural strategy of permanently changing complexity. As a framework to realize complexity change for formal systems the kenomic matrix is involved. Examples for such formal notations are given and exercises to learn more about polycontextural diamond systems are proposed.

1. Mediation in complex dynamic formal systems

It is said, that categories are distributed over the kenomic matrix.
As pointed out, dissemination is composed of distribution and mediation. How is mediation working? Again, different contextures are mediated by their proemial relationship. That is, by the proemiality of their basic terms. What are the basic terms of categories? At first, it is mentioned that categories are build as compositions of morphisms. And morphisms are mappings with domains and codomains,conceived as objects. Hence, composition of two morphisms is ruled by the matching conditions of the codomains and domains of morphisms.
With such a proposition of the scenario, the pre-conditions of categories are not reflected. That is, there locatedness is left to the mental activities of the categorist.

How to find a mediation?
Therefore, the foremost step of distributed category theories, and not only distributed single categories, is to find possibility to encounter category theories at other loci in the kenomic grid, able to get into an interaction and getting mediated. On such a path, i.e. journey, categories might occur which are structurally not prepared for mediation. Hence, strategies have to be developed to find the adequate setting of categories or contextures and reflectional techniques might be applied to adapt automatically to the situation.
Most attempts to interact will fail. Some will succeed only partially, some, probably small systems, will succeed totally. All levels of possible interactions have to be accepted and studied.

Such a selection to find a winning mediation is not unfamiliar in the theory of formal systems. Formal systems are build on a syntactic selection, cut, between correct and non-correctly composed sign sequences. Here too, most sign sequences which are possible as combinations of the signs of the pre-given sign set (repertoire, alphabet) are not accepted as formulas. Hence, the accepted formulas are a small subset of the free monoid over the alphabet.

Mediators
"A new kind of interpreters appears to the programmer in ConTeXtures, the mediator-interpreter*. This kind of interpreter has to collect, control and to establish the mediation of different programs written by different programers at different locations at different times.*

In contrast to existing compilers and interpreters with their hierarchic tectonics, the new situation can be defined as heterarchic*. Heterarchic compilers/interpreters have to manage the mediation of the different hierarchic approaches of programming. This concept of heterarchic compilers opens up a new kind of* societal *collaborations."* (Kaehr, ConTeXtures, 2005)

Today, such a societal collaboration might be called Web 2.0 selection, compilation and interpretation. It might help to design the idea of a *heterarchic* Web Computing Paradigm which is not reduced to data or service sharing (cloud and grid computing). *Mediation* is not sharing but *creation*. Hence, a collective system production is not a collective sharing system but should be conceived as a genuine societal computing paradigm. This is not only intended to surpassing the deadly anachronism of Big Corporations but also the individualistic limitations of Open Source strategies.

2. A remainder from Chinese Ontology

"Traveler, there are no path. Path are made by walking." Antonio Machado

"A good mathematician is one who is good at expanding categories or kinds (tong lei)."

The Chinese philosopher Jinmei Yuan has given some crucial hints to the understanding of ancient Chinese mathematical thinking:

Chinese mathematical art aims to clarify practical problems by examining their relations; it puts problems and answers in a system of mutual relation--a yin-yang structure for all the things in a changing world. The mutual relations are determined by the lei (kind), which represents a group of associations, and the lei (kind) is determined by certain kinds of mutual relations.

"Chinese logicians in ancient times presupposed no fixed order in the world. Things are changing all the time. If this is true, then universal rules that aim to represent fixed order in the world for all time are not possible." (Jinmei Yuan)

An Aperçu
Chinese ontology (cosmology) can be put into two main statements:
A. *Everything in the world is changing.*
B. *The world, in which everything is changing, doesn't change.*
This two main statements are designing a paradoxical constellation.

Polycontexturality is complementing this ancient Chinese world model of *harmony* by dynamizing the concept of world-models:
C. *A multitude of worlds are interplaying together.*

The paradox to formulate mathematical rules in an ever changing world is very puzzling.
Many attempts to shed some light into it or even to solve the problem had been proposed.

It is not my intention to solve this 'unsolvable' problem.

Polycontextural logic attempts to formulate formal laws for an ever changing world. Nevertheless, we first have to abandon a Western interpretation of 'change'. The Book of Change has nothing to do with Heraklit's or Leibniz's flux of things.

Many aspects about a philosophy of *logic and time* had been studied profoundly by the philosopher Gotthard Gunther. The connection of time and logic in polycontextural systems is not to confuse with any attempts of time or tense logics or physical time systems of any kind.

My own attempt to deal with the formal structure of changing first-order ontologies can be reduced, at this place, to two propositions:

Strategies of change
1. *Diamond strategies*: Each move is involved with its simultaneous counter-move.
2. *Complexity strategies*: Each move has to decide (elect/select) its intra-/trans-contextural continuation depending on the actual complexity encountered or created.

Intra-contextural continuation is supposing that the logic-structural complexity (of logic, arithmetic, semiotics, ontology) is stable and hasn't changed, hence *selects* its next step.
Trans-contextural continuation has to reflect the possible change in complexity and has to chose, i.e. *elect* its contextures, i.e. its contextural environment for its next steps to select.
In classical arithmetics, the step from n to n+1 is unambiguously defined by the arithmetical rules or axioms. In contrast, polycontextural arithmetics is involved always, in at least, two actions, election and addition, producing a kind of a 2-dimensional tabular continuation:

m – successor rule

$$n_{1.1} \Rightarrow n_{1.1} +_{1.1} 1$$
$$\Rightarrow n_{1.1} +_{1.2} 1$$
$$\Rightarrow n_{1.1} +_{2.1} 1$$
$$\Rightarrow n_{1.1} +_{2.2} 1$$

Because the strategies of change happens on the most fundamental levels of formal systems (logic, arithmetic, mathematics, ontology, semiotics, computability) a real combination of the antagonistic features of permanent change and formal operativity is opened up and accessible to realization.

One mechanism to realize change is given by the proemiality or chiasm between intra-contextural 'parts' and trans-contextural 'whole'. A predicate defined inside a contexture can become the criteria for a new contexture which is augmenting the complexity of the contextural constellation.

For the sake of simplicity, 3 constellations of change are considered:
a) *balanced* constellation between formalism and application, with equal complexity for the formalism and the system to be formalized: compl(Form) = compl(System),
b) *under-balanced* constellation, with compl(Form) <= compl(System) and
c) *over-balanced* constellation, with compl(Form) >= compl(System).

For classical Western thinking, based, shortly, on ontology and logic, only the balanced constellation with minimal complexity is available. Change is accessible in formal systems as change of complexion only. This strategy might be extremely sophisticated but it remains stable in respect to the logico-structural complexity of its paradigm.

Hence, not only every move (composition, concatenation, combination) in polycontextural diamond systems is accompanied by its hetero-morphic counter-movement but each movement is additionally determined by its polycontextural complexity-decision by *election* and *selection*.

In other words, in such a dynamic formalism, it easily can happen, that in the middle of a formal *transformation* (derivation, deduction, description, modeling) the complexity of the framework within those transformations happens might be changed, enlarged or reduced to legitimate a more reasonable and viable continuation of the transformations.

2.1. Exercises

2.1.1. Collect arguments - pros and cons, and beyond- and articles given in my Blog and elsewhere, which might support or reject the 'Apercu' of a Chinese Ontology and a Diamond World Model.

2.1.2. How are those thoughts connected to the project of Derrida's Grammatology and the deconstruction of phono-logo-centrism in formal systems? Read and comment original texts only (if necessary translations)!

2.1.3. What can you learn from the sketches to a new rationality based on polycontexturality and the concept of Chinese scriptural paradigm for the understanding of the decline of the Western Hegemony?

2.1.4. What are the immanent limits of Western thinking and how might they influence the economic and financial crash? Connect your insights with the proposals given in my "The Logic of Bailout Strategies".

2.1.5. Create more questions and answer of this kind.

2.1.6. A good exercise to experience the patterns and strategies of polycontextural and diamond thinking for more familiar topics, like ethics, human rights, identity, pluricentrism, Web 2.0 etc. might be the reading of the 'exercises' I have written in the collection *"Short Studies 2008"*.
All answers to the exercises can be written in English, German or French and posted to my Blogs. Chinese and Japanese proposals are welcomed.

3. Notational notes

3.1. The kenomic matrix

The kenomic matrix was introduced in ConTeXtures to offer a general notational approach to the dissemination of formal systems (logics, programming languages, semiotics) considering the modi of dissemination (identity, permutation, replication, iteration, bifurcation) as strategies to implement computability, reflectionability and interactionability into formal systems.

The term "kenomic" refers to keno, greek for empty. The matrices should be read as empty of logical and mathematical presumptions. That is, their mathematical features are not considered as important for the definition of the dissemination of formal systems. Such a reflection on the epistemological status the matrices would deserve a own contemplation. Metaphorically, matrices are not more than the naked shell of the turtle in the story of Lo Shu.

Hence, the kenomic matrix consists of empty places which might be occupied by formal systems or not. Like in kenogrammatics, kenoms are not linearly ordered but are inscribed in a tabular manner.

This openness allows to interpret kenomic matrixes naturally for reflectional and interactional constellations.

3.1.1. Computational, reflectional and interactional constellations

An empty kenomic matrix with $m = 6,\ n = 3$: $\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$

Framed empty matrix for $m = n = 3$: $\begin{bmatrix} PM & O1 & O2 & O3 \\ M1 & \emptyset & \emptyset & \emptyset \\ M2 & \emptyset & \emptyset & \emptyset \\ M3 & \emptyset & \emptyset & \emptyset \end{bmatrix}$

Reflectional matrix for $refl_1$ and $refl_3$: $\begin{bmatrix} PM & O1 & O2 & O3 \\ M1 & S_1 & \emptyset & \emptyset \\ M2 & S_1 & S_2 & S_3 \\ M3 & \emptyset & \emptyset & S_3 \end{bmatrix}$

Interactional matrix for $interact_2$ and $interact_3$ on S_1 : $\begin{bmatrix} \textbf{[bif, id, id]} & O_1 & O_2 & O_3 \\ M_1 & S_{1.1} & x & x \\ M_2 & S_{2.1} & S_{2.2} & x \\ M_3 & S_{3.1} & x & S_{3.3} \end{bmatrix}$

3.1.2. Dissemination of logical particles

Regarding the sketched patterns for kenomic matrices as general place-holders for formal systems, applications for the distribution of the syntax of specific systems are following naturally.
Distribution of logical connectors and quantifiers and their complex variables are constructed along the frame of the involved matrices.

3.2. Balanced formulas

As an example of the use of the matrix approach for composed formulas, the first-order formula for categorical composition might be involved.

(C3) : **mono - contextural composition**

$$\forall x \forall y\ ((\exists z : K(x, y, z) \equiv (C(x) = D(y)))$$

The short version of the formula in a 3-contextural situation, involving a transactional quantifier Q, too, is given below. Such short versions, presented usually in a Guntherian context, are working only for very simple cases and are mostly misleading.

(C3'): 3-contextural, short

$$\left(Q^1 \,\forall^2\, \forall^3\right)\left(x^1\, x^2\, x^3\right)$$

$$\left(Q^1 \,\forall^2\, \forall^3\right)\left(y^1\, y^2\, y^3\right)$$

$$\left(\exists^{(3)}\left(z^1\, z^2\, z^3\right)\right):$$

$$K_{cat}{}^{(3)}(x, y, z) \equiv \left(C^{(3)}(x) = D^{(3)}(y)\right).$$

$$\overline{Q^1} = \left[Q^{1.1}\, Q^{2.1}\, Q^{3.1}\right]$$

$$\forall^2 = \left[\forall^{2.2}\right], \quad \forall^3 = \left[\forall^{3.3}\right]$$

Hence, we have to take the burden and offer an explicit notation form of the formula. It is easy to understand the distribution of all elements involved: *variables* x, y, *predicates* C, D, K, *quantifiers* ∀, ∃, Q. The distribution of equality (=) is omitted.

(C3''): 3-contextural, explicit

$$\begin{bmatrix} Q^{1.1} & - & - \\ Q^{2.1} & \forall^{2.2} & - \\ Q^{3.1} & - & \forall^{3.3} \end{bmatrix} \begin{pmatrix} x^{1.1} & - & - \\ x^{2.1} & x^{2.2} & - \\ x^{3.1} & - & x^{3.3} \end{pmatrix}$$

$$\begin{bmatrix} Q^{1.1} & - & - \\ Q^{2.1} & \forall^{2.2} & - \\ Q^{3.1} & - & \forall^{3.3} \end{bmatrix} \begin{pmatrix} y^{1.1} & - & - \\ y^{2.1} & y^{2.2} & - \\ y^{3.1} & - & y^{3.3} \end{pmatrix}$$

$$\begin{bmatrix} \exists^{1.1} & - & - \\ \exists^{2.1} & \exists^{2.2} & - \\ \exists^{3.1} & - & \exists^{3.3} \end{bmatrix} \begin{pmatrix} z^{1.1} & - & - \\ z^{2.1} & z^{2.2} & - \\ z^{3.1} & - & z^{3.3} \end{pmatrix}:$$

$$\begin{bmatrix} K^{1.1} & - & - \\ K^{2.1} & K^{2.2} & - \\ K^{3.1} & - & K^{3.3} \end{bmatrix}$$

$$\left(\begin{pmatrix} x^{1.1} & - & - \\ x^{2.1} & x^{2.2} & - \\ x^{3.1} & - & x^{3.3} \end{pmatrix}, \begin{pmatrix} y^{1.1} & - & - \\ y^{2.1} & y^{2.2} & - \\ y^{3.1} & - & y^{3.3} \end{pmatrix}, \begin{pmatrix} z^{1.1} & - & - \\ z^{2.1} & z^{2.2} & - \\ z^{3.1} & - & z^{3.3} \end{pmatrix}\right)$$

$$\equiv\ \equiv\ \equiv$$

$$\begin{bmatrix} C^{1.1} & - & - \\ C^{2.1} & C^{2.2} & - \\ C^{3.1} & - & C^{3.3} \end{bmatrix} \begin{pmatrix} x^{1.1} & - & - \\ x^{2.1} & x^{2.2} & - \\ x^{3.1} & - & x^{3.3} \end{pmatrix}$$

$$===$$

$$\begin{bmatrix} D^{1.1} & - & - \\ D^{2.1} & D^{2.2} & - \\ D^{3.1} & - & D^{3.3} \end{bmatrix} \begin{pmatrix} y^{1.1} & - & - \\ y^{2.1} & y^{2.2} & - \\ y^{3.1} & - & y^{3.3} \end{pmatrix}$$

Formula notation

short:

$$[bif_1, id_2, id_3](log_1, log_2, log_3) = ((log_{1.1}, log_{2.1}, log_{3.1}), log_{2.2}, log_{3.3})$$

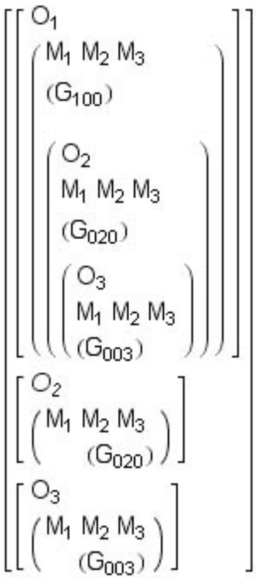
explicit:

$$op_{[bif_1, id_2, id_3]}((log_1, x, x), (x, log_2, x), (x, x, log_3))$$
$$[===]$$
$$((log_{1.1}, log_{2.1}, log_{3.1}), (x, log_{2.2}, x), (x, x\, log_{3.3}))$$

Table notation for [bif, id, id]

[bif, id, id]	O_1	O_2	O_3
M_1	$log_{1.1}$	x	x
M_2	$log_{2.1}$	$log_{2.2}$	x
M_3	$log_{3.1}$	x	$log_{3.3}$

Bracket notation for [bif, id, id]



3.3. Non-balanced formulas

This is not the place to go in deeper details of polycontextural syntactic notational systems. It is easily to see that most of the combinatorial possibilities are not well-balanced. Again, such situations are not unusual, they appear in a much simpler combinatorics in classical formal systems too.

$$\begin{bmatrix} \wedge^{1.1} & - & - \\ \wedge^{2.1} & \vee^{2.2} & - \\ \wedge^{3.1} & - & \vee^{3.3} \end{bmatrix} \left(\begin{pmatrix} x^{1.1} & - & - \\ x^{2.1} & x^{2.2} & - \\ x^{3.1} & - & x^{3.3} \end{pmatrix}, \begin{pmatrix} y^{1.1} & - & y^{1.3} \\ - & y^{2.2} & - \\ - & - & y^{3.3} \end{pmatrix} \right) \Longrightarrow$$

$$\begin{pmatrix} x^{1.1} \wedge y^{1.1} & - & y^{1.3} \\ x^{2.1} \wedge ?? & x^{2.2} \vee y^{2.2} & - \\ x^{3.1} \wedge ?? & - & x^{3.3} \vee y^{3.3} \end{pmatrix} \Longrightarrow$$

$$\begin{pmatrix} x^{1.1} \wedge y^{1.1} & - & y^{1.3} \\ x^{2.1} & x^{2.2} \vee y^{2.2} & - \\ x^{3.1} & - & x^{3.3} \vee y^{3.3} \end{pmatrix}$$

3.4. Exercises

3.4.1. Write an overview of typical notational constellations for balanced formulas. Use the sketches given in *ConTeXtures* and *From Ruby to Rudy*.

3.4.2. Program features of balanced (m,n)-contextural notational systems for junctional, transjunctional connectors and quantifiers.

3.4.3. Try to define and program more efficient and 'ergonomic' notational approaches to general tabular syntactics.

4. Sketch for complexity changes

4.1. Extending constellations by accretion

Hence, e.g., a logical constellation as for *table-a,* with complexity (3,3), can be changed to different tables depending on the type of the required complexity and complication with complexity (3, 4) , (4,3) and (4,4). Such changes are involving the formal systems as a whole. An example for a change of complexity concerning the *quantifiers* only of polycontextural logics is given below.

Table – a

$[Q^1 \forall^2 \forall^3]$	O_1	O_2	O_3
M_1	$log_{1.1}$	x	x
M_2	$log_{2.1}$	$log_{2.2}$	x
M_3	$log_{3.1}$	x	$log_{3.3}$

Table – *b*

[ooo]	O_1	O_2	O_3
M_1	$log_{1.1}$	x	x
M_2	$log_{2.1}$	$log_{2.2}$	x
M_3	$log_{3.1}$	x	$log_{3.3}$
M_4	$log_{4.1}$	$log_{2.4}$	$log_{3.4}$

Table – *c*

[oooo]	O_1	O_2	O_3	O_4
M_1	$log_{1.1}$	x	x	$log_{4.1}$
M_2	$log_{2.1}$	$log_{2.2}$	x	x
M_3	$log_{3.1}$	x	$log_{3.3}$	$log_{4.3}$

Table – *d*

[oooo]	O_1	O_2	O_3	O_4
M_1	$log_{1.1}$	x	x	$log_{4.1}$
M_2	$log_{2.1}$	$log_{2.2}$	x	x
M_3	$log_{3.1}$	x	$log_{3.3}$	x
M_4	$log_{4.1}$	x	$log_{3.4}$	$log_{4.4}$

Example of complexity elections for quantifiers

$$\begin{bmatrix} Q^{1.1} & - & - \\ Q^{2.1} & \forall^{2.2} & - \\ Q^{3.1} & - & \forall^{3.3} \end{bmatrix} \xrightarrow{elect(4,3)} \begin{bmatrix} Q^{1.1} & - & - \\ Q^{2.1} & \forall^{2.2} & - \\ Q^{3.1} & - & \forall^{3.3} \\ Q^{4.1} & \exists^{2.4} & \forall^{3.4} \end{bmatrix}$$

$$\Downarrow elect(3,4) \qquad\qquad \Downarrow elect(4,4)$$

$$\begin{bmatrix} Q^{1.1} & - & - & \exists^{4.1} \\ Q^{2.1} & \forall^{2.2} & - & - \\ Q^{3.1} & - & \forall^{3.3} & \forall^{4.3} \end{bmatrix} \xrightarrow{elect(4,4)} \begin{bmatrix} Q^{1.1} & - & - & \exists^{4.1} \\ Q^{2.1} & \forall^{2.2} & - & - \\ Q^{3.1} & - & \forall^{3.3} & \forall^{4.3} \\ Q^{4.1} & \exists^{2.4} & \forall^{3.4} & \exists^{4.4} \end{bmatrix}$$

$\xrightarrow{elect(4,3)}$ introduces the quantifiers : $\left(Q^{4.1} \quad \exists^{2.4} \quad \forall^{3.4}\right)$ in $Log^{(3,4)}$

$\xrightarrow{elect(3,4)}$ introduces the quantifiers : $\left(\exists^{4.1} \quad - \quad \forall^{4.3}\right)$ in $Log^{(4,3)}$

$\xrightarrow{elect(4,4)}$ introduces the quantifiers : $\left(Q^{4.1} \quad \exists^{2.4} \quad \forall^{3.4}\ \exists^{4.4}\right)$ in $Log^{(4,4)}$

4.2. Extending constellations by iterations and replications

Change for polycontextural systems has many faces. Additional to accretive extensions, a system might extend its scope by reflection into itself, self-reflection and introspection. This gets a formal representation by the super-operators *iter*, for iteration and *repl*, for replication.

- *Iteration* is augmenting complexity iteratively from $S_{i.j}$ to $S_{i+1.j}$
- *Replication* is deepening complexity without augmentation from $S_{i.j}$ to $S_{i.j+1}$.

In the example, system S_1 is involved into iteration, leading from S_1 at $(O_1 M_1)$ to S_1 at $(O_1 M_2)$ and into replication, leading to $S_{1.1.1.1.1}$ and $S_{2.1.1.1}$

Formula notation

$PM = \left(id_1\left(rep_1\ rep_1\ rep_1\ rep_1\right),\ iter_1\left(rep_1\ rep_1\ rep_1\right), x\right), \left(iter_2, id_2, iter_2\right), \left(x, iter_3\ id_3\right)\Big)$

$PM = \left(id_1\left(rep_{1.1.1}\right),\ iter_1\left(rep_{1.1.1}\right), x\right), \left(iter_2, id_2, iter_2\right), \left(x, iter_3, id_3\right)\Big)$

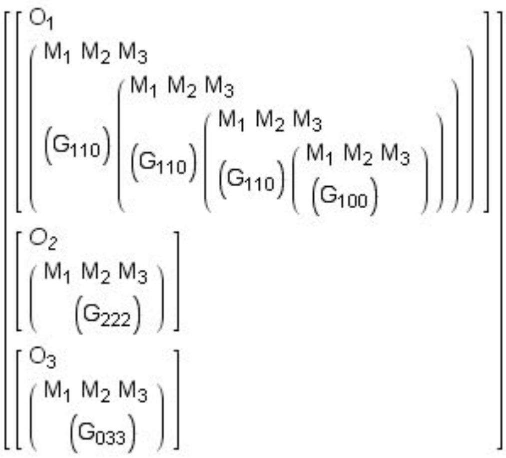
Table notation

PM	O_1	O_2	O_3
M_1	$S_{1.1.1.1.1}$	$S_{1.2}$	$\emptyset$
M_2	$S_{2.1.1.1}$	$S_{2.2}$	$S_{2.3}$
M_3	$\emptyset$	$S_{3.2}$	$S_{3.3}$

, short :

PM	O_1	O_2	O_3
M_1	$S_{1.1.1.1}$	S_2	$\emptyset$
M_2	$S_{1.1.1}$	S_2	S_3
M_3	$\emptyset$	S_2	S_3

Bracket notation



4.3. Exercises

4.3.1. Collect the arguments and constructions given in my articles and build a systematic model of the dynamic interplay of interactionality/reflectionality and interventionality in formal systems.
Recommended articles: *ConTeXtures. Programming Dynamic Complexity, Godel's Games, Actors and Objects, From Ruby to Rudy, How to compose?*

4.3.2. Compare those polycontextural and diamond models with models from modal logic, cognitive science, theory of reflection (Levebvre), reflectional programming (Smith, Maes) - and others.

4.3.3. Play around with your own ideas. Would it make fun to simulate polycontextural diamond dynamics with cellular automata models? What could we learn from such modeling, simulation and implementation? What would be lost?

4.3.4. Dynamics based on the 'kenomic matrix' might be studied for logical, arithmetical, categorical and semiotic systems by applying the materials proposed by now.

4.3.5. What are the structural consequences of contextural change for diamond category theory?

5. Metamorphic changes

5.1. Metamorphosis of topics

A transition from one contextural complexity to another doesn't presuppose a pre-given existence of the new contextures. What might be presupposed is the possibility of change. And this possibility is realized by an application of the proemial mechanism between intra- and trans-contextural decisions.
An intra-contextural topic might become contextural prominence as a new contexture associated with the previous contextural constellation.

Reflection might change the meaning of an object by applying rules of chiastic metamorphosis.
Reflection is using the statement defining the object and this usage is defining the meaning of the object. Reflection and contemplation or introspection of an object can produce the insight that the meaning of the object under consideration is changing. Reflection as replication, thus, is augmenting the deepeness of the contextural

complexity by a replicative, self-thematizing way. Reflection as iteration, is augmenting contextural complexity by an iterative, self-reproducing way. Alternatively, a reflection could change to an interactional augmentation of the contextural complexity. Both together, reflectional and interactional changes, are defining replicative, iterative and accretive contextural complexity of a polycontextural system.

The example below shows that the beginning reflection is interpreting an object as the number *zero* belonging to the topic *numerals*. This situation is implemented in a 1-contextural programming language. A second reflection considers the same object not as a numeral but as *nil* belonging to the topic of *lists*. Reflection has not to come to an end and can go further and with the interpretation and might realize that the object can be understood as belonging to the topic *Booleans* and appearing as the truth-value *true*.

Therefore the introduced *syntactical object* in its neutrality, observed and represented by an "external observer" in $log_{3.3,}$ is conceived as having simultaneously a *numerical* (in $log_{1.1}$), a *symbolic* (in $log_{1.2}$) and a *Boolean* (in $log_{1.3}$) meaning. Hence, there is a chain of metamorphic replication from the topic *Numerals*, *Lists* to *Booleans and a notation of the* 'neutral' syntactic object "object" of *Syntax*. It starts with a reflection of the object "*zero*" of *Numerals,* ends with the *Boolean* "true" and gets a contextural abstraction as syntactic "object" in *Syntax*.

The example is designed for reflectional poly-topics in the experimental programming language *ConTeXtures*.

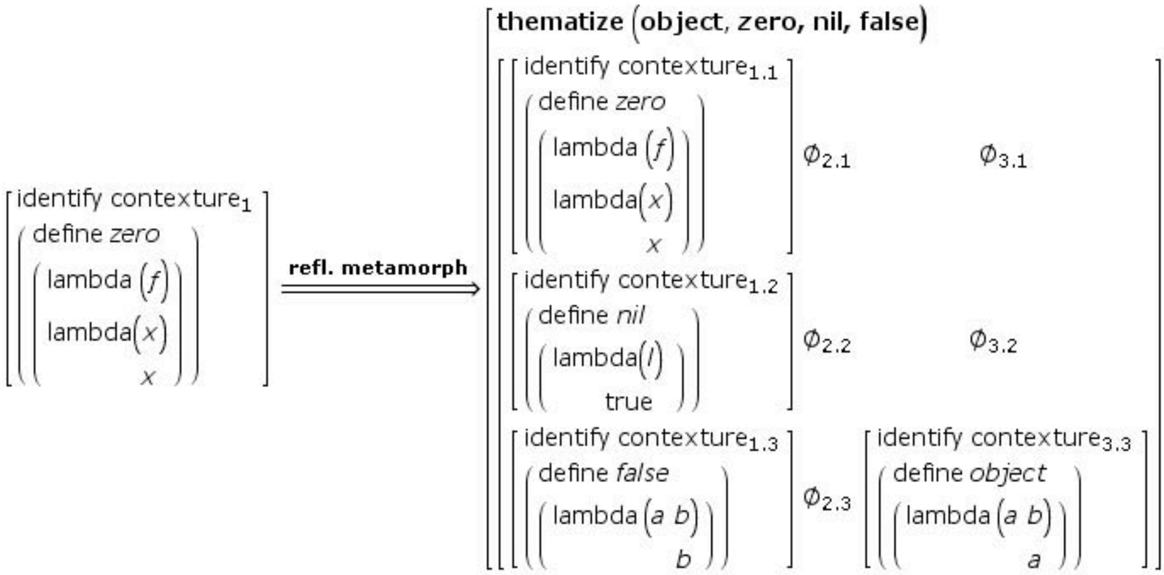


Table notation

[zero]	O_1
M_1	$log_{1.1}$

refl. metamorphosis ⟹

[$iter^3$, x, id]	O_1	O_2	O_3
M_1	$log_{1.1}$	x	x
M_2	$log_{1.2}$	x	x
M_3	$log_{1.3}$	x	$log_{3.3}$

As the example shows, the reflectional distribution of the topics Number, List, Boolean, building the category "poly-topics" , is introduced as (*zero, nil, false*) at the locus O_1. *Thematize (zero, nil, false)* is distributed reflectionally over 3 places by the super-operator *replication* (repl) and neutrally represented by the syntactical object "object" at the place O_3. In this case, the positions at place O_2 remain empty.

Exchange relations:
- "*define zero*" is "define zero as zero", as the start of the levels.
  as: define *zero* in contexture1.1 as zero in contexture1.1
- "*define nil*" is "define zero as nil",
  as: define zero from contexture1.1 as *nil* in contexture1.2
- "*define false*" is "define nil as false".
  as: define nil from contexture1.2 as *false* in contexture1.3.

This change of identity of the topics from one contexture to another by reflection/replication is producing a *chiastic* chain guaranteeing the connectedness of the step-wise reflection of the whole. Levels and meta-levels of reflection are connected by means of proemiality realizing its structural rules of exchange, order and categorial correctness (coincidence).
Thus, *define name* is an abbreviation for *"define $name_i$ as $name_j$"* with i=j.

5.2. Exercises

5.2.1. Construct examples for *reflectional*, *interactional* and *interventional* constellations for poly-topics in the framework of ConTeXtures.

5.2.2. Construct further examples in the framework of ConTeXtures with topics like semiotics, logic, arithmetics.

5.2.3. Describe ‘empirical’ situations where such contextural changes of augmenting or reducing complexity seems to be unavoidable.

5.2.4. Try to develop a polycontextural measure for complexity.

Interpretations of the kenomic matrix

Exercises to the topics of Poly-Change

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Abstract

Examples for the exercises, § 5.2, of the recent article *"Poly-Change"* are given, concerning the logical, computational and semiotic interpretation of the kenomic matrix.
http://www.thinkartlab.com/pkl/lola/Polychange/Polychange.html

1. Exercises for matrices and brackets

1.1. Table and bracket notation for diagonal mxn-matrix

Table and bracket notation for diagonal 3 x3 – matrix

PM	O_1	O_2	O_3
M_1	$S_{1.1}$	–	–
M_2	–	$S_{2.2}$	–
M_3	–	–	$S_{3.3}$

$$\left[\begin{matrix}\left[O_1\left(\begin{matrix}M_1\ M_2\ M_3\\ (G_{100})\end{matrix}\right)\right]\\ \left[O_2\left(\begin{matrix}M_1\ M_2\ M_3\\ (G_{020})\end{matrix}\right)\right]\\ \left[O_3\left(\begin{matrix}M_1\ M_2\ M_3\\ (G_{003})\end{matrix}\right)\right]\end{matrix}\right] \quad \textbf{dual}\text{ presentation}: \quad \left[\begin{matrix}\left[M_1\left(\begin{matrix}O_1\ O_2\ O_3\\ (G_{100})\end{matrix}\right)\right]\\ \left[M_2\left(\begin{matrix}O_1\ O_2\ O_3\\ (G_{020})\end{matrix}\right)\right]\\ \left[M_3\left(\begin{matrix}O_1\ O_2\ O_3\\ (G_{003})\end{matrix}\right)\right]\end{matrix}\right]$$

Table and bracket notation for diagonal 4 x4 – matrix

$$\left[\begin{array}{l}\left[\begin{array}{l} O_1 \\ \left(\begin{array}{r} M_1\ M_2\ M_3\ M_4 \\ (G_{1000}) \end{array}\right)\end{array}\right] \\ \left[\begin{array}{l} O_2 \\ \left(\begin{array}{r} M_1\ M_2\ M_3\ M_4 \\ (G_{0200}) \end{array}\right)\end{array}\right] \\ \left[\begin{array}{l} O_3 \\ \left(\begin{array}{r} M_1\ M_2\ M_3\ M_4 \\ (G_{0030}) \end{array}\right)\end{array}\right] \\ \left[\begin{array}{l} O_4 \\ \left(\begin{array}{r} M_1\ M_2\ M_3\ M_4 \\ (G_{0004}) \end{array}\right)\end{array}\right]\end{array}\right]$$

P M	O_1	O_2	O_3	O_4
M_1	$S_{1.1}$	–	–	–
M_2	–	$S_{2.2}$	–	–
M_3	–	–	$S_{3.3}$	–
M_4	–	–	–	$S_{4.4}$

Simplification

$$\left[\begin{array}{ll}\left[\begin{array}{l} O_1 \\ (G_{1000}) \end{array}\right] & \left[\begin{array}{l} O_2 \\ (G_{0200}) \end{array}\right] \\ \left[\begin{array}{l} O_3 \\ (G_{0030}) \end{array}\right] & \left[\begin{array}{l} O_4 \\ (G_{0004}) \end{array}\right]\end{array}\right]$$

1.2. Reflection

Bracket notation for reflectional change from 3 x3 to 5 x3 matrix

$$\left[\begin{array}{l}\left[\begin{array}{l} O_1 \\ \left(\begin{array}{r} M_1\ M_2\ M_3\ M_4\ M_5 \\ (G_{11111}) \end{array}\right)\end{array}\right] \\ \left[\begin{array}{l} O_2 \\ \left(\begin{array}{r} M_1\ M_2\ M_3\ M_4\ M_5 \\ (G_{22200}) \end{array}\right)\end{array}\right] \\ \left[\begin{array}{l} O_3 \\ \left(\begin{array}{r} M_1\ M_2\ M_3\ M_4\ M_5 \\ (G_{03303}) \end{array}\right)\end{array}\right]\end{array}\right]$$

P M	O_1	O_2	O_3
M_1	$S_{1.1}$	$S_{1.2}$	–
M_2	$S_{2.1}$	$S_{2.2}$	$S_{2.3}$
M_3	$S_{3.1}$	$S_{3.2}$	$S_{3.3}$
M_4	$S_{4.1}$	–	–
M_5	$S_{5.1}$	–	$S_{5.3}$

Bracket notation for reflectional / replicational change from 3 x3 to 3 x3 matrix

$$\left[\left[O_1 \left(M_1\ M_2\ M_3\ (G_{110}) \left(M_1\ M_2\ M_3\ (G_{110}) \left(M_1\ M_2\ M_3\ (G_{110}) \left(M_1\ M_2\ M_3\ (G_{100})\right)\right)\right)\right)\right] \left[O_2 \left(M_1\ M_2\ M_3\ (G_{222})\right)\right] \left[O_3 \left(M_1\ M_2\ M_3\ (G_{033})\right)\right]\right]$$

P M	O_1	O_2	O_3
M_1	$S_{1.1.1.1.1}$	$S_{1.2}$	∅
M_2	$S_{2.1.1.1}$	$S_{2.2}$	$S_{2.3}$
M_3	∅	$S_{3.2}$	$S_{3.3}$

Bracket notation for reflectional / replicational change for 5 x3

$$\left[\left[O_1 \left(M_1\ M_2\ M_3\ M_4\ M_5\ (G_{1.1.1.1.1}) \left(M_1\ M_2\ M_3\ M_4\ M_5\ (G_{10000}) \left(M_1\ M_2\ M_3\ M_4\ M_5\ (G_{10110}) \left(M_1\ M_2\ M_3\ M_4\ M_5\ (G_{10000}) \left(M_1\ M_2\ M_3\ M_4\ M_5\ (G_{11111})\right)\right)\right)\right)\right)\right] \left[O_2 \left(M_1\ M_2\ M_3\ M_4\ M_5\ (G_{22200})\right)\right] \left[O_3 \left(M_1\ M_2\ M_3\ M_4\ M_5\ (G_{03303})\right)\right]\right]$$

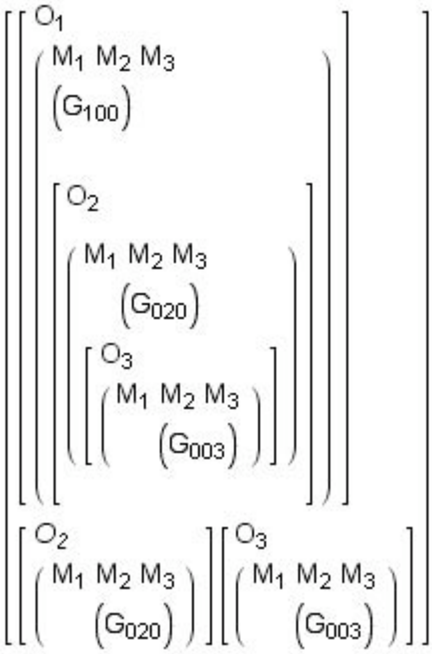
P M	O_1	O_2	O_3
M_1	$S_{1.1.1.1.1}$	$S_{1.2}$	–
M_2	$S_{2.1}$	$S_{2.2}$	$S_{2.3}$
M_3	$S_{3.1.1}$	$S_{3.2}$	$S_{3.3}$
M_4	$S_{4.1}$	–	–
M_5	$S_{5.1.1.1.1}$	–	$S_{5.3}$

Alternative notation for reflectional / replicational change for 5 x3

$$
\left[\begin{array}{l}
\left[\begin{array}{l} O_1 \\ \left(\begin{array}{l} M^{(5)} \\ \left(G_{1.1\ .1\ .1\ .1}\right) \\ \left(\begin{array}{l} M^{(5)} \\ \left(G_{10000}\right) \\ \left(\begin{array}{l} M^{(5)} \\ \left(G_{10110}\right) \\ \left(\begin{array}{l} M^{(5)} \\ \left(G_{10000}\right) \\ \left(\begin{array}{l} M^{(5)} \\ \left(G_{11111}\right) \end{array}\right) \end{array}\right) \end{array}\right) \end{array}\right) \end{array}\right) \end{array}\right] \\
\left[\begin{array}{l} O_2 \\ \left(\begin{array}{l} M^{(5)} \\ \left(G_{22200}\right) \end{array}\right) \end{array}\right] \left[\begin{array}{l} O_3 \\ \left(\begin{array}{l} M^{(5)} \\ \left(G_{03303}\right) \end{array}\right) \end{array}\right]
\end{array}\right]
$$

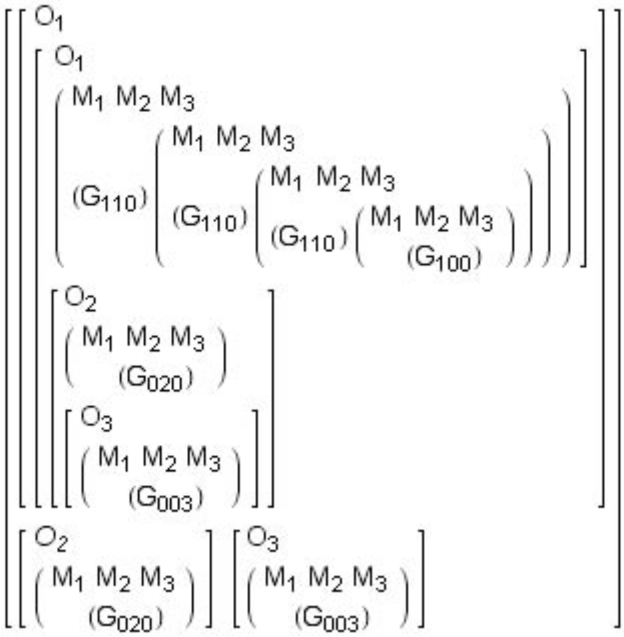
1.3. Interaction

Bracket notation for interactional change for 3 x3



[bif, id, id]	O_1	O_2	O_3
M_1	$S_{1.1}$	x	x
M_2	$S_{2.1}$	$S_{2.2}$	x
M_3	$S_{3.1}$	x	$S_{3.3}$

Bracket notation for interactional and reflectional / replicational change to 3 x3



[bif, id, id]	O_1	O_2	O_3
M_1	$S_{1.1\,.1\,.1}$	x	x
M_2	$S_{2.1}$	$S_{2.2}$	x
M_3	$S_{3.1}$	x	$S_{3.3}$

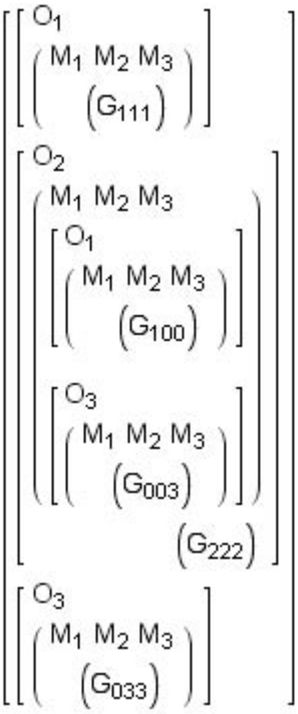
Bracket notation for interactional and reflectional / replicational change to 4 x5

[ ]	O_1	O_2	O_3	O_4	O_5
M_1	$S_{1.1\,.1\,.1}$	–	–	$S_{1.4}$	□
M_2	$S_{5.1}$	$S_{2.2}$	–	–	$S_{2.5}$
M_3	$S_{4.1}$	–	$S_{3.3}$	–	–
M_4	$S_{3.1}$	$S_{2.4}$	–	$S_{4.4}$	$S_{4.5}$

$$\begin{bmatrix} \mathrm{repl}_1^{1.2.3.4}\left(\mathrm{iter}_2^{2.4}\left(\mathrm{id}_3^3\left(\mathrm{iter}_4^{1.4}\left(\mathrm{iter}_5^{2.4}\left(\left[MO^{(4,5)}\right]\right)\right)\right)\right)\right) \\ \begin{bmatrix} O_1 \\ \begin{bmatrix} O_1 \\ \begin{pmatrix} M_1\ M_2\ M_3\ M_4 \\ (G_{1000}) \begin{pmatrix} M_1\ M_2\ M_3\ M_4 \\ (G_{1000}) \begin{pmatrix} M_1\ M_2\ M_3\ M_4 \\ (G_{1000}) \begin{pmatrix} M_1\ M_2\ M_3\ M_4 \\ (G_{1000}) \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{bmatrix} \\ \begin{bmatrix} O_5 \\ \begin{pmatrix} M_1\ M_2\ M_3\ M_4 \\ (G_{0500}) \end{pmatrix} \end{bmatrix} \begin{bmatrix} O_4 \\ \begin{pmatrix} M_1\ M_2\ M_3\ M_4 \\ (G_{0040}) \end{pmatrix} \end{bmatrix} \\ \begin{bmatrix} O_3 \\ \begin{pmatrix} M_1\ M_2\ M_3\ M_4 \\ (G_{0003}) \end{pmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} O_2 \\ \begin{pmatrix} M_1\ M_2\ M_3\ M_4 \\ (G_{0202}) \end{pmatrix} \end{bmatrix} \begin{bmatrix} O_3 \\ \begin{pmatrix} M_1\ M_2\ M_3\ M_4 \\ (G_{0030}) \end{pmatrix} \end{bmatrix} \\ \begin{bmatrix} O_4 \\ \begin{pmatrix} M_1\ M_2\ M_3\ M_4 \\ (G_{4004}) \end{pmatrix} \end{bmatrix} \begin{bmatrix} O_5 \\ \begin{pmatrix} M_1\ M_2\ M_3\ M_4 \\ (G_{0505}) \end{pmatrix} \end{bmatrix} \end{bmatrix}$$

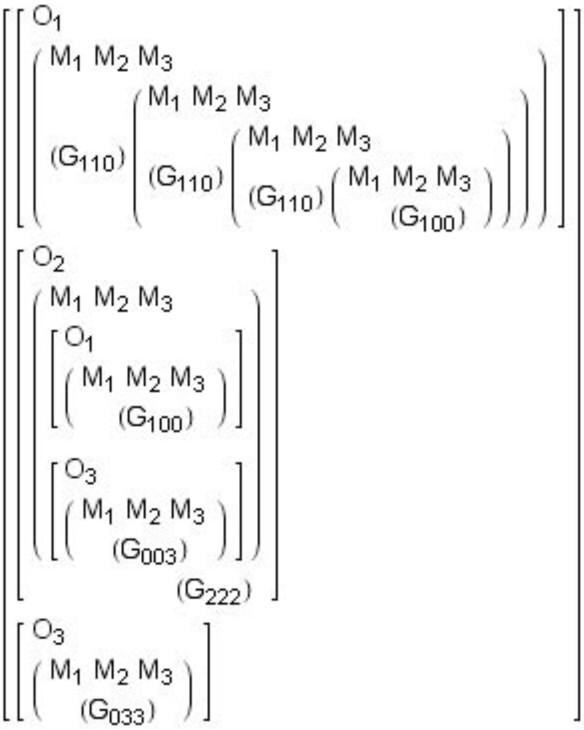
1.4. Interplay between interactionality and reflectionality

Mixing freely reflectional and interactional pattern are leading to local iterations and recursions of the general scheme producing a fractalization of the general scheme.
The examples shows:
At the locus O_2 we have a full reflection G_{222} and an interaction from the locus O_1 into the locus O_2 producing additionally to G_{222} at O_2 the interactional pattern G_{100} and an interaction from the locus O_3 into the locus O_2 producing the interactional pattern G_{003}.
Hence, the whole reflectional/interactional pattern of the example is: [G_{111}, $G_{222/003/100}$, G_{033}].



[ ]	O_1	O_2	O_3
M_1	$S_{1.1}$	$S_{2.1.1}$	–
M_2	$S_{2.1}$	$S_{2.2.0}$	$S_{3.2}$
M_3	$S_{3.1}$	$S_{2.3.3}$	$S_{3.3}$

Interplay between interactionality, reflectionality and replicativity
Additional to the example above for interactionality and reflectionality, a pattern of *replicativity* or introspection is involved at O_1 with $G_{1.1.1.1.1}$ for M_1 and $G_{2.1.1.1}$ for M_2.



[ ]	O_1	O_2	O_3
M_1	$S_{1.1.1.1.1}$	$S_{2.1.1}$	–
M_2	$S_{2.1.1.1}$	$S_{2.2.0}$	$S_{3.2}$
M_3	$S_{1.3}$	$S_{2.3.3}$	$S_{3.3}$

1.5. Permutations

Permutative patterns, produced by the super-operator *perm*, are behind those visits to other systems and back to the start again. The journey might start simultaneously in $system_1$ and $system_3$, both visiting $system_2$ at their offered locations, and back again.
The table represents more the static pattern, while the bracket notation the dynamics

of this permutation.

$$\begin{bmatrix} \begin{bmatrix} \begin{bmatrix} O_1 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ (G_{100}) \\ \begin{bmatrix} O_2 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ (G_{100}) \end{pmatrix} \end{bmatrix} \end{pmatrix} \end{bmatrix} \\ \begin{bmatrix} O_1 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ (G_{100}) \end{pmatrix} \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} O_3 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ (G_{003}) \\ \begin{bmatrix} O_2 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ (G_{003}) \end{pmatrix} \end{bmatrix} \end{pmatrix} \end{bmatrix} \\ \begin{bmatrix} O_3 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ (G_{003}) \end{pmatrix} \end{bmatrix} \end{bmatrix} \\ \multicolumn{2}{c}{\begin{bmatrix} O_2 \\ \begin{pmatrix} M_1\ M_2\ M_3 \\ (G_{123}) \end{pmatrix} \end{bmatrix}} \end{bmatrix}$$

[perm, id, perm]	O_1	O_2	O_3
M_1	$S_{1.1}$	$S_{2.1}$	–
M_2	–	$S_{2.2}$	–
M_3	–	$S_{2.3}$	$S_{3.3}$

2. Logical interpretations

2.1. The kenomic matrix and polycontextural functions

The importance of the kenomic matrix for the interpretation and organization of polycontextural functions has to be emphasized. The classical treatment of polycontextural logical functions is based on set-theoretic functions and their decomposition, i.e. interpretation.

In this exercise of mapping logical systems onto the kenomic matrix, only *bi-valent* (dyadic, dichotomic, dual) logical systems are involved. As it is shown for semiotic systems, arbitrary *contextural bases* of dyadic, triadic and tetradic up to n-adic bases have to be considered. In the literature there is nearly nothing to read about the distribution mechanisms for genuine triadic m-contextural logical systems. First combinatorial concepts occur, nevertheless as early as 1962 in Na' s work.

Super – operators for the mapping of logical systems onto the matrix

$$\text{Logic}^{(m)}: \left[\text{Logic}^{(m)}\right]_{\text{refl, act}} \xrightarrow[\text{sops}]{} \left[\text{Logic}^{(m)}\right]_{\text{refl, act}}$$

$$\text{sops} = \{\text{id, perm, red, bif, repl}\}$$

$$\textbf{id}: \forall\, i, j \in s(m): \quad (\text{Logic}^{i,j}) \xrightarrow[\text{id}]{} (\text{Logic}^{i,j})$$

$$\textbf{perm}(i, j): \forall\, i, j \in s(m): (\text{Logic}^{i}, \text{Logic}^{j}) \xrightarrow[\text{perm}]{} (\text{Logic}^{j}, \text{Logic}^{i})$$

$$\textbf{red}(i, j): \forall\, i, j \in s(m): (\text{Logic}^{i}, \text{Logic}^{j}) \xrightarrow[\text{red}]{} (\text{Logic}^{i}, \text{Logic}^{i})$$

$$\textbf{bif}(i, j): \forall\, i, j \in s(m): (\text{Logic}^{i}, \text{Logic}^{j}) \xrightarrow[\text{bif}]{} ((\text{Logic}^{i} \,\|\, \text{Logic}^{j}), \text{Logic}^{j})$$

$$\textbf{repl}(i, j): \forall\, i, j \in s(m): (\text{Logic}^{i}, \text{Logic}^{j}) \xrightarrow[\text{repl}]{} ((\text{Logic}^{i} \,|\, \text{Logic}^{j}), \text{Logic}^{j})$$

2.1.1. Positions

Positioning or placing (Setzung), realized by the super-operator *id* (identity), is well studied in the polycontextural literature. But it is only applicable to a very small set of constellations. They have a natural interpretation by the main diagonal of the kenomic matrix, which is also producing the matching conditions MC.
Conjunctions and disjunctions as introduced by Gunther and their DeMorgan formulas are typical. But it is working for balanced negational systems only.

Examples of the positional mapping of junctions

pattern : [id, id, id]

$$(JJJ) : L^{(3)} * L^{(3)} \longrightarrow L^{(3)} : [L_1, L_2, L_3]$$

$$\begin{bmatrix} Log_1 : L_1 * L_1 \xrightarrow[\text{junct } J]{} L_1 \\ Log_2 : L_2 * L_2 \xrightarrow[\text{junct } J]{} L_2 \\ Log_3 : L_3 * L_3 \xrightarrow[\text{junct } J]{} L_3 \end{bmatrix}, \quad \left[\begin{bmatrix} O_1 \\ \begin{pmatrix} M_1 \; M_2 \; M_3 \\ (Log_{100}) \end{pmatrix} \end{bmatrix} \begin{bmatrix} O_2 \\ \begin{pmatrix} M_1 \; M_2 \; M_3 \\ (Log_{020}) \end{pmatrix} \end{bmatrix} \begin{bmatrix} O_3 \\ \begin{pmatrix} M_1 \; M_2 \; M_3 \\ (Log_{003}) \end{pmatrix} \end{bmatrix} \right]$$

[JJJ]	O_1	O_2	O_3
M_1	J	–	–
M_2	–	J	–
M_3	–	–	J

$$J = \{\wedge, \vee\}$$

$$\frac{t_1\ X \wedge\wedge\wedge Y}{t_1\ X \quad t_1\ Y} \qquad \frac{f_1\ X \wedge\wedge\wedge Y}{f_1\ X \mid f_1\ Y} \qquad \frac{t_1\ X \vee\wedge\vee Y}{t_1\ X \mid t_1\ Y} \qquad \frac{f_1\ X \vee\wedge\vee Y}{f_1\ X \quad f_1\ Y}$$

$$\frac{t_2\ X \wedge\wedge\wedge Y}{t_2\ X \quad t_2\ Y} \qquad \frac{f_2\ X \wedge\wedge\wedge Y}{f_2\ X \mid f_2\ Y} \qquad \frac{t_2\ X \vee\wedge\vee Y}{t_2\ X \quad t_2\ Y} \qquad \frac{f_2\ X \vee\wedge\vee Y}{f_2\ X \mid f_2\ Y}$$

$$\frac{t_3\ X \wedge\wedge\wedge Y}{t_3\ X \quad t_3\ Y} \qquad \frac{f_3\ X \wedge\wedge\wedge Y}{f_3\ X \mid f_3\ Y} \qquad \frac{t_3\ X \vee\wedge\vee Y}{t_3\ X \mid t_3\ Y} \qquad \frac{f_3\ X \vee\wedge\vee Y}{f_3\ X \quad f_3\ Y}$$

2.1.2. Interactions

First concepts of logical interactions goes back to Gunther's morphogrammatic *transjunctions* (1962).

Example for the bifurcational mapping of transjunctions

pattern : [bif, id, id]

$$(<>\vee\vee): L^{(3)} * L^{(3)} \longrightarrow L^{(3)} : [L_1, (L_2 \| L_1), (L_3 \| L_1)]$$

$$\left[\begin{array}{l} Log_1 : L_1 * L_1 \xrightarrow[\text{transjunct } <>]{} L_1 : \begin{cases} f_1 * t_1,\ t_1 * f_1 \longrightarrow f_2,\ f_3 \\ t_1 * t_1 \longrightarrow t_1,\ t_3 \\ f_1 * f_1 \longrightarrow f_1,\ t_2 \end{cases} \\ Log_2 : L_2 * L_2 \xrightarrow[\text{conjunction}]{} L_2 \| L_1 \\ Log_3 : L_3 * L_3 \xrightarrow[\text{conjunction}]{} L_3 \| L_1 \end{array}\right.$$

[<> ∨ ∨]	O_1	O_2	O_3
M_1	trans	trans	trans
M_2	–	and	–
M_3	–	–	and

$$\frac{t_1\ X\ <>\vee\vee\ Y}{\begin{array}{c} t_1\ X \\ t_1\ Y \end{array}} \qquad \frac{f_1\ X\ <>\vee\vee\ Y}{\begin{array}{c} f_1\ X \\ f_1\ Y \end{array}}$$

$$\frac{t_2\ X\ <>\vee\vee\ Y}{\begin{array}{c|c} t_2\ X & f_1\ X \\ t_2\ Y & f_1\ Y \end{array}} \qquad \frac{f_2\ X\ <>\vee\vee\ Y}{f_2\ X \mid f_2\ Y \left\| \begin{array}{c|c} f_1\ X & t_1\ X \\ t_1\ Y & f_1\ Y \end{array}\right.}$$

$$\frac{t_3\ X\ <>\vee\vee\ Y}{\begin{array}{c} t_3\ X \\ t_3\ Y \end{array} \left\| \begin{array}{c} t_1\ X \\ t_1\ Y \end{array}\right.} \qquad \frac{f_3\ X\ <>\vee\vee\ Y}{f_3\ X \mid f_3\ Y \left\| \begin{array}{c|c} f_1\ X & t_1\ X \\ t_1\ Y & f_1\ Y \end{array}\right.}$$

2.1.3. Reflections

Reflectional patterns appeared first as interpretations of implicational constellations they might be modeled as reductions.

What's the kenomic matrix for?
"It wasn't unknown to Gunther that there is a little problem of distribution/mediation which needs a special explanation. Gunther's solution insisted correctly that the value-sequence of sub-system S_3 is still a *disjunction* because it is based in the morphogram [1] for disjunction. But it was slightly shifted to a value-sequences corresponding to a value-sequence of sub-system S_1. To solve this point, an *interpretation* was introduced: it was called a *disjunctive disjunction*. Such *interpretative* solutions had been widely used to justify logical functions in place-valued logics. But they are in no way operational.
In polylogical systems such problems are solved naturally by distribution over the *polycontextural matrix."*

Example for the reductional mapping of junctions

pattern: $\left[\text{id}, \ \text{id}, \ \text{red}\right]$

$$(\vee \longrightarrow \vee): L^{(3)} * L^{(3)} \longrightarrow L^{(3)}: \left[(L_1 \middle| L_3), \ L_2, \ \emptyset\right]$$

$$\left[\begin{array}{l} \text{Log}_1 : L_1 * L_1 \xrightarrow[\text{disjunction } \vee]{} L_1 \\ \text{Log}_2 : L_2 * L_2 \xrightarrow[\text{imlication } \longrightarrow]{} L_2 \\ \text{Log}_3 : L_3 * L_3 \xrightarrow[\text{disjunction } \vee]{} L_1 \end{array}\right.$$

$[\vee \longrightarrow \vee]$	O_1	O_2	O_3
M_1	or	–	–
M_2	–	impl	–
M_3	or	–	–

$$\frac{t_1 \ X \ \vee \longrightarrow \vee \ Y}{f_1 \ X \,|\, f_1 \ Y \,\|\, f_3 \ X \,|\, f_3 \ Y} \qquad \frac{f_1 \ X \ \vee \longrightarrow \vee \ Y}{\begin{array}{c} f_1 \ X \\ f_1 \ Y \end{array} \Big\| \begin{array}{c} f_3 \ X \\ f_3 \ Y \end{array}}$$

$$\frac{t_2 \ X \ \vee \longrightarrow \vee \ Y}{f_2 \ X \,|\, t_2 \ Y} \qquad \frac{f_2 \ X \ \vee \longrightarrow \vee \ Y}{\begin{array}{c} t_2 \ X \\ f_2 \ Y \end{array}}$$

2.1.4. Replications

Replicative constellations don't have an appearance in polycontextural logics as it was sketched by Gunther. It seems that replications don't have a direct representation in 'propositional' polycontextural logic. They might have a natural interpretation on a quantificational meta-level.
An example might be a 'quotational' system as a kind of intrinsic introspections. Represented as tables, replications are introducing an additional dimension to the 2-dimensional tabular structure.

3. Computational interpretations

3.1. General scheme of ConTeXtures

$$\mathit{sketch} - \text{horizon}^{(m,\,n)}$$

$$\left[\begin{array}{l} \mathit{build} - \text{architectures} \\ \left[\begin{array}{l} \mathit{thematize} - \text{szenario} \\ \left[\begin{array}{l} \mathit{identify} - \text{contextures} \\ \left(\begin{array}{l} \mathit{define} - \text{operations} \\ \left(\begin{array}{l} \mathit{abstract} - \text{functions} \\ \left\{\mathit{propose} - \text{statements}\right\} \end{array}\right) \end{array}\right) \end{array}\right] \end{array}\right] \end{array}\right]$$

3.2. Different modi of replication

Replication into the ' name – space ' of a contexture

$$\text{samba}^{(3,3)}(\text{repl, id, id}) - \text{horizon}$$

$$\begin{bmatrix} \text{thematize (topics}^{(3)}) \\ \begin{bmatrix} \begin{bmatrix} \text{identify contextures}_{1.1} \\ \begin{bmatrix} \text{define name} \\ \begin{bmatrix} \begin{pmatrix} \text{lambda}_{1.1} \\ \{\text{statements}\} \end{pmatrix} \\ \begin{pmatrix} \text{lambda}_{1.2} \\ \{\text{statements}\} \end{pmatrix} \\ \begin{pmatrix} \text{lambda}_{1.3} \\ \{\text{statements}\} \end{pmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} & & \emptyset \\ & \begin{bmatrix} \text{identify contexture}_{2.2} \\ \begin{pmatrix} \text{define name} \\ \begin{pmatrix} \text{lambda} \\ \{\text{statements}\} \end{pmatrix} \end{pmatrix} \end{bmatrix} & \emptyset \\ & & \begin{bmatrix} \text{identify contexture}_{3.3} \\ \begin{pmatrix} \text{define name} \\ \begin{pmatrix} \text{lambda} \\ \{\text{statements}\} \end{pmatrix} \end{pmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

Replication of a name into a contexture

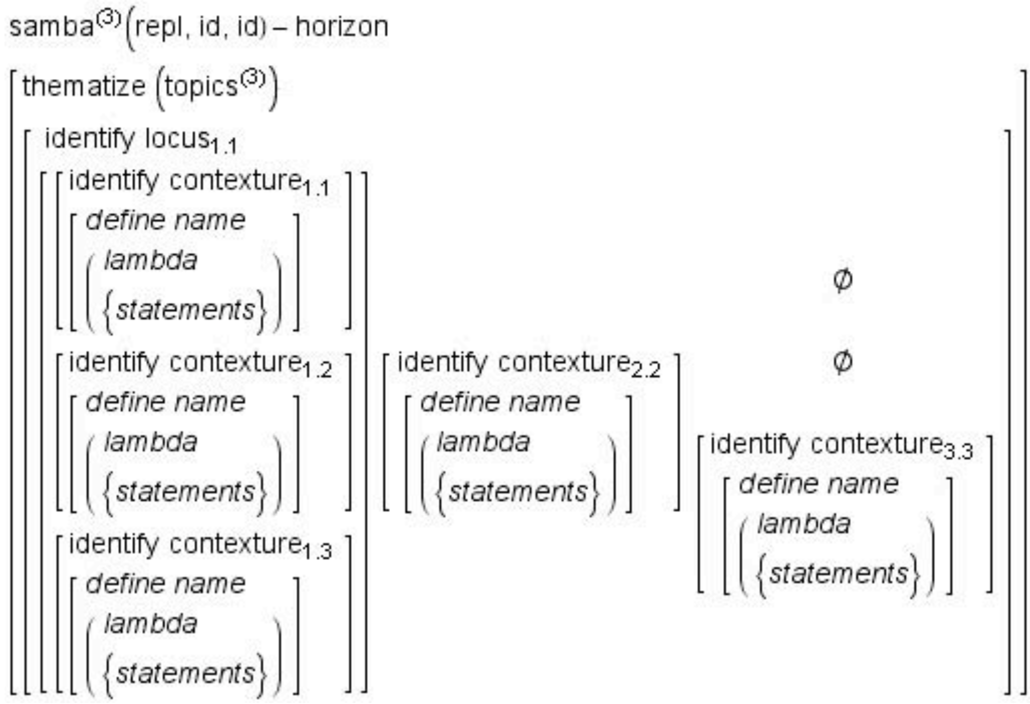
$$\text{samba}^{(3,4)}(\text{repl, id, id}) - \text{horizon}$$

$$\begin{bmatrix} \text{thematize (topics}^{(3)}) \\ \begin{bmatrix} \begin{bmatrix} \text{identify contextures}_{1.1} \\ \begin{bmatrix} \text{define name}_{1.1} \\ \begin{bmatrix} \begin{pmatrix} \text{lambda} \\ \{\text{statements}\} \end{pmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \text{define name}_{1.2} \\ \begin{bmatrix} \begin{pmatrix} \text{lambda} \\ \{\text{statements}\} \end{pmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \text{define name}_{1.3} \\ \begin{bmatrix} \begin{pmatrix} \text{lambda} \\ \{\text{statements}\} \end{pmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \text{define name}_{1.4} \\ \begin{bmatrix} \begin{pmatrix} \text{lambda} \\ \{\text{statements}\} \end{pmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} & & \emptyset \\ & \begin{bmatrix} \text{identify contexture}_{2.2} \\ \begin{pmatrix} \text{define name} \\ \begin{pmatrix} \text{lambda} \\ \{\text{statements}\} \end{pmatrix} \end{pmatrix} \end{bmatrix} & \emptyset \\ & & \begin{bmatrix} \text{identify contexture}_{3.3} \\ \begin{pmatrix} \text{define name} \\ \begin{pmatrix} \text{lambda} \\ \{\text{statements}\} \end{pmatrix} \end{pmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

Null

Replication of a contexture into itself at an elected locus

This kind of replication of *a* contexture changes into an *iterative* reflection.



3.3. Mediators

"In a more generous setting the systems can be distributed over a network, say Internet, and mediators similar to compilers would have to mediate the distributed programming on mediated poly-processor computing systems. Mediators *would have to parse the different programming approaches in respect to their* mediability. *That is, conditions of mediation would have to be checked, optimized and debugged. Thus, the chain of realization from programing to compiling has first to be augmented by a system of mediation. The new paradigm of realization now is* programming-mediating-compiling *in distributed and mediated programming languages."*

4. Semiotic interpretations

4.1. Positions

Positioning (Setzung) semiotic systems isn't well studied in the polycontextural literature.
Nevertheless, semiotics, i.e. semiotic systems, like Peirce-Bense-Toth systems, might naturally be distributed and mediated over the kenomic matrix. Classical semiotics is distributed, *per se*, over a single kenomic locus which isn't accessible to semiotics by semiotic means.

The examples for the exercises shows two sorts of modelings of the kenomic position matrix: a distribution of mediated semiotic *dyads* and a distribution of mediated semiotic *triads*.

In the *first* case, triadic-trichotomic semiotics, i.e. the matrix of sign classes, is understood as a mediation of dyads. The decomposition of the Peircean triads into mono-contextural dyads corresponds to the Bensean interpretation of semiotics and its semiotic Cartesian matrix.

Bense' s approach is not including a mediation of the dyads neither a contextural interpretation of the dyads. The dyads are composed by some set- or category theoretical operations. A mediative interpretation would automatically lead to a kind of a place-valued system for semiotics and inherit all its formal problems.
Mediation of dyads in the sense of polycontexturality was introduced by my own papers and published recently at this place.

The *second* case is distributing and mediating full *triads* over the kenomic matrix. Hence, the mediation is concerned with triads and not with their internal structure, i.e. dyads, like the first example. This example of a different modeling, is mirrored by the different indexes involved.

Epistemological cuts
The decision to chose an epistemological paradigm of arbitrary complexity should be free. Unfortunately, there are only a few accessible. There is no special need to believe in dyads, triads tetrads, etc. or in monads. The question is, does it work? It works for dyads. It rarely works for triads. And there is no accepted formalism for tetrads. Obviously, n-adic relations of algebraic relation theory and relational logic are based on dyads, and their n-ads are always reducible to dyads. What's lost with this manoeuvre isn't told in general.

"To iterate is human ... but to recurse is divine." (Alfred Inselberg)
Hence, to di(s)rempt must be devilish?

Therefore, it should be a question of a free decision to develop semiotics as founded in dyads, triads or tetrads, or generally in non-reducible n-ads.

To go further with this **exercise**, study the paper *"Transjunctional semiotics"*.

4.1.1. Triadic semiotics as mediations of dyads

What does it mean to choose a triadic foundation of semiotics?
As sketched before, triadicity as a mediation of dyads, hence, has to be realized on all levels of thematization. That is, a triadic matrix alone doesn't mean much if it is not based simultaneously on all needed triadic formal systems, like logics, arithmetic, category theory, etc.
On the other hand, the mechanism of *mediation* of dyads has not to stop with the construction of triads. All kind of n-ads, based on mediated dyads, might be constructed.

Table notation

PM	O_1	O_2	O_3
M_1	$Sem_{1.1}$	–	–
M_2	–	$Sem_{2.2}$	–
M_3	–	–	$Sem_{3.3}$

Bracket notation

$$\left[\begin{array}{l} \left[\begin{array}{l} O_1 \\ \left(\begin{array}{l} M_1\ M_2\ M_3 \\ \qquad \left(Sem_{100}\right) \end{array}\right) \end{array}\right] \\ \left[\begin{array}{l} O_2 \\ \left(\begin{array}{l} M_1\ M_2\ M_3 \\ \qquad \left(Sem_{020}\right) \end{array}\right) \end{array}\right] \\ \left[\begin{array}{l} O_3 \\ \left(\begin{array}{l} M_1\ M_2\ M_3 \\ \qquad \left(Sem_{003}\right) \end{array}\right) \end{array}\right] \end{array}\right]$$

Scheme of $\mathbf{Sem^{(3,2)}}$ **:**

$$\text{Semiotics}^{(3,2)} = \begin{bmatrix} (1.1)_{1.3} & \longrightarrow & (1.2)_{1} & \longrightarrow & (1.3)_{3} \\ \downarrow & x & \downarrow & x & \downarrow \\ (2.1)_{1} & \longrightarrow & (2.2)_{1.2} & \longrightarrow & (2.3)_{2} \\ \downarrow & x & \downarrow & x & \downarrow \\ (3.1)_{3} & \longrightarrow & (3.2)_{2} & \longrightarrow & (3.3)_{2.3} \end{bmatrix}$$

Sub – system decomposition of $\mathbf{Sem^{(3,2)}}$ **:**

$$\text{sub-system}_1 = \begin{bmatrix} (1.1) & \longrightarrow & (1.2) \\ \downarrow & x & \downarrow \\ (2.1) & \longrightarrow & (2.2) \end{bmatrix}$$

$$\text{sub-system}_2 = \begin{bmatrix} (2.2) & \longrightarrow & (2.3) \\ \downarrow & x & \downarrow \\ (3.2) & \longrightarrow & (3.3) \end{bmatrix}$$

$$\text{sub-system}_3 = \begin{bmatrix} (1.1) & \longrightarrow & (1.3) \\ \downarrow & x & \downarrow \\ (3.1) & \longrightarrow & (3.3) \end{bmatrix}$$

Positions

3 – contextural semiotic matrix

$$\text{Sem}^{(3,2)} = \begin{pmatrix} MM & .1_{1.3} & .2_{1.2} & .3_{2.3} \\ 1_{1.3} & \mathbf{1.1_{1.3}} & \mathbf{1.2_{1}} & \mathbf{1.3_{3}} \\ 2_{1.2} & \mathbf{2.1_{1}} & \mathbf{2.2_{1.2}} & \mathbf{2.3_{2}} \\ 3_{2.3} & \mathbf{3.1_{3}} & \mathbf{3.2_{2}} & \mathbf{3.3_{2.3}} \end{pmatrix}$$

Reflections

3 – contextural semiotic matrix [id, red, id]

$$\text{Sem}^{(3,2)}_{[id,red,id]} = \begin{pmatrix} MM & .1_{1.3} & .2_{1.2} & .3_{2.3} \\ 1_{1.3} & \mathbf{1.1_{1.3}} & \mathbf{1.2_{1}} & \mathbf{1.3_{3}} \\ 2_{1.2} & \mathbf{2.1_{1}} & \mathbf{2.2_{1.1}} & \mathbf{1.2_{1}} \\ 3_{2.3} & \mathbf{3.1_{3}} & \mathbf{2.1_{1}} & \mathbf{1.1_{1.3}} \end{pmatrix}$$

Interactions

3 – contextural semiotic matrix [bif, id, id]

$$Sem^{(3,2,2)}_{(bif,\,id,\,id)} = \begin{pmatrix} [\blacklozenge,\circ,\circ] & 1 & 2 & 3 \\ 1 & \mathbf{1.1}_{1.3} & \mathbf{2.3}_{2.3} & \mathbf{1.3}_{3} \\ 2 & \mathbf{3.2}_{2.3} & \mathbf{2.2}_{1.2} & \mathbf{2.3}_{2} \\ 3 & \mathbf{3.1}_{3} & \mathbf{3.2}_{2} & \mathbf{3.3}_{2.3} \end{pmatrix}$$

Replications

3 – contextural semiotic matrix [repl, id, id]

$$Sem^{(3,2,2)}_{(repl,\,id,\,id)} = \begin{pmatrix} MM & .1_{1.3} & .2_{1.2} & .3_{2.3} \\ 1_{1.3} & \mathbf{1.1}_{1.1.3} & \mathbf{1.2}_{1.1} & \mathbf{1.3}_{3} \\ 2_{1.2} & \mathbf{2.1}_{1.1} & \mathbf{2.2}_{1.1.2} & \mathbf{2.3}_{2} \\ 3_{2.3} & \mathbf{3.1}_{3} & \mathbf{3.2}_{2} & \mathbf{3.3}_{2.3} \end{pmatrix}$$

4.1.2. Tetradic semiotics as mediations of dyads

Positions

4 – contextural 2 – semiotic matrix

$$Sem^{(4,2,3)} = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1.3.6} & 1.2_{1} & 1.3_{3} & 1.4_{6} \\ 2 & 2.1_{1} & 2.2_{1.2.5} & 2.3_{2} & 2.4_{5} \\ 3 & 3.1_{3} & 3.2_{2} & 3.3_{2.3.4} & 3.4_{4} \\ 4 & 4.1_{6} & 4.2_{5} & 4.3_{4} & 4.4_{4.5.6} \end{pmatrix}$$

$$val(Sem^{(4,1,2)} \, x \, Sem^{(4,1,2)}) =$$

$$(1_{1.3.6},\ 2_{1.2.5},\ 3_{2.3.4},\ 4_{4.5.6}) \, x \, (1_{1.3.6},\ 2_{1.2.5},\ 3_{2.3.4},\ 4_{4.5.6})$$

with:

$$val(Sem^1 \, x \, Sem^1) = (1, 2)_1 \, x \, (1, 2)_1$$

$$val(Sem^2 \, x \, Sem^2) = (2, 3)_2 \, x \, (2, 3)_2$$

$$val(Sem^3 \, x \, Sem^3) = (1, 3)_3 \, x \, (1, 3)_3$$

$$val(Sem^4 \, x \, Sem^4) = (3, 4)_4 \, x \, (3, 4)_4$$

$$val(Sem^5 \, x \, Sem^5) = (2, 4)_5 \, x \, (2, 4)_5$$

$$val(Sem^6 \, x \, Sem^6) = (1, 4)_6 \, x \, (1, 4)_6.$$

4.1.3. Pentadic semiotics as mediations of triads

As an example we shall study the mediation of two triadic-trichotomic semiotic basic systems, Sem^1 and Sem^2. Both semiotic systems are not decomposed into dyadic relations but kept together as triadic systems. A ‘concatenational’ composition of two genuine triadic systems results in a pentadic semiotic system as much as a ‘concatenational’ composition of dyads results in a composed triad.

With the composition formula:
$\text{compl}(\text{Sem}^{(5,3)}) = \text{compl}(\text{Sem}^1 \otimes \text{Sem}^2) = \text{compl}(\text{Sem}^1) + \text{compl}(\text{Sem}^2) - 1$,
hence: 3+3-1=5.

$$\text{Sem}^1 = \begin{bmatrix} 1.1_1 & 1.2_1 & 1.3_1 \\ 2.1_1 & 2.2_1 & 2.3_1 \\ 3.1_1 & 3.2_1 & 3.3_1 \end{bmatrix}, \ \text{Sem}^2 = \begin{bmatrix} 3.3_2 & 3.4_2 & 3.5_2 \\ 4.3_2 & 4.4_2 & 4.5_2 \\ 5.3_2 & 5.4_2 & 5.5_2 \end{bmatrix}$$

$$\text{Sem}^{(5,3,2)} = \begin{bmatrix} MM & 1 & 2 & 3 & 4 & 5 \\ 1 & 1.1 & 1.2 & 1.3 & 1.4 & 1.5 \\ 2 & 2.1 & 2.2 & 2.3 & 2.4 & 2.5 \\ 3 & 3.1 & 3.2 & 3.3 & 3.4 & 3.5 \\ 4 & 4.1 & 4.2 & 4.3 & 4.4 & 4.5 \\ 5 & 5.1 & 5.2 & 5.3 & 5.4 & 5.5 \end{bmatrix}$$

The sub – system indices of the matrix values are omitted.

$$\textbf{Partions}_{(5,3)} = \left\{ \begin{matrix} (1, 2, 3), (3, 4, 5), \\ (1, 2, 4), (1, 2, 5), (1, 3, 4), (1, 3, 5), (1, 4, 5), \\ (2, 3, 4), (2, 3, 5), (2, 4, 5) \end{matrix} \right\}$$

Sign class subsystems

$SR^1 = (1, 2, 3)$, $SR^2 = (3, 4, 5)$,

$SR^3 = (1, 2, 4)$, $SR^4 = (1, 2, 5)$, $SR^5 = (1, 3, 4)$, $SR^6 = (1, 3, 5)$, $SR^7 = (1, 4, 5)$

$SR^8 = (2, 3, 4)$, $SR^9 = (2, 3, 5)$, $SR^{10} = (2, 4, 5)$.

Mediation Conditions MC

$$MC(SR^1 \otimes SR^2) = \{3.3_1 \equiv 3.3_2\}$$

$$MC(SR^3 \otimes SR^4 \otimes SR^5 \otimes SR^6 \otimes SR^7) = \left\{ \begin{matrix} 1.1_3 \equiv 1.1_4 \equiv 1.1_5 \equiv 1.1_6 \equiv 1.1_7, \\ 4.4_3 \equiv 4.4_5, \\ 5.5_4 \equiv 5.5_6 \equiv 5.5_7 \end{matrix} \right\}$$

$$MC(SR^8 \otimes SR^9 \otimes SR^{10}) = \left\{ \begin{matrix} 2.2_8 \equiv 2.2_9 \equiv 2.2_{10} \\ 5.5_9 \equiv 5.5_{10} \\ 4.4_8 \equiv 4.4_3 \equiv 4.4_{5,} \end{matrix} \right\}$$

$$MC(SR^{(5)}) = \left\{ \begin{matrix} 1.1_3 \equiv 1.1_4 \equiv 1.1_5 \equiv 1.1_6 \equiv 1.1_7, \\ 2.2_8 \equiv 2.2_9 \equiv 2.2_{10}, \\ 3.3_1 \equiv 3.3_2, \\ 4.4_3 \equiv 4.4_5 \equiv 4.4_{8,} \\ 5.5_4 \equiv 5.5_6 \equiv 5.5_7 \; 5.5_9 \equiv 5.5_{10} \end{matrix} \right\}$$

Sub – system decomposition (2 – sub – systems inherited , indices have to be adjusted)

$$SR^{1}{}_{(1,2,3)} = \begin{bmatrix} (1.1)_{1.3} & \longrightarrow & (1.2)_{1} & \longrightarrow & (1.3)_{3} \\ \downarrow & x & \downarrow & x & \downarrow \\ (2.1)_{1} & \longrightarrow & (2.2)_{1.2} & \longrightarrow & (2.3)_{2} \\ \downarrow & x & \downarrow & x & \downarrow \\ (3.1)_{3} & \longrightarrow & (3.2)_{2} & \longrightarrow & (3.3)_{2.3} \end{bmatrix}$$

$$SR^{2}{}_{(3,4,5)} = \begin{bmatrix} (3.3)_{1.3} & \longrightarrow & (3.4)_{1} & \longrightarrow & (3.5)_{3} \\ \downarrow & x & \downarrow & x & \downarrow \\ (4.3)_{1} & \longrightarrow & (4.4)_{1.2} & \longrightarrow & (4.5)_{2} \\ \downarrow & x & \downarrow & x & \downarrow \\ (5.3)_{3} & \longrightarrow & (5.4)_{2} & \longrightarrow & (5.5)_{2.3} \end{bmatrix}$$

$$SR^{3}{}_{(1,2,4)} = \begin{bmatrix} (1.1)_{1.3} & \longrightarrow & (1.2)_{1} & \longrightarrow & (1.4)_{3} \\ \downarrow & x & \downarrow & x & \downarrow \\ (3.1)_{1} & \longrightarrow & (3.2)_{1.2} & \longrightarrow & (3.4)_{2} \\ \downarrow & x & \downarrow & x & \downarrow \\ (4.1)_{3} & \longrightarrow & (4.2)_{2} & \longrightarrow & (4.4)_{2.3} \end{bmatrix}$$

$$SR^{4}{}_{(1,2,5)} = \begin{bmatrix} (1.1)_{1.3} & \longrightarrow & (1.2)_{1} & \longrightarrow & (1.5)_{3} \\ \downarrow & x & \downarrow & x & \downarrow \\ (2.1)_{1} & \longrightarrow & (2.2)_{1.2} & \longrightarrow & (2.5)_{2} \\ \downarrow & x & \downarrow & x & \downarrow \\ (5.1)_{3} & \longrightarrow & (5.2)_{2} & \longrightarrow & (5.5)_{2.3} \end{bmatrix}$$

$$SR^{5}{}_{(1,3,4)} = \begin{bmatrix} (1.1)_{1.3} & \longrightarrow & (1.3)_{1} & \longrightarrow & (1.4)_{3} \\ \downarrow & x & \downarrow & x & \downarrow \\ (3.1)_{1} & \longrightarrow & (3.3)_{1.2} & \longrightarrow & (3.4)_{2} \\ \downarrow & x & \downarrow & x & \downarrow \\ (4.1)_{3} & \longrightarrow & (4.3)_{2} & \longrightarrow & (4.4)_{2.3} \end{bmatrix}$$

$$SR^{6}{}_{(1,3,5)} = \begin{bmatrix} (1.1)_{1.3} & \longrightarrow & (1.3)_{1} & \longrightarrow & (1.5)_{3} \\ \downarrow & x & \downarrow & x & \downarrow \\ (3.1)_{1} & \longrightarrow & (3.3)_{1.2} & \longrightarrow & (3.5)_{2} \\ \downarrow & x & \downarrow & x & \downarrow \\ (5.1)_{3} & \longrightarrow & (5.3)_{2} & \longrightarrow & (5.5)_{2.3} \end{bmatrix}$$

$$SR^{7}{}_{(1,4,5)} = \begin{bmatrix} (1.1)_{1.3} & \longrightarrow & (1.4)_{1} & \longrightarrow & (1.5)_{3} \\ \downarrow & x & \downarrow & x & \downarrow \\ (4.1)_{1} & \longrightarrow & (4.4)_{1.2} & \longrightarrow & (4.5)_{2} \\ \downarrow & x & \downarrow & x & \downarrow \\ (5.1)_{3} & \longrightarrow & (5.4)_{2} & \longrightarrow & (5.5)_{2.3} \end{bmatrix}$$

Reduction of Sem_2 to Sem_1

$\mathrm{red}(Sem^{(5,3,2)}) =$

MM	1	2	3	4	5
1	1.1	1.2	1.3	1.4	1.5
2	2.1	2.2	2.3	2.4	2.5
3	3.1	3.2	3.3	3.2	1.3
4	4.1	4.2	2.3	2.2	1.2
5	5.1	5.2	3.1	2.1	1.1

Replication of Sem_1 to $Sem_{1.1}$

$\mathrm{repl}(Sem^{(5,3,2)}) =$

MM	1	2	3	4	5
1	$1.1_{1.1}$	$1.2_{1.1}$	$1.3_{1.1}$	1.4	1.5
2	$2.1_{1.1}$	$2.2_{1.1}$	$2.3_{1.1}$	2.4	2.5
3	$3.1_{1,1}$	$3.2_{1.1}$	$3.3_{1.1.2}$	3.4_2	3.5_2
4	4.1	4.2	4.3_2	4.4_2	4.5_2
5	5.1	5.2	5.3_2	5.4_2	5.5_2

Interaction between Sem_2 and Sem_1

$\mathrm{inter}(Sem^{(5,3,2)}) =$

MM	1	2	3	4	5
1	1.1_1	3.4_2	3.5_2	1.4	1.5
2	3.4_2	2.2_1	2.3_1	2.4	2.5
3	3.5_2	3.2_1	$3.3_{1.2}$	3.4_2	3.5_2
4	4.1	4.2	3.4_2	4.4_2	4.5_2
5	5.1	5.2	3.5_2	5.4_s	5.5_2

4.1.4. What is the practical use of that fuss?

If there is any practical use for triadic-trichotomic semiotics, as Toth and others demonstrated *in extenso*, any extension of triadicity might open up some more complexity to deal with real-world matters in an operative and not reducing manner.

In sociology, cultural theory, international law, legitimations for torture and killing innocent people for good and accepted reasons, we encounter, in short, only *two* structural models of reasoning and acting. One is reducing complexity of what ever domain to a *binary* and *dichotomic* pattern. The other extreme is dissolving complexity into a *multitude* of autonomous isolated and and not-mediated dichotomous systems.

The first has the advantage of maximal *operativity* in technological and juridical systems, supporting nearly fully-automated surveillance systems and killing procedures.

The second is hopelessly non-operative and still based on humanistic propaganda for a better world - and even for Change.

"The genius of Michelangelo is like the genius of the Talmud, with several layers of meaning*, one on top of another. So you can interpret it in terms of Christianity and Judaism, sociologically, historically and artistically. We are just adding one level that has either been ignored or covered up over the centuries."* Cathryn Drake, Did Michelangelo Have a Hidden Agenda? http://online.wsj.com/article/SB122661765227326251.html

"For the third millennium, the struggle against semantic disorder and perversions of the intellect should supersede, precede and be sustained in all cultures, religions, systems of thought and political systems whenever there is a historical necessity to initiate a war of liberation from oppression, domination and exclusion." Mohammed Arkoun, ISLAM: To reform or to subvert?, The rule of law and civil society in Muslim context, Beyond Dualist Thinking, 2006, p. 381

Hence, the academic question still remains:
Wouldn't it be worth to support a developement of a cultural paradigm in which pluriversity *and* operativity *could co-operate together?*

The logic of bailout strategies

The end of capitalism or the end of the state?

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Abstract

Some thoughts about/of the logic, blend, chiasm and diamond of bailout strategies. Eliciting aspects of the maxim: *"Without insurrection, no resurrection"*.

1. Duality, Blends, Chiasm or Diamonds?

"The end of the year is approaching" and *"We are coming to the end of the year."*

"Notice that in the first, time is moving, while in the second the deictic reference point (conventionally called "ego") is moving while time is fixed." (J. Goguen)

Category

The category of approaching.

The concept of "approaching" is categorically closed. Compositions of "approaching" terms are producing composed "approaching", and nothing else. Steps of approaching are building morphisms. Approaching morphisms are composable and are ruled by identity, commutativity and associativity. Hence, designing the category of "approaching"

The category of coming.

The concept of "coming" is categorically closed. Compositions of "coming" terms are producing composed "comings", and nothing else.

Dual to the category "approaching, the category "becoming" is defined by the steps of approaching. "Becoming" morphisms are composable and are ruled by identity, commutativity and associativity. Hence, designing the category of "becoming".

Duality and inversion

A *dualism* between "approaching"- and "coming"-terms might be constructed.

In another setting, between "approaching" and "coming" terms an *inversion* might be considered.

Dualism and inversion are categorically closed, too.

The dual or opposite of an "approaching" or a "coming" category is itself a category.

Thus, dualization and inversion are not unifying the concepts of "coming" and "approaching", both concepts are left in alternating isolation.

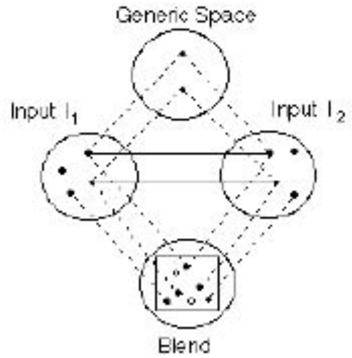
Possible interactions of "approaching" and "coming" are tackled by the mechanisms of *blending*, *chiasm* and *diamonds*.

Blending

Blending (blend, mélange, mixture, integration) together "approaching" and "coming".

This strategy is well introduced by Goguen's *"Semiotic Morphisms"* approach to complex and polysemic systems. Blends are structured by categorical morphisms and composition.

Blending is conceptualized on the base of a single *external* observer for whom blending is offering a kind of a *holistic* unification of both tendencies, suggesting something new: a *blend of both*, here, with "coming" and "approaching" mélanged together. But to add emergent features to the blending process, some ingredients have to be spent.



"*Blending* two conceptual spaces yields a new space that combines parts of the given spaces, and may also have *emergent* structure." (Goguen)

Chiasm
Chiastic inter-change of "approaching" and "coming".
A chiastic *interactive* modeling is involving different points of view for different observers to describe the conceptual interaction between both as observer-depending.
What is "approaching" for one position is "coming" for the other position, and *vice versa.*
Both positions are aware about their inter-dependency. They are not observing in separation, each for itself, arbitrarily, different phenomenons but a complexion of "movements".

Chiasms, consisting of order, coincidence and exchange relations, are framing such inter-dependency.

Between "coming" and "approaching" a kind of an exchange relation, i.e. a *duality* happens.
The coincidence relation is guaranteeing a kind of "togetherness" of both categories, "coming" and "approaching". It also gives the possibility of measuring their conceptual distance. And obviously, the objects of the categories "coming" and "approaching" are ruled, *per se*, by order relations, i.e. they are representing morphisms.

What is emerging as something new in the process of chiastic mediation is a complex phenomenon requiring different positions of observation to be recognized or constructed. The new is given by the features of the chiastic mechanism itself and not by any additional elements from the outside of the conceptual mechanism.

In contrast to blending, the processes involved in chiastic mediation retain their autonomy and are not mélanged together into a holistic wholeness, including elements from nowhere or left in isolation, like in the case of dualities and inversions.

Diamond
Diamond interplay of "approaching" and "coming".
The diamond approach is liberating involved processes even more than the chiastic conceptualization, and is radicalizing the interactional approach of chiasms to an *interplay* between categorical and saltatorical systems.

Both conceptualizations, the categorical and the saltatorical, of a diamond are not only autonomous but are reflecting their antagonistic movements, i.e. they are involved in an *antidromic* and *parallax* interplay.

2. Bailout logic

In economics, a bailout is an act of loaning or giving capital to a failing business in order to save it from bankruptcy, insolvency, or total liquidation and ruin.

Detailed material and description about the complex aspects of the USA bailout is summarized at Emergency_Economic_Stabilization_Act_of_2008. Or watch the video:Why Wont The Bail Out Work?

There are funny discussions about the nature, probably it is better to call it, the ideology and stratagemes, of what's going on today in the economic and financial world.

Without surprise, there is also an *"illogic of the bailout":*

"Are the American people waiting for their own bailout*? It's never going to come. You are the* bailer, *not the* bailee.
This is the biggest, fastest wealth-transfer scheme in the history of the world. And no one is marching *on Washington. Explain this to me."* (J. Farah)

The Coming Global Insurrection

But despite the propagandistic silence of the media, there is some uproar and insurgency globally to observe. To the global capitalism , its state doctrin and its cyclic collapse, a *global insurrection* is emerging out of suppression and ruins.

Without insurrection, no resurrection.

On the level of debates

The funniest chapter is the emergence of an ever growing *debate* about the transformation of the relationship between capital, economy, market on one side and state, governments, administrations, bureaucracy on the other side of this dichotomous distinction or antagonism..

Things are not as funny as they could be. The biggest economic crises since the last big crash is producing serious global poverty and will become a good reason for further wars.

What's annoying me is that the same stupidity of our ruler and their academic adviser is going on without any interruption or critical reflection on what happened and still is happening.

The same politicians and Nobel Prize Winners are on stage.

Do we have to enter this debate?
There is no need to get messed up about their opinions.
It seems to be good enough to think about the most simple structure of the whole manoeuvre to understand its logic and strategy.

The state, of whatever governmental form, from the Swiss democracy to Gordon Brown's British surveillance administration, the USA to China, the state is asked for or is offering a *bail-out* of companies, corporations, institutions which are running into bankruptcy and other dysfunctionality.

The bail-out is paid by so called tax payers money. Hence, the state will take over such companies to some degree in ownership and regulations. It is seen as a reversal of the process of privatization. Some, are happy to interpret it as the symptoms of the end of capitalism. Unfortunately, the tax payers are not the players in this play. They are set into *silence*, by the state, the capital and their ideologues, parties and unions.

There are others, not many, for good reasons, which are more cynical and are understanding the bail-out manoeuvre of the state as a *coup* of the capital to overtake the state with its tax payer's money and its power of regulation.

This position in the debate is still hidden in the background. It would be too dangerous to defend such a complementary position explicitly and with the proper intensity.

It is also not easy to conceptualize such a coup, because the categories of state and capital are getting involved into a transformation, which is beyond classical political theories and which is also transcending logical and dialectical understanding.

E.g., Gordon Brown, the British Prime Minster, a politician, who thinks from himself that he is saving the world, at least the Western world, with his programs of the Third Way, is becoming as politician a capitalist, overtaking banks and industries with the means of tax payers money. Meanwhile the heads of some Scotish banks, as capitalists, are becoming politicians in the banks ruled by the government.

The *silencing* of the voice of the people as consumers and workers is perfect. The ideologues of both sides are telling us, *"Don't worry, it's only transitional and temporary. Everything will be back to normal, soon"*.

It is said that the state will take shares of the companies and will use more control over them. Does it matter? There will be bankers and managers from the side of the capital who will enter the save heaven of governmental offices to do the job. Hence, the capitalist bankers are becoming administrators and the governmental administrators are becoming bankers.

The governmental bankers, which had been in charge to control the capitalist banks, are as much involved in the crash as their colleagues from the so called private sector.

Both positions of the debate, surely, are demanding for themselves unique truth of their interpretations. Only debaters with some secured positions are liberal or tolerant enough to accept, at least, the existence and relative reasonability of the complementary position. But that doesn't matter, now.

Hence, we are at the beginning, again. The crisis is declared as much too serious to allow the luxury of philosophical reflections and distinctions and is only weakening, argumentatively, the severity of the global situation.

In fact, there is, up to now, no debate at all. The opposite position to ones position in this virtual debate is declared as mislead and for empirical and logical reasons as wrong.

It is still the dominating position that the government has to save the failing industries (banks, car, insurance, etc.) with the help of bailout strategies. But it is only consistent, and is in fact on the way to happen, that the state is offering itself a bailout to survive.

In other words, the bailout strategies to save the economy will be applied to save the administration, military, police, and all the surveillance and controlling institutions of Web2.0 and the building industry to build more prisons.

The government declares, it will use the tax payers money properly, fulfilling highest standards of economical thinking and ethics. But who in the government is, by definition, able to do this honorable job? Nobody else than bankers have by profession the knowledge and experience to be able to such a capitalist job. Hence, the bankers are in the government and are controlling the state banks.

Does it matter, where the money comes from, directly from the national tax payer or indirectly, via China e.g. ? The government wasn't elected to spend this money especially for bailouts, anyway.

Supposed distinctions:
private/public,
state/capital
market/company
state/economy
administration/democracy

Is the tax payers money private or public?
Is a tax payer private? What happens if the so called worker is his own capitalist? A shareholder of "his" company for which he is now on the way to pay his bailout with the generous help of the government? And the capitalist, e.g. the manager his own (self-)exploiter?

Hence, the tax payer is paying the bailout of his company where he is a shareholder and a worker at once, which is exploiting him and makes him, at the same time, an owner of the company, which is, together with him, on the way to bankruptcy. This surely has to be prevented, otherwise the tax payer gets unemployed and is losing his status as a shareholder and as a worker of his company.

It also has to be prevented because the tax payer could start a *rebellion* against the whole system, paid on the base of his private money he put aside. But how could he, and where?

There are no capitalists nor workers, anymore. Both are intertwined into the complex reality of globalization and the self-exploitation by anonymous corporations.

That is, public money from the private tax payer has to save the private company owned by the capital. The state wants to become a part-owner of the capital with the money of the private shareholders of the company.
The mission is to save employment for the private shareholders.
This sounds humanitarian and is in harmony with a progressive protestant work ethic.

But this is only one side of the coin.

Is it not better for the public capital and the markets to get as much capital by the state's private capital to be fit to survive against the consequences of mismanagement and global competition?

In fact, and this will become, in the future, more and more obvious, the capital has to be made fit against the cultural limitations of Western science and technology and their decline.

The so called nationalization of markets is in fact a disguised *overtaking* of the state by the capital.

The state, complementary, is hallucinating a control and annexation of the markets and the capital. He wants to become owner of the banks, etc.
The bailout *''Promotes centralized bureaucracy by allowing government powers to choose the terms of the bailout.''* (WiKi)

The state is playing the rescuer of the markets to save its own existence.
The capital is overtaking in disguise the state to save its own existence.

Therefore, the whole bailout saga is a secret *coup*:
coup d'´etat and coup de capitale.

The common of both is the commotion and the threat of their proper existence.
Both forms of existence are fundamentally out of date and obsolete.

The epistemological problem is:
The (bailout) situation is polycontextural and self-referential, and our mathematical and computational paradigms, ideologies and tools are mono-contextural and linear.

3. Neither nationalization nor privatization

3.1. The Big Joke of the Third Way

The Big Joke of the Third Way is: Who is overtaking whom?

Circular situations should be read in both directions.
Hence, the *"withering away of the state"* (Lenin)[6] has to go *"hand in hand"* with its inverse movement of a *dissolution* of the capital.

3.1.1. Bailout of the capital

The state is supporting the capital.
The capital is using the state'.s capital

The government is governmentalizing the capital.
The capital is capitalizing the state.

CSSC:
Nationalization: capital --> state
Privatization: state --> capital.

Der Staat unterstützt Banken (Verstaatlichung des Kapitals).
Die Banken bedienen sich des Staates. (Verkapitalisierung des Staates)

3.1.2. Bailout of the state

The state is using the capital's capital.
The capital is supporting the state.

The state is governmentalizing the capital.
The capital is capitalizing the state.

SCCS:
Governmentalization: state --> capital
Capitalization: capital --> state.

3.2. Blending of bailout

The blending interpretation of the bailout is blinding for the fact that the emergent features of whatever mélange between capital and state has first to be generated and paid.
But a blending approach,with its undecided mix, is best prepared to offer the necessary structural vagueness and non-transparency for ever growing new departments in the opacity of both administrations, the state and the capital.

3.3. Chiasm of bailout

Inter-dependencies of both, capital and state, still intertwined and reciprocatively dependent, but at least a holistic and processual conceptualization and understanding of the mechanism is uncovered and conceived by the chiastic thematization of the bailout..

The chiastic approach of the bailout is emphasizing the *complicity* of both movements, the privatization and the nationalization, as belonging to the same reality.

Hence, any controversial debate, like with the logical, contradictorily or antagonistic, modeling, which is understanding the parts as singular or in a reflective turn as dual, is obsolete within the chiastic understanding.

What has to be studied is the inter-relational complicity of both interpretations, their chiastic relationality, like the coincidence and exchange relations. To function as a whole of interdependency, the exchange relations between the opposite, but common terms have to be adapted by the coincidence relations between the similar but distributed terms.

The *dualistic* interpretation of the situation is conflictive and is not offering a tribune for negotiation. One, and only one interpretation is accepted by the defenders as adequate and true. On the base of such blindness, only ethical and moralizing judgements and the cry for more interventional actions are available.

The *chiastic* interpretation is offering an insight into the very mechanism of the conflictive situations. The mediating contextures of the chiasm is placing the structural possibility of negotiation and resolution, albeit inside of the framework of the scenario.

Both positions, the dualistic and the chiastic, are accepting the situation as it is. This is reasonable for descriptive and analytical motives. Despite its non-classical conceptuality, the chiastic model is not yet offering any structural strategies to overcome and reject the structural fundaments of the whole situation.

As a result, a kind of a humanitarian harmony of the antagonism remains as the ultimate aim. This solution of the problem is guaranteeing a safe return of the problem on a new, more complex and reflected level of development, securing an even deeper and broader stage-management of the *"eternal recurrence"* of booms, bubbles and crashes.

3.4. Diamond of bailout

The diamond approach is not denying the correctness of the chiastic modeling of the antagonistic situation but is trying to reject the whole construction in favor of a future-oriented transformation, where the components or "objects" of the chiasm, state and capital, are dissolved.

The diamond approach, with its complementarity of *acceptional* and *rejectional* thematizations, is separating the antagonistic aspects from their intertwining complicity. Both are conceived as autonomous societal movements, crossing at some parts, historically, and disappearing into other situational interactions.

Their complicity is historic and there is no necessity to reduce social life to it.

Because of the autonomic interplay between acceptional tendencies, framed by *categories* and rejectional tendencies, framed by *saltatories*, a chance to separate both structurations (of societal structures and movements) is conceived and accessible to realize.

3.5. The bailout of the bailout

Rejection of the figure of bailouts by *dissemination* and *subversion.*

The bailout of banks and industries by the governments is a big sandbox game: moving money, power and control from one societal heap to another societal heap of a national and/or global economy framed by the opposition of capital and state.

3.5.1. Dissemination: Polycontexturality of society

Polycontexturality of society is dissolving such terminological identifications like 'state' and 'capital'. Terms, like 'state' and 'capital', are not observer-independent identifications, like 'potato' or 'herring'. They are depending on observations and are set into multiple perspectives, which are dissolving their a-historical and nominalistic identity.

Polycontextural logics are prepared to describe, formalize and implement such complexities in an adequate way.

Gunther Teubner is describing the challenges for law and society and its understanding by polycontextural thematizations.

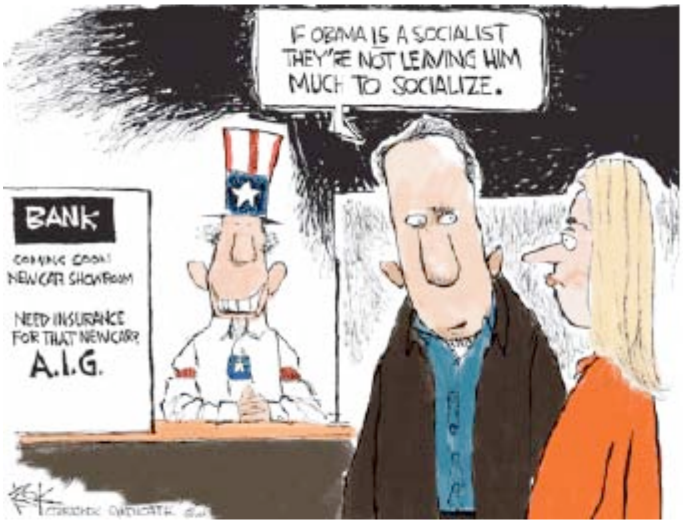
"In Habermas' "ideal speech situation", formal procedures are supposed to guarantee the undistorted reciprocal expression of individual interests as well as their universalization into morally just norms. However, polycontexturality, one of the most disturbing experiences of our times, thoroughly discredits these recent variations of a Kantian concept of justice.

"With polycontexturality understood as the emergence of highly fragmented intermediary social structures based on

binary distinctions, society can no longer be thought of as directly resulting from individual interactions, and justice can no longer be plausibly based on universalizing the principle of reciprocity between individuals."

"In these perspectives, irreconcilable incompatibilities result from colliding social practices each of them endowed with their own rationality and normativity and with an enormous potential for mutually-inflicted damage.

The highest degree of abstraction has been reached by Gotthard Günther *who radicalizes polycentricity into a more threatening polycontexturality, that is, a plurality of mutually exclusive perspectives which are constituted by binary distinctions. They are not compatible with one another and can be overcome only by rejection values which in their turn lead to nothing but to different binary distinctions." (Teubner, p. 4/5)*



3.5.2. Subversion: Morphogrammatics of sociality

A morphogrammatic subversion of the understanding of society is rejecting their leading concepts and models of monetary and phono-logical interpretations.
Subversion, hence is not rejection *"which in their turn lead to nothing but to different binary distinctions."* Binary distinctions discovered by rejections are establishing, again, contextures albeit new ones, and thus there is, in this strategy, no escape and nothing left except of contextures, and contextures of contextures.

There is not much to tell about such a morphogrammatic turn or abstraction, i.e. subversion, and it is hard to write and to inscribe how to subvert the surface structures of society to 'enlight' its hidden actional structuration by morphogrammatics.

Morphogrammatics is abstracting even from *"the highest abstraction"* (Teubner) of the contextures of polycontexturality.

To try it with metaphors, it seems to be reasonable, in what ever logic or rationality, that contextures too, are taking *place*, are *positioned* and *localized*, where?, in a kind of space(s). Such a space might be called an inscriptional space or even more metaphorically a (meta-/proto)conceptual space, giving space and loci for *éspacement* (spacing) and *temporalisiation* of positioned contextures and their interplay. Such a space is empty of all kinds of conceptual characterizations but it is nevertheless not a vague void, but structured, organized, beyond the dictatorship of order and chaos, axioms and rules.

That bailouts for state and capital can happen in a specific societal space, which has to be spaced and temporalized by actions and activities before/after capital and state can happen on/off historical stage of history. Bailouts to save living space and future(s) have to be discovered and invented beyond state and capital - and bailouts.

Without fundamental *change(s)* nothing will be changed for the future.

References&Notes

1 http : // www.cse.ucsd.edu / ~goguen / pps / taspm.pdf

2

XANADU's textemes

Diamond theoretical reflections on hypertextuality

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Abstract

Xanadu is still not yet realized. Nevertheless, it is appropriate, not only to understand its principles and its radical difference to established Web hypertext and multimedia, but to try to think and design even more advanced concepts of non-traditional interactions. One interesting extension of identity-oriented thematizations is opened up by polycontextural, kenogrammatic and diamond approaches to text theory; proposed recently as textemes (or textems). Textemes are based on the interplay of anchored semiotic diamonds and are delivering necessary environments for transclusions. Transclusions and transjunctions are modeled additionally in a polycontextural setting. The characteristics of '*electronic*' text in contrast to '*physical*' paper texts are emphasized.

1. Xanadu: Nelson's still new principles

1.1. Hyper-textuality

Since some decades, everybody knows Xanadu and nearly nobody ever has seen it working.
Most people think of it as a special kind of a hypertext project with two-way links and connected with projects like Hypercard. Hence, the focus is on the machinery of links.

Personally I had a similar perception and therefore wasn't specially interested in it.
But there is a very crucial distinction at place which makes a profound difference to all kind of linking systems. It is Nelson's insistence on the difference of '*physical*' and '*electronic*' documents. At the first glance this seems to be obvious and trivial too, but it isn't at all.

There is a lot of postmodern writing about the virtuality and simulacrum of electronic media. Nevertheless I couldn't find any conceptually and technically useful elaborations.

With such a change, from the '*physical* to the '*electronic*', in the ontological and epistemological status of documents and texts, the whole topic of links (transclusions, deep links, content links, etc.) appears as a 'natural' consequence of the new understanding of text ('electronic', digital', 'virtual'). Quite obviously, the term '*electronic*' is historic and utterly metaphoric. Xanalogical concepts are not about electronics, information processing, etc., neither about 'virtuality'.

A good example for a *conservative* and common understanding of the Web and its hypertext links is demonstrated with Michael Wesch's *"The Machine is us/ing Us"* Youtube entry.
http://mediatedcultures.net/ksudigg/?p=78

"Digital hypertext is above all... hypertext can link..."
"Digital text can do it better. Form and content can be separated."
"XML was designed to do just that."

1.1.1. Ted Nelson's Xanadu

"To Project Xanadu, that means enacting two types of connection: profuse and unbreakable ****deep links**** *to embody the arbitrary connections that may be made by many authors throughout the world (content links); and *a system of visible, principled* ***re-use*****, showing the origins and context of quotations, excerpts and anthologized materials, and content transiting between versions (transclusions).*

*This may be simplified to: connections between things which are *different*, and connections between things which are *the same*. They must be implemented differently and orthogonally, in order that linked materials may be transcluded and vice versa. This double structure of abstracted literary connection --* ****content links**** *and* ****transclusion**** *-- constitute xanalogical structure."*

Transclusion
*"Transclusion is what quotation, copying and cross-referencing merely attempt: they are ways that people have had to *imitate* transclusion, which is the true abstract relationship that paper cannot show. Transclusions are not copies and they are not instances, but *the same thing knowably and visibly in more than once place*. This is a simple point which is remarkably difficult to get across. While copies and cross-reference are workarounds in place of transclusion, aliases and caches are *forms* of transclusion."*

Text is not simply text
"Nelson always meant hypermedia when he said hypertext, it's one of the things that people get wrong about Nelson. They think that they've invented ***hypermedia*** *and he only invented hypertext. He meant 'text' in the sense of corpus, not text in the sense of characters. I know this for a fact because we've talked about it many times (van Dam 1999, interview)."*

Hypertextuality in the sense of the **Web** and its WEB-0.X-mythology, is restricted to a unidirectional exchange of signs as data without environments. Web links are not only uni-directional by definition but they have only two logical states: *broken/unbroken*.

It would by great to enjoy a more dynamic bi-directional Web connectivity in the sense of *transclusions* (Ted Nelson). But Xanadu links are postulated as UNBREAKABLE. Does it matter if they are one- or two-way links if they are not qualified to *perish*? http://www.xanadu.com/xuTheModel/

What's an 'electronic' text?

It isn't easy to characterize properly 'electronic' or 'digital' texts and documents in the sense of Nelson's intentions.
A *digital text* in the common sense is easy defined. It is a digitally codified mapping of a text (media) by a binary code into an electronic representation. This is realized for all media (text, sound, picture, graphics, etc.).

One hint to understand the difference to the common understanding of hypertext is given by the distinction of "*same*" and "*different*" instead of '*equal*' and '*unequal*' of textual 'things'.
" ... connections between things which are *different*, *and connections between things which are* *the same*."
A further hint to the different epistemological character of 'electronic' texts is given by the necessity of '*orthogonal*' structures. "*They must be implemented differently and orthogonally, in order that linked materials may be transcluded and vice versa.*"
Furthermore, 'electronic' texts are charactericed by a complementarity of polar distinctions, i.e. by a double structure of '*content links*' and '*transclusions*'.
"*This double structure of abstracted literary connection --* ****content links**** *and* ****transclusion**** *-- constitute xanalogical structure."*

Some more distinctions
Some more distinctions might help to grasp the specific character of 'electronic' texts.
1. The mainstream understanding of text is still dominated by the *sentence*-model. A text is a composition of sentences (phrases, statements, etc.). A sentence is ideally a well-formed statement with a clear meaning.

2. Hypertext in the mainstream understanding is a text of a text. As a meta-level a markup language is constructed to link textual elements of the primary text.

"*In a classical node-link hypertext, a graph can be constructed on the set of nodes where each edge is identified with a link and structure discussions typically take place with respect to this graph." (Neumuller, p.89)*
"The Web link is in essence little more than a goto or a jump instruction to the Web browser to retrieve and display a new document." (ibd., p. 149)

3. And to give the whole thing some meaning, a markup language of a markup language of the ordinary text is introduced. This is the concept of a multi-layered text, which still remains syntactically restricted, is introduced in an ontology-based *Semantic Web.*

4. Nelson's *Docuverse*, "deep electronic literature", virtual documents

"...transclusions are hard to formalize in graph theory: are they nodes themselves? If they are, they would transform trees into directed graphs. I have included them in this section, as they seem to mark a breakpoint of graph theory." (ibd., p.90)

The same at different places, without 'physical' representation by copy-and-paste.

"*Transclusions* are not copies and they are not instances, but '*the same thing knowably and visibly in more than one place'*." (Nelson)

Key Concepts
- *Parallel Documents*
- *The Big three : Transpointing, Transclusion and Transcopyright.*
- *Transpublishing.*

Hence a further aspect of the epistemology of 'electronic' texts, i.e. xanalogical texts, is the fact that they have to be *placed*, that they have to take *place* in a textual space. There is no such thing in classical text theory as a textual place or *locus.* In other words, classical texts are anchored in uniqueness, hence the unique anchor can be lifted and omitted. A procedure which is producing specific speculations, illusions and phantasm about otherness, void and omnipotence. This observation of a missing localization of classical textuality shouldn't be confused with the triviality that in classical text theory all kinds of topologies, hodologies and super-graphs might be used to explain, model and formalize classical texts as complex objects.
In more recent publications at the University of Southhampton, Nelson introduced further distinctions and broader self-interpretations.
The concept of *anchored* semiotics, diamonds and textemes offers a simple but radical mechanism of epistemic localizations of documents.

Table of concepts

(linear) text	sentences	first – order logic
hypertext	marked sentences	modal logic, graph theory
semantic hypertext	2 – level marked texts	higher – order modal logic
xanalogical docuverse	transclusions, transpointing	?? ?

Nevertheless it seems therefore that, despite the contrary narratives, the idea of Xanadu in itself is strictly different, not only from established hypertext systems but also from Vannevar Bush's Memex.

Because elaborated conceptuality is missing, the whole Xanadu projects seems to be lost in metaphors and intentions.

Founding background
Behind many of the most important inventions in computer technology, realized or

conceptualized, by Doug Engelhart, Ted Nelson, Heinz von Foerster and Gotthard Gunther, was the enthusiastic help of the assistant of Harold Wooster, Rowena Swanson, of the US Air Force Office of Scientific Research.

"Long ago we considered on-line documents. One of the first questions we asked was: "How can computers improve on paper documents?" Our principal answer: ***"By keeping every quotation connected to its source."*** *We still believe this. However, those who created today's computer world didn't get that documents should be different now. They imitated paper. We see this as retrograde, like the buggy-whip sockets on the early horseless carriates." (Ted Nelson) http://www.xanadu.com.au/transquoter/*

What exactly does it mean: *"keeping quotations connected"* to it's sources, if there are no sources but only quotations?

Perishing links and textemes
Perishing links are neither breakable nor unbreakable, they are enabling such differences, uni- and pluri-directional. Hence, a perishing link is not a killed link. And an non-perishing link is not an endless link, like an endless non-terminating process. Because of the polycontextural complexity of the xanalogical link structures, with its chiasm of 'originals' and 'copies', a broken (micro-)link is not breaking the link as such. Redundancy, self-repair and learning is included in the conceptuality of complex links, i.e. interactions.

Textemes with their environments and chiastic interactions are enabling links to *perish*, to be, as reductions, ordinary links, which might be broken/unbroken or even unbreakable. Nevertheless, actions in textemes and between textemes are not links but *interactions*, able of interactionality, reflectionality and interventionality. Hence, they have their life.

Hypertextuality in the sense of **Xanadu** might find a scientific model by the interplay of *internal* and *external* environments of *textemes*. That is, links refer to the external environment and are connected with the internal *environment* of neighboring textemes, and vice versa.

Common understanding of links

"Let us now try to use those notions for analysing the main features of Web pages. Web pages are so-called ***hypertexts****, that is, texts with some of their components (words or sentences), possibly linked to other (hyper)texts, and so on and so forth. The reader can navigate through the whole text in a non-linear manner, by activating so-called hot links or anchor points that are linking some piece of text to some other.*
These links are an obvious example of ***indexes****, with a word pointing to (refering to) its definition or to some related piece of information. The WWW merely extends the basic notions of hypertext by making it possible for one index to refer to some physically-distant location on a remote computer somewhere else on the Internet, together, of course, with the ability to link to and therefore communicate images and sound. However in order to act as an index, a* ***sign*** *has to be recognized as such, i.e. the index has to exhibit itself as a* ***reference****. This is done in hypertext by marking the* ***hot links*** *in blue ink, in order to make the reader aware that he can jump to another piece of hypertext or image, therefore using a conventional* ***symbol*** *in order to ''show'' the index as such." Philippe Codognet, THE SEMIOTICS OF THE WEB, http://pauillac.inria.fr/~codognet/web.html*

1.2. Xanadu and XML

Is there any chance to realize the Xanadu document concept in the framework of XML?
I would like to stipulate that this question has to be connected with the problem of *identification.* Identification is basic on all levels of computation (Lambda Calculus) and understanding (naming). XML is strictly restricted to identity; and as a consequence to hierarchy.

It would be an artificial and tedius project to model, formalize and to try to realize the specifics of Xanadu in the framework and language of XML. But there are people who found a fundation for that.

Obviously, Xanadu's link concept, especially the "two-way links" are not part of the Web-link nomenclatura simply because Web links are linking *identifiable* documents together in different modes, e.g. direct, indirect, reciprocal, one-way, multi-way, and even incestous, over- and under-linking, etc.

XML simulations of Xanadu
Academic implementations of Xanadu as simulation are presented at some universities. As long as *simulation* is not confused with *realization* nothing is wrong about such achievements and much can be learned. XML is

modeled along the 'physical' concept of documents. Hence, a construction of genuine *xanalogical* concepts with XML methods is not producing more than a *simulation* instead a realization. As much as any simulation of an earthquake isn't the real earthquake, a simulation of Xanadu concepts isn't the real thing. Presupposed, obviously, there exists as such a thing.

"With the XML Pointer Language (XPointer) fragments of XML documents can be identified and addressed as well. Thus, a combination of XML, XLink and XPointer can be employed.‘'
Josef Kolbitsch, Hermann Maurer, Transclusions in an HTML-Based Environment

1.2.1. Abstractness of documents

The great step of Xanadu seems to not be primarily in the new modes of linkage but in the radically new and more abstract concept of an *"electronic document or text"*, positioned conceptually on a very different *epistemological* level than, say XML-documents which are represented as syntactical trees. And thus, on the base of this new abstraction of textuality from the physical to the 'electronical', a transformation of the ordinary link concepts follows 'naturally'.

"Like Engelbart, Nelson believes the technical system moves in paradigms, and that the current era is bound to paper as a central metaphor. We need to be forced from our collective tricycles. 'I deal with new ***paradigms****' (Nelson 1999, interview)." (Belinda Barnet)*
http://www.latrobe.edu.au/screeningthepast/firstrelease/fr_18/BBfr18a.html

On the base of the classical 'electronic document' concept, mirroring the main characteristica of physical texts (text, sound, graphis, pictures, odeurs, haptics, etc.) only a specific kind of links are possible.

"The term ***multi-way*** *simply refers to the fact that the link exchange is between 3 or more websites, however each link is singular by only pointing to one other website."*

"A ***typed*** *link in a hypertext system is a link to another document or part of a document that includes information about the character of the link."*

*"Nelson coined the term "****transclusion****," as well as "hypertext" and "hypermedia", in his 1982 book, Literary Machines. Part of his proposal was the idea that micropayments could be automatically exacted from the reader for all the text, no matter how many snippets of content are taken from various places"*

"In computer science, transclusion is the inclusion of part of a document into another document by reference. It is a feature of substitution templates." (WiKi, transclusion)

Different identity relations
"Let me talk identic relationships. The term identic you might enjoy looking up in the dictionary. I hope it does not have some mathematical definition. I am just trying use it here to mean some relationship showing the two data structures are the same.
The number of different identic relationships in the computer field. A copy is in identic relationship with its original. An instance is in identic relationship with its original. A cached copy is in identic relationship with its original. A counted reference is in identic relationship with the places, the context, that reference it. So these are different identic relationships with different properties. Write-through cache, write-back cache.

So now I want to tell you about another identic relationship. I am calling it ***transclusion****. Think of it as hypersharing if you like. What it is is this. There is only one copy, one master copy of anything. Let's call it a cosmic original. Every other copy you see is a manifestation of this cosmic original.*

I use these terms because I don't believe they are currently in use. So when you see the Lord Shiva over the road, is it a copy of Lord Shiva? Of course not, it is the real guy. And so it should be with all text. We should never have to type anything twice.

Transclusion: you are simulating and enacting and bringing about a situation in which all instances can be regarded as the master. Naturally there must be many copies and this is a point that many people have missed because of the emphasis on the original."

Generalized Links, Micropayment and Transcopyright
http://www.almaden.ibm.com/almaden/npuc97/1996/tnelson.htm

Transclusions in HTML
Transclusions are an advanced technique for the inclusion of existing content into new documents without the need to duplicate it.
Josef Kolbitsch , Hermann Maurer, Transclusions in an HTML-Based Environment
http://cit.zesoi.fer.hr/browseIssue.php?issue=25

Hence, such nice concepts in Webology, like n-way linking, have absolutely nothing to do with the 'two-way linking' of a *"double structure of abstracted literary connection* -- *content links* *and* *transclusion* -- *constitute xanalogical structure."* (Nelson) http://www.kolbitsch.org/research/transclusions/

1.2.2. Locatedness of documents

Again, the same at different places, without 'physical' representation by copy-and-paste.
"*Transclusions* are not copies and they are not instances, but *'the same thing knowably and visibly in more than one place'."* (Nelson)
In contrast, *"As the nodes need not have a fixed place in a spatial order to form this network of text (and other hypermedia), hypertext structure is commonly analyzed by means of graph theory."* (Kolbitsch)

1.2.3. Accessibility: Abstractness and locatedness

There is an interesting antagonism between abstractness and locatedness of documents. It could be expected that the abstractness of the text model is covered by a topological space and its graph theoretical representations where places don't matter and documents are represented abstractly and place-free.

Hence the type of abstractness of 'electronic' (xanalogical) documents is of a different kind as the abstractness of a topological space. The difference might be termed as that of "connectivity of links" in contrast to the *"deep connectivity"* of transclusions.

"As Nelson is fond of saying, all this means is making and maintaining connections between things that are the same (Nelson 1995), or ***'deep connectivity'*** *as the Udanax community term it. Remote instances remain part of the same virtual object, wherever they are."* (Belinda Barnet)

This might be one of the difficulties to explain properly *xanalogical* concepts. Their abstractness is concretization, while ordinary abstractness of texts is 'generalization'.

1.3. Xanadu and semiotics

A semiotic reconstruction in the framework of the purely syntactic XML wouldn't change much, neither. Semiotics in its triadic-trichotomic form (Peirce, Bense), or even in its tetradic extension by Toth, is still framed, bracket and caged by the decision and necessity of *identification* and *uniqueness*. It seems that semiotic based approaches are missing the point.

1.3.1. Xanadu semiotics, citations

"Let us now try to use those notions for analysing the main features of Web pages. Web pages are so-called hypertexts, that is, texts with some of their components (words or sentences), possibly linked to other (hyper)texts, and so on and so forth. The reader can navigate through the whole text in a non-linear manner, by activating so-called hot links or anchor points that are linking some piece of text to some other.

"These links are an obvious example of ***indexes****, with a word pointing to (refering to) its definition or to some related piece of information. The WWW merely extends the basic notions of hypertext by making it possible for one index to refer to some physically-distant location on a remote computer somewhere else on the Internet, together, of course, with the ability to link to and therefore communicate images and sound.*
However in order to act as an index, a sign has to be recognized as such, i.e. the index has to exhibit itself as a reference*.*
This is done in hypertext by marking the ***hot*** *links in* ***blue*** *ink, in order to make the reader aware that he can jump to another piece of hypertext or image, therefore using a conventional symbol in order to ''show'' the index as such. [...]*
As in all semiotic systems, we have seen that the web is a ***mesh*** *of icons, indexes and symbols, with each type of the trichotomy indeed depending on the others, even for its own definition."*
Philippe Codognet, THE SEMIOTICS OF THE WEB
http://pauillac.inria.fr/~codognet/web.html

*"****Intertextuality*** *is a term introduced by Julia Kristeva and widely adopted by literary theorists to designate the complex ways in which a given text is related to other texts.*
Just as there is no sign apart from other signs, there are no texts apart from other texts.
In Kristeva's words, "every text is constructed as a mosaic of other texts, every text is an absorption and transformation of other texts."

"Nielsen points out that "the fact that a system is multimedia-based does not make it hypertext. [. . .] Only when users ***interactively*** *take control of a set of dynamic links among units of information does a system get to be hypertext,"*
Moritz Neumüller, Hypertext Semiotics in the Commercialized Internet
http://sammelpunkt.philo.at:8080/23/2/ht_semiotics.pdf

"Xanadu is a system for registered and owned content with thin document shells, re-usable by reference, connectable and intercomparable sideways over a vast address space (Nelson 1999, interview)."

1.3.2. Identity, copy, original

The main strategy of classical attempts to implement Xanadu concepts in the framework of HTML, XML, XPath and others is quite straitforward but in full denial of the difference of original-based identity texts and Xanadu texts, which, whatever it means, are conceived as origin-free.

"Transclusion*: you are simulating and enacting and bringing about a situation in which all instances can be regarded as the master."*

There are two identity concepts in the game, one, the classical, is an ontological and logical identity concept based on subject-free, i.e. user-free, existence of objects, i.e. texts. Here, there is a strict hierarchy between the original and its copy; and the plagiat-police is well equiped behind the corner. Originals are first, copies are second.

The Xanadu 'identity' concept is not an onto-logical but a *reflectional* and *cognitive* concept of the pragmatics of *using*, i.e. interaction with texts.
With taht, the plagiate-police gets jobless and confused:

"Transclusions allow authors to include portions of existing documents into their own articles without duplicating *them."*
"Transclusions are designed as complete replacement for all cut-and-paste mechanisms in use. Nelson argues that **cut-and-paste** *is not what people actually want to do but that it is a restriction imposed upon authors by the nature of paper."* (Kolbitsch, p. 162)

The mentioned sentence is a citation, accessible to TurnItin!-control. I had to copy and paste the text from CIT704.PDF to my text in progress. No transclusion accessible! It is still very difficult to grasp a textuality beyond ontological identity.

But the reasons are simple. Only, it is postulated, if texts have an identity, they can be owned by me, being my possession. Only then, I can get a degree and a patent and sell it and get rich or bankrupt.
Hence, don't promote the real thing!

Transcopyright and micropayement
This point, textual identity and authorship beyond ontological identity, was well reflected by Nelson's early concept of **transcopyright** and **micropayement** for/in Xanadu.

'Whenever a reader views a transclusion, a note about the rights associated with the transcluded content is added, and a micropayment is made to the corresponding owner. Nelson names this model ***transcopyright."*** (Kolbitsch, p.162)

"The on-line copyright problem may be resolvable by a simple, sweeping permission method. This proposed system, which anyone may use, allows broad re-use of materials in exchange for automatic tracking of ownership. Payment goes to the original publisher and credit to the original author (Nelson 1995)."

"Necessarily, a mechanism must be put in place to permit the system to charge for instances, a micropayment system which provides a bridge to the original from each instance. Critically, this bridge should never break; links should not be outdated. At the same time, the bridge must leave no trace of who bought the pieces, as this would make reading political. As such, Xanadu would require a micropayment system parallel to the docuverse." (Belinda Barnet)
http://www.latrobe.edu.au/screeningthepast/firstrelease/fr_18/BBfr18a.html

There are educational problems too. Computer scientists and engineers are not trained in humanities, like hermeneutics, rhetorics, comparatistics, etc. and their ambiguous, complex, multi-layered texts. On the other hand, cultural scientists are not trained in computer science and programming. Hence, Ted Nelson's project Xanadu was not only to early in time but trapped into mutual misunderstandings.

Madonna, neither original nor copy

In cultural history there are a lot of such paradox situations. Take a iconoclastic Madonna! She is the real Madonna! No copy, exclusive singularity. But from time to time she turns into a pictoral object, and has to be refreshed, repaired or exchanged with a more fancier one. During this procedure she surely didn't change her mode of existence into a copy. What had to be restored was the material, which has nothing to do with the Madonna as the real Madonna. Nothing at her body was lifted. In fact, to speak about an original includes a concept of the opposite, the copy. But the Madonna is the Madonna and the Madonna is nothing else than the Madonna in uniqueness. Albeit the neighbor church will claim, without any logical contradictions or ontological conflicts, the same for their own unique Madonna. Hence, uniqueness and multitude, 'original' and' copies' remain, in this world-view, in harmony.
Similar figure had been at work with the Ancient Egyptian Gods.

List of changes
Change in identity concept, chiasm of original/copy.
From physical (linguistic) to electronic (virtual, xanalogical) text.
From information to knowledge.
From links to transclusions.
From copyright to transcopyright.
From shopping to micropayement.

2. Diamonds, bi-signs and textemes

Up to now, the classical concept of a link didn't produce any serious scientific problemes, conceptually and for implementation. Links have a simple logical structure: they are broken (dead) or not-broken (alive).
Nelson's extension of the concept of links is radically abstracting from the physical and linguistic paradigm of documents. His '*transclusions*' and '*content links*' are much more abstract than syntactic links. But their logical status seems to be simple too. They are 'unbreakable' or, and this may become a problem, "breakable".

Hypertextuality in the sense of the **Web** and its WEB-0.X-mythology, is restricted to a unidirectional exchange of signs as data between identical addresses without environments. Web links are not only uni-directional by definition but they have only two logical states: *broken/unbroken.*

It would by great to have a more dynamic bi-directional Web connectivity in the sense of *transclusions* (Ted Nelson). But Xanadu links are postulated as UNBREAKABLE. Does it matter if they are one- or two-way links if they are not qualified to *perish*? http://www.xanadu.com/xuTheModel/

"Long ago we considered on-line documents. One of the first questions we asked was: "How can computers improve on paper documents?" Our principal answer: ***"By keeping every quotation connected to its source."*** *We still believe this. However, those who created today's computer world didn't get that documents should be different now. They imitated paper. We see this as retrograde, like the buggy-whip sockets on the early horseless carriates." (Ted Nelson) http://www.xanadu.com.au/transquoter/*

Perishing links are neither breakable nor unbreakable, they are enabling such differences, uni- and pluri-directional. textemes with their environments and chiastic interactions are enabling links to perish, to be, as reductions, ordinary links, which might be broken/unbroken or even unbreakable.

Nevertheless, actions in textemes and between textemes are not links but *interactions* able of interactionality, reflectionality and interventionality. Hence, they have their life.
But all that, endless self-generation of signs and contextuality of signs and texts, is well known and taught endlessly. But does it matter?

"Just as there is no sign apart from other signs, there are no texts apart from other texts.
In Kristeva's words, *"every text is constructed as a mosaic of other texts, every text is an absorption and transformation of other texts."*

As it was said at another place, an endless repetition of a sentence is not a truth-criteria. The truth-value of sentence might not be what is significant for interactions. True sentences without any *relevance* are dead.

Not only that the term "*endless*" and, e.g the metaphor "a *mosaic* of other texts" (Kristeva) is

not scientifically explained at any other semiotic considerations, its insistence runs out of relevance.
Who cares that, after, e.g Peirce and Derrida, endless iterability of signs is constitutive for sign activities. Later studies from Caputo or Gasché about *infinity* are badly hiding their weakness.

2.1. Limits of semiosis

How can a sign realize inter-activity, a prerequisite of any hypertextuality, if it is constitutionally depraved of any environment? In other words, the triadic-trichotomic concept of semiosis (sign production) with all its differentiations is not offering a single distinction, concept or mechanism to realize a semiotics for an interplay between different semiotic systems, i.e. trans-semiotic and inter-semiotic .

Textemes, as applications of polycontextural diamonds, are distributed over kenomic loci. Hence, the concept and mechanism of loci gives us a hint to understand textuality in a non-ontological and non-topological sense.

Disseminated semiotics gets their ontological orientation bracketed and neutralized. Classical semiotics is furthermore blind for its ontological anchoring in uniqueness. Textemes are *per se* anchored in different configurations of disseminated anchors. Anchors are not monolitical, they have different functionalities to anchor systems and environments, concurrently or chiastic.

Textemes are representing whole semiotic systems, i.e. semiotics. Textemes are distributed and mediated, i.e. polycontexturaly disseminated over different loci of a kenomic grid. Because there is no priviledged beginning for disseminated semiotics, there is no original semiotics as a beginning of all semiotics; all semiotics are ‘copies of copies’.

Applied to XML. If contexturally disseminated textemes (semiotics) are intra-structurally interpreted by XML, then XML gets disseminated as well; and there is no original XML left. For a classical understanding, this is utter nonsence. Classical science is conceiving this situation *manorial*: there is one and only one real XML (and its millions of dialects) but uncountable applications of it. XML for everything. Thus, there is, without surprise, a strict hierarchy between the original and the copy of XML.

All those sophisticated studies about semiotic interactivity, reflexionality and interventionality, in whatever field, Hypertext, Anthropology, Nursery, are permanently confusing *theory* and *application*.

Without doubt, semiotics, as well as logic, can be applied in many ways to model interaction and hypertexts, and more. This is in full harmony with Aristotle and later. But it is not harmonious with Ancient Chinese mathematics. There, ‘application’ is ‘theory’- and *vice versa*.

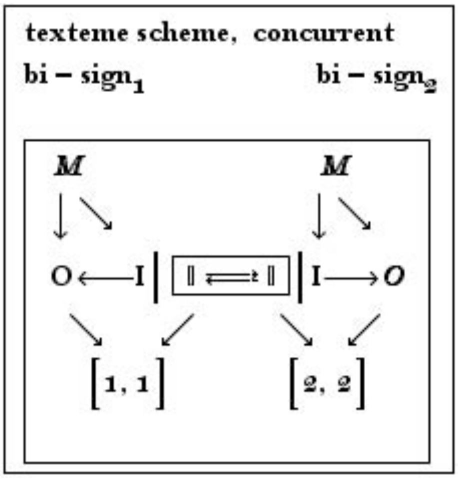
Semiotics as semiotics has no environments as semiotics.
Therefore, semiotics as such has no possibility of semiotic interaction with semiotics strange, xenomorph, to semiotics.

Hence, Sowa’s statement, *“The Internet is a giant semiotic system”*, is not the real thing at all.
Again, albeit semiotics might produce interesting insights into the character of the Web, Ontologies and Hypertext, it is fundamentally inappropriate for an understanding of the main properties of a Xanadu based Docuvers (texts, documents, semantics) with its aspects of *abstractness*, *situatedness*, *polylogic* and *liveliness* of a new Web.

2.2. The idea of textemes

As Alfred Toth, an eminent semiotician, mentioned, the introduction of *diamond category theory* had been opening up unforeseen possibilities for further developments of theoretical semiotics as a purely semiotic discipline. Hence, let’s try to apply it!

On the base of diamond theory, I introduced, as a very first step or risk, the idea of texteme*s*. This is conceptually straight forward because it is an interpretation of the mechanism of the composition and combination of anchored diamonds.
Recall, diamonds are conceived as systems with inner and outer environments *per se*.
Hence, different diamonds might be combined by their different but interacting external environments. Other modes of combination and interactions might be omitted for now.
Anchors had been omitted in recent publications to focus on the new aspects of diamonds. Anchoring was introduced in the late 90s, following a hint from Cartwell, to concretize *conceptual graphs* of operations and their proemiality.



texteme :
diamond = (sign + environment)
bi – sign = (diamond + 2 – anchor)
texteme = (composedbi – signs + chiasm).

2.3. Textemes and text theory

Classical and post-structuralistic text theory is, despite all kinds of subversion against its heritage, based on classical sémiologie and linguistics. Hence, textemes, in a post-structuralist sense, are based on the chain of

grapheme --> *phoneme* -->*morpheme* --> *signification*.

with its foundation in linguistics, i.e. the theory of *spoken* language and its conflicting concepts of writing.
Textemes for ‘*electronic*’ texts, which are multimedial, virtual and distributed, are not based on a linguistic chain of logocentric abstractions and idealizations. Textemes as proposed in this paper are epistemologically not based on semiotics but on diamond theory. In some sense, textemes can be seen as based on a diamond understanding of *systems* and their specific *environments*. More precisely, textemes are based on an interplay of diamonds mediated by their external environments.
The fact, that on a *micro-analytical* level, links might have a pointer-structure and therefore a semiotic representation as *indexes* of the sign process, is secondary, and based on the structural interplay between diamonds and bi-signs.

2.4. Textemes and interactions

If we accept the limited value of sign systems for interactivity, it seems to be interesting to apply the idea, concept and formalism of textemes to study a new concept of links as *interactions*, with all their possible properties of interactionality, reflectionality and interventionality, to mention some important features.

I will not use *'semiotic glue'* to connect different semiotic action systems together but the post-semiotic concept of an environment of *diamonds* as supported by textemes.

The fact of the *dissemination* of textemes is subverting the systemic identity of the involved semiotics.

This change in the 'ontological' status of textemes, semiotics and contextures by dissemination has to be kept in mind if the following thoughts are focusing on environments, interactions and links. Such links, environments and interactions are not thematized in the framework of cybernetic system theory or data processing of Multi-Agent Systems (MAS).

Utilizing previous constructions about reflectional interactivity in general, some diagrams from other papers might elude the mist of primary conceptualizations.

A *sign system* shall be modeled, in general, first abstracting from its semiotic properties, by its contextural subsystems S_i, $i \in \mathbb{N}$ at a structural (kenomic) place O_j, $j \in \mathbb{N}$.
Each subsystems of a sign system has its own neighbor system.

As a first step, to focus on the *environments*, the anchors shall be omitted.
With a second step, the specific semiotic characterization, i.e. M, I, O, shall be replaced by a general *contextural* scheme.

What's the meaning of anchors anyway?
Anchors don't exist in semiotics. The only classical reason could be found in the *"Satz vom zureichenden Grund"* (Leibniz) or the *"causa (forma) teleologica"* (Aristotle) of ontology and epistemology. But, because there is one and only one metaphysical reason for existence and truth postulated by classical thinking, its notation simply can be omitted.

Anchors are getting more interesting if a multitude of autonomous semiotics and their environments, i.e. texteme*s*, are accepted. Textemes might be anchored for themselves or by others. The same for environments, they might be anchored together with their semiotics or by anchors of other semiotics. This could be called the *architectonics* of anchors. But there is also dynamics involved. *Metamorphosis* between textemes might involve anchors. Hence, an anchor of one system might function as a system of another texteme.
For reasons of introduction, such complex metamorphosis of anchors shall be omitted too.

2.4.1. Elementary textemes

$$\frac{\Pi^{(1,2)}_{\substack{\textbf{Type}\\ \textbf{texteme}\\ \textbf{0-anch}}} \begin{pmatrix} M & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & O \end{pmatrix}}{\begin{bmatrix} M & \varnothing \\ I & O \end{bmatrix} \left| \left(\overline{T} \rightleftarrows \overline{T}' \right) \right| \begin{bmatrix} \varnothing & M' \\ O' & I' \end{bmatrix}}$$

Formula notation for 0 – anchored 2 – textemes

(1, 2) – texteme contextual scheme

$\text{sign}_1 = O_1$ **$\text{sign}_2 = O_2$**

$$\begin{array}{ll} S_1 & \\ \downarrow \searrow & \\ S_3 \longleftarrow S_2 & \Big| \boxed{S_4 \rightleftarrows S_4} \Big| \end{array} \begin{array}{ll} S_1 & \\ \downarrow \searrow & \\ S_2 \longrightarrow S_3 & \end{array}$$

Bracket versions:

$$\begin{bmatrix} \textbf{bi-sign}^{(1,1)} \\ \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 1, 2, 3 \end{bmatrix} \\ 4 \end{bmatrix} \\ \langle 1; 1 \rangle \end{bmatrix} \end{bmatrix}, \begin{bmatrix} \textbf{texteme}^{(2,1)} \\ \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 1, 2, 3 \end{bmatrix} \\ 4 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 1, 2, 3 \end{bmatrix} \\ 4 \end{bmatrix} \\ \left[\langle 5; 6 \rangle \;\|\; \langle 5; 6 \rangle \right] \end{bmatrix} \end{bmatrix}$$

$$\text{elementary texteme} = [[[S^1, s^1]; [S^2, s^2]] : q], \ (s^1 \simeq s^2)$$

$$\text{texteme}^{(2,1)} = [[[\text{Sem}^1 \mid \text{env}^1]; [\text{Sem}^2 \mid \text{env}^2]] : \ < \text{anch} >], \ (\text{env}^1 \simeq \text{env}^2)$$

elementary texteme

$$\text{texteme}^{(2,n)} = [[[\text{Sem}^1 \mid \text{env}^1]; [\text{Sem}^2 \mid \text{env}^2]; \ldots; [\text{Sem}^n \mid \text{env}^n]] : \ < \text{anch} >], \ (\text{env}^i \simeq \text{env}^j), \ 1 \leqq i \neq j \leqq n, \ n \in \mathbb{N}$$

composition of textemes

$$\text{texteme}^{(m,1)} = \left[\left[\begin{matrix} [\text{Sem}^1 \mid \text{env}^1] \\ [\text{Sem}^2 \mid \text{env}^2] \\ \cdots \\ [\text{Sem}^m \mid \text{env}^m] \end{matrix}\right]; < \text{anch} > \right]$$

$$(\text{env}^i \simeq \text{env}^j), \ 1 \leqq i \neq j \leqq m, \ m \in \mathbb{N}$$

mediation of textemes

2.4.2. Composed textemes

$$\frac{[(M_\alpha \longrightarrow I_\omega) \diamond (I_\alpha \longrightarrow O_\omega)]^{(1,1)} \odot [(M_\alpha \longrightarrow I_\omega) \diamond (I_\alpha \longrightarrow O_\omega)]^{(1,2)}}{(M_\alpha \longrightarrow O_\omega)^{(1,1)} \,\Big|\, ^{(2)}(\tilde{I_\omega} \Longleftarrow \tilde{I_\alpha})^{(1)} \,\Big|\, (M_\alpha \longrightarrow O_\omega)^{(1,2)}}$$

Diamond composition rule for homogeneous semiotic texteme

$$\frac{\left[(M_\alpha \longrightarrow I_\omega)\diamond(I_\alpha \longrightarrow O_\omega)\right]^{(1,1)} \odot \left[(I_\alpha \longrightarrow M_\omega)\diamond(M_\alpha \longrightarrow O_\omega)\right]^{(1,2)}}{(M_\alpha \longrightarrow O_\omega)^{(1,1)} \left\| \begin{pmatrix} \tilde{I_\omega} \longleftarrow \tilde{I_\alpha} & (1) \\ \tilde{M_\omega} \longleftarrow \tilde{M_\alpha} & (2) \end{pmatrix} \right\| (I_\alpha \longrightarrow O_\omega)^{(1,2)}}$$

Diamond composition rule for heterogeneous semiotic texteme

$$\frac{\Pi^{(1,3)}_{\substack{\text{Type}\\ \text{texteme}\\ 0-\text{anch}}} \begin{pmatrix} S_1 & & \square \\ \downarrow & \searrow & \\ S_3 & \longrightarrow & S_2 \end{pmatrix}}{\begin{bmatrix} S_1 & \varnothing \\ S_3 & S_2 \end{bmatrix}\begin{bmatrix} \varnothing & S_1 \\ S_3 & S_2 \end{bmatrix}\begin{bmatrix} \varnothing & S_1 \\ S_2 & S_3 \end{bmatrix} \left| S_4^{1.2.3} \rightleftarrows S_4^{1.2.3} \right|}$$

$$\mathbf{MC} = \left\{S_1^2 = S_1^3,\ S_1^1 = S_3^3\right\}$$

Formula notation for 0 – anchored (1, 3) – textemes

2.4.3. Mediated textemes

4-fold semiotics

A general scheme for a 4-fold, in contrast to triadic semiotics, might first be introduced as a mediation of 4 triadic semiotics. Such a construct shall then be interpreted as a genuine 4-fold structure with, e.g. the formal distinctions of o*ur-medium*, *me-interpretant*, *you-interpretant* and *our-object*, all triadic distinctions modified by the whole $\text{Sem}^{(4,1)}$ -structure of the construction.

Only with the introduction of a semiotic complexity of at least 4, mechanism of *view-points* and corresponding *as-abstractions* as differences and chiasms between two interpretants or semiotic agents are conceptually and formally realizable. Hence, an interpretation of a text as at once being an original and a copy are conceivable without logical-ontological contradictions.

$$Sem^{(4,1)} = \begin{pmatrix} M_{1,3,4} & \Longrightarrow & O_{1,3}/M_2 \\ \downarrow & x & \downarrow \\ I_{2,3,4} & \Longrightarrow & I_1/O_{2,4} \end{pmatrix}$$

with:

$sem_i = (M, O, I)_i, \; i = 1, 2, 3, 4$

and the matching conditions:

$$\begin{array}{l} M_1 \cong M_3 \cong M_4 \\ O_1 \cong M_2 \cong O_3 \\ I_1 \cong O_2 \cong O_4 \\ I_2 \cong I_3 \cong I_4 \end{array}$$

An interpretation of a 4 – contextural semiotics

$$Sem^{(4,1)} = \begin{pmatrix} M_{1,3,4} & \Longrightarrow & O_{1,3}/M_2 \\ \downarrow & x & \downarrow \\ I_{2,3,4} & \Longrightarrow & I_1/O_{2,4} \end{pmatrix},$$

$[M_{1,3,4}]$ as our – *medium* in $Sem^{(4,1)}$

$[I_1/O_{2,4}]$ as you – *interpretant* in $Sem^{(4,1)}$

$[O_{1,3}/M_2]$ as our – *object* in $Sem^{(4,1)}$

$[I_{2,3,4}]$ as me – *interpretant* in $Sem^{(4,1)}$

4-fold textemes

An the base of the shortly sketched 4-fold semiotics, diamonds and textemes are naturally introduced.

Scheme of a 4 – contextural system

$$\text{Diam}^{(4,1)} = \begin{bmatrix} S_{1,3,7} & \Longrightarrow & S_{5,6,7} \\ \downarrow & x & \downarrow \\ S_{1,2,6} & \Longrightarrow & S_{2,3,5} \end{bmatrix} \Bigg| \left[S_{4,9} \rightleftarrows S_{8,9} \right]$$

Scheme of a 4 – contextural texteme

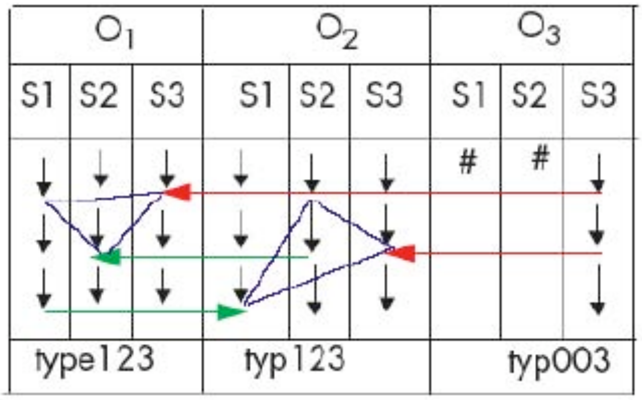
$$\bigoplus_{\substack{\textbf{Type} \\ \textbf{texteme} \\ \textbf{0-anch}}}^{(4,1)} \begin{pmatrix} S_1 & \longrightarrow & S_4 \\ \downarrow & \searrow & \downarrow \\ S_3 & \longrightarrow & S_2 \end{pmatrix}$$

$$\begin{bmatrix} S_1 & \longrightarrow & S_4 \\ \downarrow & \searrow & \downarrow \\ S_3 & \longrightarrow & S_2 \end{bmatrix} \Bigg\| \begin{pmatrix} S_4^{1.2} & \rightleftarrows & S_4^{1.2} \\ S_4^{3.4} & \rightleftarrows & S_4^{3.4} \end{pmatrix} \Bigg\| \begin{bmatrix} S_4 & \longleftarrow & S_1 \\ \downarrow & \swarrow & \downarrow \\ S_2 & \longleftarrow & S_3 \end{bmatrix}$$

$$\textbf{MC} = \left\{ S_1^1 = S_3^3,\ S_\square^2 = S_\square^\square \right\}$$

Formula notation for 0 – anchored
$(4, 1)$ – textemes

2.4.4. Interactions



PM	O_1	O_2	O_3
M_1	sem_1	env_1	x
M_2	env_2	sem_2	x
M_3	env_3	env_3	sem_3

PM	O_1	O_2	O_3
M_1	$comm_1$	put_1	x
M_2	put_2	$comm_2$	x
M_3	get_3	get_3	$comm_3$

Interaction in this case is properly playing with the difference of *inner-* and *outer*-environments of acting systems. A two-way link is an interaction, i.e. a mutual bi-directional interaction between two autonomous systems in the mode of communication between the operations "put" and "get" in relation to the environments of both systems, and mirrored in a third communicational system.
A bi-directional *link* might be modeled as a *bi-arrow (graph)* between two nodes. Obviously, *interactions* are not adequately modeled by graph theoretical concepts and methods.

2.4.5. Links

Links in identity systems are connections between two entities. Even for physical systems it isn't always easy to understand how bi-directional or two-way activity could happen. Conceptually, a source (domain, initial object) can't function at once as its opposite, a target (codomain, final object). Mathematically, there is no problem involved. Links are arrows between nodes.

The saying, *"there are no texts apart from other texts"*, which is implying some detachments from an origin, or Nelson's *"copy of copies"*, might be a post-modern move against conceptual fundamentalism but this gesture is still caged, by negation and rejection, in the logocentric understanding of negation, iteration, continuity, and origin.

"Nelson mentions that there is a fundamental difference between links *and* transclusions *yet fails to clarify and elaborate.*
• *Links* different - *Transclusions* same
• *Is the memex building trails through transclusions rather than links?"*
Xanalogical Structure, Needed Now More than Ever: Parallel Documents, Deep Links to Content, Deep Versioning, and Deep Re-Use. Qasim Hasan, Sandeep Jauhal, Sept. 18, 2004, p. 5

If there is no origin, then everything might function *as* an origin, and this is not another abstract statement but is itself involved in the formalism of chiasms of thematizations of the interplay of origins and copies **as** this and that.

Links in the paradigm of Xanadu, therefore, have to be understood in such an interplay of as-abstractions.
Again:

"Transclusion*: you are simulating and enacting and bringing about a situation in which all instances can be regarded* **as** *the master."*

"*There is only one copy, one master copy of anything. Let's call it a cosmic original. Every other copy you see is a manifestation of this cosmic original*".

And obviously, there is no cosmic original or Kantian *Ding an sich*. If it would exist, we wouldn't have been chosen to experience it..

Hence, Xanadu's text concept is *"parallel documents"*.

"Parallel documents *are everywhere, but are not generally acknowledged.*
There are relatively few explicitly *parallel documents (like Tom Stoppard's play "Rosencrantz and Guildenstern Are Dead", which is explicitly parallel to "Hamlet" -- showing events that occur offstage in "Hamlet", and vice versa).*
But implicitly *parallel documents are everywhere -- the parallelism of commentaries, the parallelism of long and short versions of reports, the parallelism of translations, the parallelism of holy books [Nelson 1998].*
It is vital that we be able to see this parallelism of documents and to intercompare and work with their side-by-side connection."

The same lack of conceptual clarity or functionality is produced by the property *"parallel documents"* of Xanadu. It can be postulated that *parallelism* of texts needs an operative concept of environments of texts and neighbor texts which is able to explain and implement the interplay between *parallel* and *orthogonal* documents. Otherwise, such parallelism is easily reduced to linearity and hierarchy.

Methods of visualizations and implementations
"transpointing windows"
http://xanadu.com.au/ted/TN/PARALUNE/paraviz.html
"All documents are parallel."
http://xanadu.com.au/ted/TN/PARALUNE/paradoxx.html
http://xanadu.com.au/ted/tedpage.html
zigzag
http://xanadu.com.au/ted/zigzag/xybrap.html

2.4.6. Logic of content links (clinks)

Web
Links in identity systems are connections between two entities, realizing the two states of a binary situation: realized/not-realized or broken/unbroken.

Textemes
Complexions of textemes are realizing corresponding complexions of logical states. Such complexions of disseminated, i.e. distributed and mediated logical states are demanding a corresponding polycontextural logic to their adequate logical modeling and implementation.

Xanadu
In a world without original, each of the many simultaneous relations between texts is entitled to its own logical status of being an original and a copy.

This might be modeled by the distinction positive (pos) as original and negative (neg) as copy (mirror) of diamond logic. The following examples proposed are restricted to diamonds. It shouldn't be a big deal to apply the results to textemes as textemes are based on an interplay of two diamonds.

Hypermedia and Unified Data Structure
Xanadu's text and document concept is multi-medial from the beginning.
Again, what does it mean and how does it work? The new paradigm is declared as *"The Age of the Unified Data Structure"*. But as with all unified and type-free declarations, circularity

and endless regress is programmed.

What kind of data structure is meant if everything is unified into a general data type?
Another point is obvious, if the distinction original/copy collapses, all is one. This is the common, post-modern understanding of the rejection of origins with the result of nil operativity.

As developed at other places and sketched in this paper, there is another understanding too. If there is no origin, there might be many origins. That is, everything might function as an origin or as a copy. In other words, the operation of negation in "*no* this-and-that" is based on distinction, and if there is no distinction left then negation is not applicable. Hence, what we need is not a logic of *universal* everything but a logic of the *mechanism of specific change*. The change from origin to copy and from copy to origin in a specific situation, distributed over all kinds of media and media data.
The Xanadu paradigm is not answering any of those questions.
Therefore, there are at least *two* interpretations legitimately possible for a further understanding and modeling of xanalogical concepts.
One is what's going on: a hidden *type-free universalism* best modeled by a tupe-free logic. The other is what I prefer to reflect on: a transparent and explicit *polycontextural logic* for the interactions of texteme*s* based on diamonds and their environments.

ALL MATERIAL IS ONE

"Transclusion is a way to include, to quote, parts of a document without losing its current (or any subsequent) contexts, and without it becoming a physical part of the new text (which could be a movie, hyperfiction document, you name it). In this fashion one might see all newly formulated or recorded texts, data, sounds, pictures as future 'boilerplate paragraphs' or fragments, available for viewing, digesting, and transclusion in new works." (Ian Feldman) http://xanadu.com.au/media/nelson90.html

"All the contents on all of the Xanadu storage servers act as a ***single pool****. You can send for any part of any document or link to or quote any part of any document."*
http://www.aus.xanadu.com/xanadu/future.html

"5 Every document can consist of ***any*** *number of parts each of which may be of any data type.*
6 Every document can contain links of any type including virtual copies ("transclusions") to any other document in the system accessible to its owner.
7 Links are visible and can be followed from all endpoints."

"Transliterary structure is meant to be the fullest ***generalization*** *of documents. This means being able to represent* ***all*** *possible document structures, and to deal with the vicissitudes of change, versioning and copyright.*
Transliterary *structure is the latest version of the Xanadu project.*
"Documents" are packages of content constructed by authors from new and old text, audio and video, in any arrangement and pieces and desired appearance."
http://xanadu.com/XanaduSpace/btf.htm

2.4.7. Negations for logical diamonds

Logified diamond$^{(3)}$

Logified elementary diamond

$$D^{(3)} = \begin{bmatrix} id_4 \\ id_1 \; id_2 \\ id_3 \end{bmatrix} = \begin{bmatrix} & & neg_4 \xleftarrow{rej} pos_4 & & \\ & & | \qquad\quad | & & \\ pos_1 & \xrightarrow{prop} & neg_1 \Updownarrow pos_2 & \xrightarrow{opp} & neg_2 \\ | & & & & | \\ pos_3 & & \xrightarrow{acc} & & neg_3 \end{bmatrix}$$

Negation *non₁*

$$non_1\left(D^{(3)}\right) = \begin{bmatrix} id_4 \\ non_1 \; id_2 \\ id_3 \end{bmatrix}$$

$$= \begin{bmatrix} neg_4 - neg_1 \longleftarrow pos_1 & pos_3 \longrightarrow neg_3 \\ \uparrow \qquad \Updownarrow & | \\ pos_4 - pos_2 \longrightarrow & neg_2 \end{bmatrix}$$

Negation *non₂*

$$non_2\left(D^{(3)}\right) = \begin{bmatrix} id_4 \\ id_1 \; non_2 \\ id_3 \end{bmatrix}$$

$$= \begin{bmatrix} pos_3 \longrightarrow neg_3 & neg_2 \longleftarrow pos_2 - pos_4 \\ | & \Updownarrow \qquad \downarrow \\ pos_1 \longrightarrow & neg_1 - neg_4 \end{bmatrix}$$

System notation for negations non_i, $i = 1, 2, 3, 4$

$$\begin{bmatrix} id_4 \\ non_1 \; id \ldots \end{bmatrix} : \begin{bmatrix} S_4 \\ S \mid S \end{bmatrix} \xrightarrow{perm_1} \begin{bmatrix} \bar{S}_4 \\ \bar{S} \mid S \end{bmatrix}$$

$$\boxed{\mathrm{neg}_1\left(\mathrm{neg}_2\left(\mathrm{neg}_1\left(D^{(3)}\right)\right)\right) = \mathrm{neg}_2\left(\mathrm{neg}_1\left(\mathrm{neg}_2\left(D^{(3)}\right)\right)\right)}$$

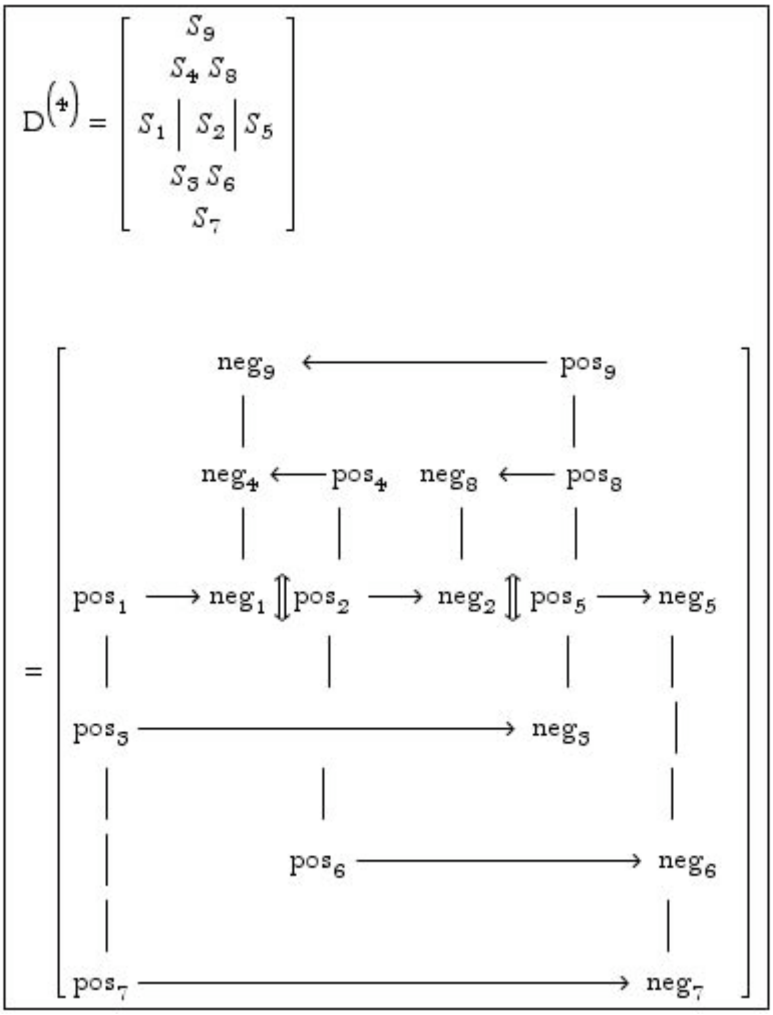
$$neg_1\left(neg_2\left(neg_1\, D^{(3)}\right)\right) =$$

$$\begin{bmatrix} & S_4 & \\ S_1 & | & S_{2.} \\ & S_3 & \end{bmatrix} \xrightarrow{\mathrm{perm}_1} \begin{bmatrix} & \bar{S}_4 & \\ \bar{S}_1 & | & S_{3.} \\ & S_2 & \end{bmatrix} \xrightarrow{\mathrm{perm}_2} \begin{bmatrix} & \bar{S}_4 & \\ \bar{S}_3 & | & S_1 \\ & \bar{S}_{2.} & \end{bmatrix} \xrightarrow{\mathrm{perm}_1} \begin{bmatrix} & \bar{S}_4 & \\ \bar{S}_2 & | & \bar{S}_1 \\ & \bar{S}_3 & \end{bmatrix} \bullet$$

$$neg_2\left(neg_1\left(neg_2\, D^{(3)}\right)\right) =$$

$$\begin{bmatrix} & S_4 & \\ S_1 & | & S_{2.} \\ & S_3 & \end{bmatrix} \xrightarrow{\mathrm{perm}_2} \begin{bmatrix} & S_4 & \\ S_3 & | & \bar{S}_{2.} \\ & S_1 & \end{bmatrix} \xrightarrow{\mathrm{perm}_1} \begin{bmatrix} & \bar{S}_4 & \\ S_{2.} & | & \bar{S}_3 \\ & \bar{S}_1 & \end{bmatrix} \xrightarrow{\mathrm{perm}_2} \begin{bmatrix} & \bar{S}_4 & \\ \bar{S}_2 & | & \bar{S}_1 \\ & \bar{S}_3 & \end{bmatrix} \bullet$$

Logified diamond$^{(4)}$



2.4.8. Transclusions and transjunctions

What is a transclusion logically? Is there any similarity, on a logical level, between *transclusions* and *transjunctions*?

"A transclusion is the reuse in whole or in part of another node in one node's rendering. That is, one node including another node when it is being displayed. A transclusion is different from pure copying, however, in that only a reference to the foreign material is stored."
http://www.usemod.com/cgi-bin/mb.pl?TransClusion

Transclusions are including data from other places without copying and duplicating the data.
Logical transjunctions are mirroring logical data from logical functions of other places into their own logical domain. That is, transjunctions are including logical values of other logical systems, without copying them into their domain. Transjunctions are polylogical functions representing interactions between different logical systems. Such representations and inclusions are not set theoretical operations.

The example shows an interaction in a 3-contextural logic between Logic_1 and Logic_2, Logic_3.
Logic_1 is including parts of Logic_2 and Logic_3.

Distribution tables for (trans, conj, conj) in Logic$^{(3)}$

PM	O1	O2	O3
M1	log1	log1	log1
M2	∅	log2	∅
M3	∅	∅	log3

PM	O1	O2	O3
M1	trans	trans	trans
M2	∅	and	∅
M3	∅	∅	and

Logical function for (trans, conj, conj)

$$(\oplus\wedge\wedge): L^{(3)} * L^{(3)} \xrightarrow[(\oplus\wedge\wedge)]{} L^{(3)}: [L_1, (L_2 \| L_1), (L_3 \| L_1)]$$

$$\left[\begin{array}{l} \mathrm{Log}_1: L_1 * L_1 \xrightarrow[\text{transjunct } \oplus]{} L_1: \begin{cases} f_1 * t_1,\ t_1 * f_1 \longrightarrow f_2,\ f_3 \\ t_1 * t_1 \longrightarrow t_1,\ t_3 \\ f_1 * f_1 \longrightarrow f_1,\ t_2 \end{cases} \\ \mathrm{Log}_2: L_2 * L_2 \xrightarrow[\text{conjunction } \wedge]{} L_2 \| L_1 \\ \mathrm{Log}_3: L_3 * L_3 \xrightarrow[\text{conjunction } \wedge]{} L_3 \| L_1 \end{array}\right.$$

Tableaux rules for transjunction with conjunctions

$$\frac{f_1\ X \oplus\wedge\wedge\ Y}{\begin{array}{c} f_1\ X \\ f_1\ Y \end{array}} \qquad \frac{f_1\ X \oplus\bigwedge\bigwedge\ Y}{\begin{array}{c} f_1\ X \\ f_1\ Y \end{array}}$$

$$\frac{t_2\ X \oplus\wedge\wedge\ Y}{\begin{array}{c|c} t_2\ X & f_1\ X \\ t_2\ Y & f_1\ Y \end{array}} \qquad \frac{f_2\ X \oplus\wedge\wedge\ Y}{f_2\ X \,\big|\, f_2\ Y \,\Big\|\, \begin{array}{c|c} f_1\ X & t_1\ X \\ t_1\ Y & f_1\ Y \end{array}}$$

$$\frac{t_3\ X \oplus\wedge\wedge\ Y}{\begin{array}{c||c} t_3\ X & t_1\ X \\ t_3\ Y & t_1\ Y \end{array}} \qquad \frac{f_3\ X \oplus\wedge\wedge\ Y}{f_3\ X \,\big|\, f_3\ Y \,\Big\|\, \begin{array}{c|c} f_1\ X & t_1\ X \\ t_1\ Y & f_1\ Y \end{array}}$$

2.4.9. Transjunctions in textemes

Transjunction are naturally modeled in semiotics and textemes following the strategy sketched above. The conditions for transjunctions and transclusionss in general are distributed and mediated systems, like logics, semiotics, diamonds and textemes. Textemes are taking place, occupying a structural locus, this enables interactions, transclusions and transjunctions, between autonomous systems.

transjunction in Semiotics(3)

PM	$O1$	$O2$	$O3$
M1	sem1	sem1	sem1
$M2$	$\emptyset$	sem2	$\emptyset$
$M3$	$\emptyset$	$\emptyset$	sem3

2.5. Structure of environments for transclusions in textemes

The structural possibilities of environments are now offering different realizations of interactions, concretized as links, transclusions and other interactions.

Depending on the complexity of interplaying textemes, different structural possibilities for interaction for interactionality, reflextionality to interventionality are accessible for implementation.

Depending on the structure of the common environments, actions like reflection, interaction and intervention are available and supported for interplaying textemes.

Reflection: Bidirectional environments are offering minimal requisites for mutual reflection .
Interaction: Mutual autonomy of different environments are enabling interaction.
Intervention: Different antidromic environments offer the possibility of intervention between the different environmental systems.

The bilateral interaction between the two isomorphic environments is a new topic added to the unilateral environment of diamonds.

$$(1): \boxed{\tilde{I}_\omega \leftleftarrows \tilde{I}_\alpha}$$

In this case (1), both actors are agreeing to accept a common environment.

$$(2): \boxed{\tilde{I}_\omega \rightleftarrows \tilde{I}_\alpha}$$

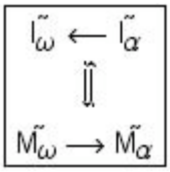
In this case (2), both actors agree in the autonomy and simultaneity of their environments, which are accepted as their common environment.

$$(3): \boxed{\begin{array}{c} \tilde{I}_\omega \leftarrow \tilde{I}_\alpha \\ \tilde{M}_\omega \leftarrow \tilde{M}_\alpha \end{array}}$$

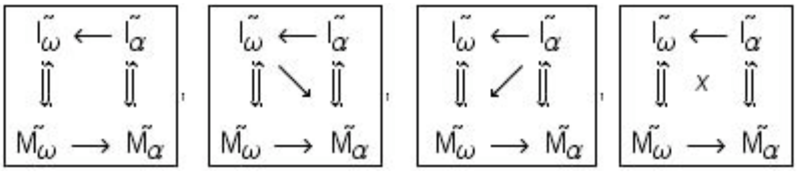
In this case (3), two different environments are accepted as the common environment. A further task would be to analyze their intra-environmental structure of cooperation.

$$(4): \boxed{\begin{array}{c} \tilde{I}_\omega \leftarrow \tilde{I}_\alpha \\ \tilde{M}_\omega \longrightarrow \tilde{M}_\alpha \end{array}}$$

In this case (4), two different, antidromic autonomous environments are accepted as the common environment of the texteme.

(5): 

In this case (5), two different and antidromic environments are accepted as the common environment of the texteme and *interventional* activities between both environments are possible.

(6): 

In this cases (6), new combinations of textemes are required to realize parallel and orthogonal interactions. Further combinations might be introduced as applications of different patterns.

Category of Glue, Part II

Is there any glue to stop the decline of Western culture?

Rudolf Kaehr Dr.[@]

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Abstract

Part I:

A typology of different categories of glue (ordinary, super-, para-, proto-, trans-glue) are glued together with different strategies of gluing (set and category theory, combining logics, bi-category with (co)spans, polycontexturality and diamond theory). Interpretations of "interactional glue", "nerve glue", "logical glue" are sketched. Keywords of the dissemination of the concept of "glue" in history (Hegel, Marx, Lenin, Gunther, Derrida, Obama) and strategies (Glue, Opium, Mediation) of gluing them together under a general parapluie (ontology, society, solidarity, fear) are critically sketched.
The economical question is: *Can we still afford to glue interactions together?*
The category of glue isn't blue. Categories are clueless to interaction and are banking unsecured resources.

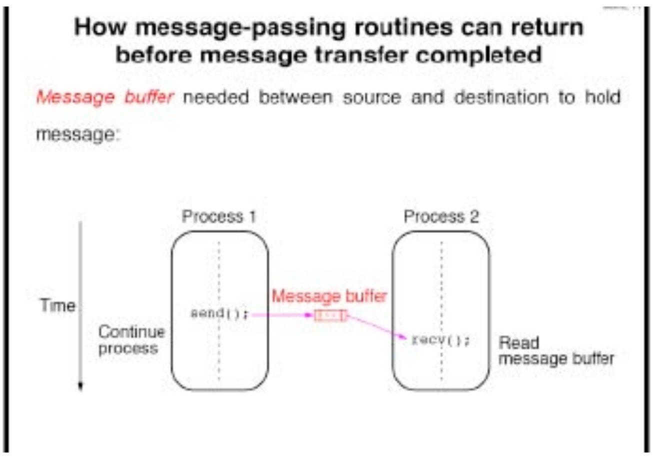
Part II:

How to get rid of glue? From gluing to jumping. A new abstraction, the *as-abstraction*, and a subversion, the morpho-abstraction, has to be risked to avoid the complicity of category theory with the unavoidable exploitation of (conceptual) resources by the Western approach to interaction and communication in computer science.
To overcome the limitations of the category "glue", contexturalization and mediation in a chiastic and diamond framework has to be elaborated and achieved to create chances to surpass and subvert such cultural and technological limitations.

1. Diamond theory of interactivity

1.1. Buffering super glue

1.1.1. Gluing information



"Whatever its nature (radio waves, wires, laser beams), the carrier is called a *transmission line* or wave. At the other end of this line the *message* is decoded and transcribed into *information* that has *meaning* for the person to whom it is addressed. But in order for the recipient to recognize and use the information, there must already have been information *memorized* that can be compared with the message just received. A final and important point is that *disturbances* occurring in the transmission line (the "noise") can alter the message and change its meaning." (de Rosnay)
Joël de Rosnay, THE MACROSCOPE, A NEW WORLD SCIENTIFIC SYSTEM
http://pcp.vub.ac.be/macroscope/chap4.html

1.1.2. Circularity of buffering information

There is obviously a kind of a well known circularity involved with the buffer paradigm.
A sender is sending a message to a receiver, but this procedure is not working directly from sender to receiver, but indirectly via a buffer. Hence, the buffer is a receiver too, albeit a semi-receiver, but in its functionality to act as a buffer it has to receive the message which has to be buffered. Therefore, the buffer to work as a receiver needs a buffer, his own semi-buffer, which is enabling the main buffer to buffer the massage for a receiver. Again, this is only the beginning of an infinite regress. Our semi-buffer needs a semi-semi-buffer to semi-buffer the buffer to buffer the message for a receiver. And so on!

In other words: Super Glue isn't enough. What is needed is the *ultimate* super glue, the super glue of the super glue.

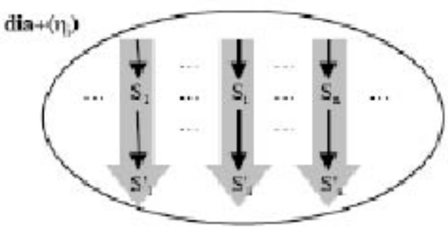
Some Hello!
"Those links may make you pull your hair out! The "Glue" article is all over the place, but, I think, working around the problem you're addressing." (John Powers)

The style of this "Glue"-text might succeed to reflect, "all over the place" (John Powers), the lack of strict and save interconnections between its heterogeneous parts. Also, as much is still glued together by the metaphor "glue", the lack of continuity, deductive or explicative, is marking the *gaps* and the chances or challenges to choreograph jumps and salti to "bridge" together that what doesn't belong together. What belongs together, and might be glued for ever, like mankind, nation, family, identity, doesn't need to be bridged. It remains well placed and accessible in the labyrinth of the human cage.

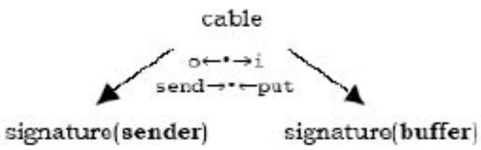
especially:

1.2. Streching super glue

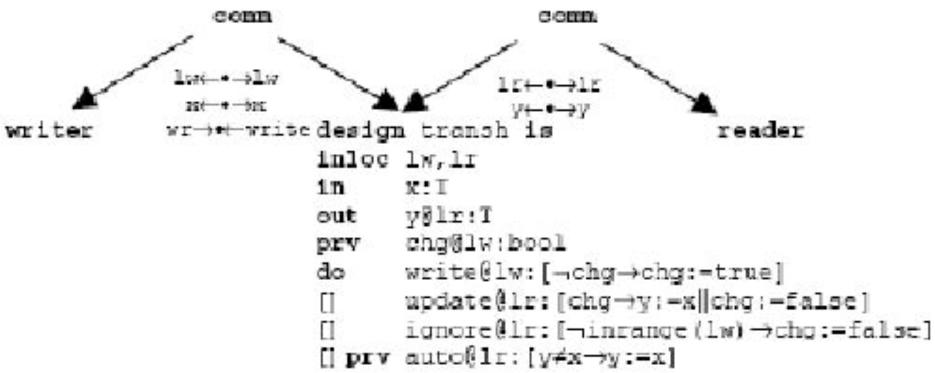
1.2.1. Horizontally: Meta-pattern



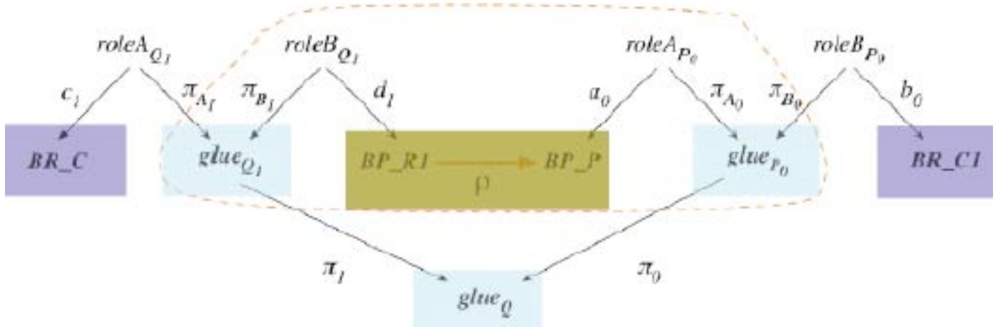
"For instance, a configuration in which the messages from a sender component are sent (to a receiver) through a bounded buffer defines the following diagram:



The node labelled *cable* is the representation of the set of bindings.” (Cat, p. 146)
http://www.cs.le.ac.uk/people/jfiadeiro/



Glue in action (Fiadeiro, *p.* 19, 2008)



1.2.2. Pfalzgraf’s Fibered Glue

Another strategy of streching and distributing glue is given by the general techniques of fibering and indexing categories. Where is the glue of fibered distribution, e.g. of agents in Multi-Agent Systems (MAS)?

Despite a great clarification of meta-glued fusions and confusions, meta-glue is still at work and necessary to enable interaction between distributed agents in the fibered polycontextural model of MAS. Fibered distribution is modeled in the framework of *bundle* categories along a general *index* fiber, i.e a *“base manifold”* (Pfalzgraf). The index fiber is gluing all those distributed free fibers together. Hence, fibered distribution is based on glue. This glue is gluing together all the fibers of stretched glue-fibres. The proposed technniques are higly sophisticated: the super glue is meta-gluing itself iinfinitely. That is, the base manifold or index set might be fibered itself. The constuction is “recursive, fractal and self-referential" (Pfalzgraf), hence bottom-less.
http://www.cosy.sbg.ac.at/~jpfalz/ACCAT-Tutorial-KI2004.html

"Actually, when listening to the experts speaking about *polycontextural* logics, he was reminded of the concept of *fiber spaces.*

This led him to the idea of introducing *“logical fiberings”* , i. e. taking the abstract concept of fiber bundles and combine it with logical spaces as fibers all put together over a *base manifold* (which acts as *index set* with a particular structure). Thus over each point of the base space there sits ('resides’) a fiber which can be interpreted as the *local state space* of that point ('agent’).

The fiberings method is found to be very useful in modeling communication and interaction between cooperating agents, due to the possibility to switch between a *local/global* point of view which is inherent to this framework."

Pfalzgraf et al, Towards a General Approach for Modeling Actions and Change in Cooperating Agents Scenarios, 1996
PFALZGRAF et al. Logic Jnl IGPL.1996; 4: 445-472 , http://jigpal.oxfordjournals.org/cgi/content/abstract/4/3/445

A more generalized approach to fibred logics is given recently by Pfalzgraf/Soboll as a *"base diagram"*, which is introduced with: "Fundamental important for our work is the observation that the general *communication* and *cooperation* structure of a MAS can be represented by a corresponding arrow diagram, called *base diagram* of the MAS."

"To each MAS we associate such a *base diagram*, which represents the complete relational structure (i.e. communication in the general sense). The nodes of this arrow diagram represent *agents*, the arrows (and paths of arrows) hold the *communication* and *cooperation* information. This gives a category by its own right, more precisely a typed category.
In a MAS communication and cooperation (in general relations) between agents can change. This fact gives rise to the definition of the category **MAS** of all MAS where the objects are *base diagrams* of Multiagent Systems and the morphisms are **MAS** morphisms i.e. structure preserving maps between base diagrams.
Based on this category **MAS** a transformation system for Multiagent Systems can be established by applying the double pushout approach to Multiagent Systems.
[C]hecking the so called *"gluing condition"* solves this problem, in this paper we introduce an alternative algorithm."
[cf. DPO: Double Push Out!]
Thomas Soboll, On the Construction of Transformation Steps in the Category of Multiagent Systems
http://www.portal.acm.org/citation.cfm?id=1428606

"The scenarios of cooperating robot agents were originally devised to demonstrate how the concept of a Logical Fibering can be used in a natural way to assign a system of Distributed Logics to a MAS, where every agent has an individual local logical state
space (fiber), the collection of all the fibers forms the global state space (fiber bundle) of the MAS.
This Fiber Bundle aspect can naturally be extended and generalized to introduce fibers of various structure types, modeling corresponding state space properties which are of relevance to model the complete state space of an agent, consisting of various modules (fibers) definig the complete type of an agent." (Pfalzgraf, p.34)

1.2.3. What are the aims of glued interactions?

Service-oriented approach
"Services add a ***social*** layer of abstraction over a component infrastructure in sense that they structure the process of interconnection (*programmed interconnections*).
- Services should be published at a level of abstraction that corresponds to a real-world activity or
  recognisable business function (which is where social complexity can be understood)
- Systems should be ***socially-reflective***."
(Fiadeiro, 2008, p. 4)
Semantics of Service Discovery and Binding,
http://www-lipn.univ-paris13.fr/~choppy/IFIP/URBANA-CHAMPAIGN/URBANA-DATA/Fiadeiro-Urbar

"For every activity *a*, a homomorphism *B(a)* of graphs between the body of *B(a)* and *SF*.
(This homomorphism makes configurations ***reflective***.)" (ibd., p.9)

"Therefore, we decided to look for algebraic mechanisms of interconnection that can capture **peer-to-peer** interactions among autonomous components. That is why, in this paper, we report on the use of co-spans - pairs $<f_A,f_B>$ where f_A:A->S and f_B:B->S are morphisms of a category D." (Fiadeiro, CALCO'07, p. 195)
J. L. Fiadeiro, V. Schmitt (2007) Structured co-spans: an algebra of interaction protocols. In T. Mossakowski, U. Montanari, M. Haveraaen (eds) Algebra and Coalgebra in Computer Science. LNCS, vol 4624. Springer, Berlin Heidelberg, pp 194-208

Glue is gravitational, it holds things together by forcing them down.

1.3. Inhaling glue

Along the symbolic interaction metaphor of "I" and "Me" of a "Self", a duplication of an agent into itself and his inner-environment might open up the possibility for a *reflectional* modeling. Such a

reflectional distinction is reasonable only for a *society* of agents and is of no relevancy for a solipsistic concept of agents.
The inhaling concept of reflectional glue is highly solipsistic and is celebrating gluish hedonism.
Therefore, to stretch further the antropomorhic metaphor of "I" and "Me" of a "Self" for societal interaction, a new distinction has to be sketched: the distinction between the homogeneity of (empirically) different "Selfs"s and a heterogeneity between such "Selfs" and "Thou". This move is quickly over-stretching ordinary glue. Between Herbert Mead's *Self* as I and Me, and Gotthard Gunther's *Thou* as a fundamentally different "I " and "Me", a chiasm demands challenging flexibility to any glue:

$$\text{Chiasm}\left(\left(\text{I, Me}\right)_{\text{Self}} \Updownarrow \left(\text{I, Me}\right)_{\text{Thou}}\right).$$

Such over-stretching is avoided by the denial of a dialogical difference between Self (or I) and Thou.

That is, the functionality of a buffer for sender and receiver could be *incorporated* from the outer-environment (channel) into the inner-environment of an agent as a reflectional part of the sender and receiver interaction. The inner-environment of an agent (Me) is taking the agent-specific elements of glue into its domain. Glue gets inhaled and with this incorporation it is loosing its procrastinating function as a non-computing coordinative buffer. Inhaled glue becomes part of the agent and its computational facilities.

This approach to reflectionality is leading strictly into meta-circular iterations and regresses. Quickly, the incorporated glue is starting to glue itself by incorporating its incorporated glue, endlessly.

A *monocontextural* modeling is drawing a strict distinction between the three identical, i.e. non-reflectional entities:
Sender, Buffer, Receiver with the activities: send, receive, put, get. That is, send-->put, receive<--get.
Therefore, there is no flexibility accessible to model reflectional incorporation of the functionality of buffers into the instances "sender" and "receiver".

A *polycontextural* modeling is drawing a distinction between two positions and their chiastic functionality as Sender and as Receiver. Hence, distinctions like *"buffer as receiver"* and *"receiver as buffer"* and also "*buffer as sender"* and *"sender as buffer"* are accessible.

Such a movement towards an *internalization* of the functionality of buffers is reducing the costs of interactions.
Because of its chiastic structure, such an internalization is also free of bad circularity, like the infinite regress of buffering buffers.
Hence, the simple distinction between active and passive processuality of an agent (computation) and buffer (coordination) has to be transformed into an active/passive activity of agents.

2. Getting rid of glue

2.1. Interfaces

2.1.1. Interfaces as mutual representation

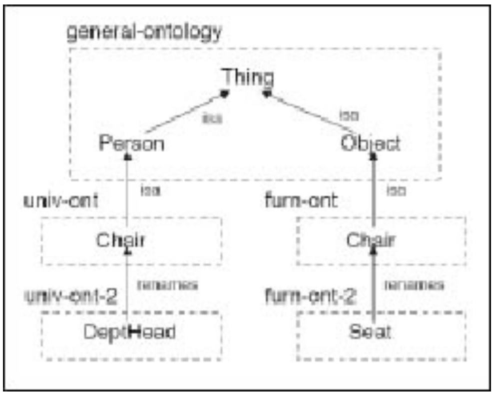
Glue-free mediation is still challenging the clue-less approaches of scientific modeling, formalization and implemention of interactivity and interactionality.

As sketched before, interactions mediated by buffers and their glue are interacting in the ontological-semiotic mode of *is-abstractions* (is-a, has-a) accepting basic presupositions of identity-driven conceptualization and implementation, i.e. ontology, semiotics and logic.

One of the main problems of modeling in the mode of the is-abstraction is the well known *polysemy* of semantics and its multiple inherence. Despite the amount of academic solutions, the problem remains as long as its identity presuposition, i.e. disambiguation, isn't changed.

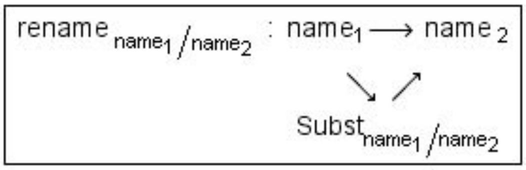
The sold solutions are based on direct or indirect *ad-hoc* principles and *one-step*-thinking.

The example below is base on the ingeneous procedure "*rename*" (Hendle). After renaming the term, the general ontology might be prolongated from the general ontology [Thing, Person] with its university-ontology to the furniture-ontology. With this procedure, no conflict will arise. At least as long as nobody starts thinking one step further.

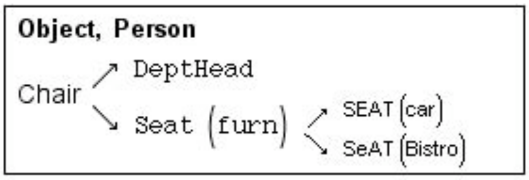


Cost of disambiguation and buffering
Communication of disambiguated data is cost-free. But it is naive to think that it comes without costs. In contrary, the costs of disambiguation, necessary before any procedures can happen, are enormous. But disambiguation is arbitrarily stopped and is depending on a context. With a change of context, disambiguation has to start again.

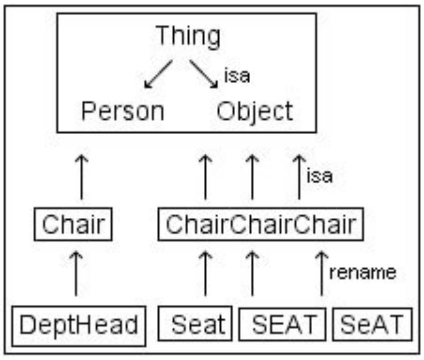
Renaming is itself a kind of a buffering, i.e. gluing procedure, lacking any interactivity:



Example
Semantics of*Chair*:



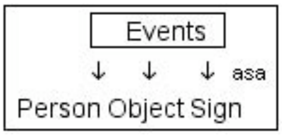
Some renamed and extended ontologies



Extensions by renaming are ad-hoc and have to be repeated endlessly through the semantic universe, which is not restricted to the English language only. Even to stop its regress in concrete computing scenarios isn't cost-free. Fixed solutions, glued by chance, have to be re-fixed after the slightest context-change of the fixture. For complex, flexible and mobile computing, simply a disaster.

It is typical for the design of ontologies for semantic purposes in computation, like the Semantic

Web, that the medium of its thematization, i.e. language, sign systems, is not mirrored in the General Ontology Language (GOL). GOL is designed in the framework of set theory which is mirroring Aristotelian ontology. There is no place for reflection, self-reflection and interaction between autonomous subjects left.
Reflectional approaches are much involved with the, at least, triadic-trichotomic semiotic in the sense of Peirce (Goguen).
Therefore, to the whatever Aristotelian ontology, at least the “ontology” of the vocabulary “Sign” has to be added for a *conservative* extension of the GOL. Hence, events or phenomena are occuring as Person, Object or Sign in an (semiotically) extended ontology.



Again:
“No self-reference is possible unless a system acquires a certain degree of freedom. But any system is only free insofar as it is capable of interpreting *its environment and choose for the regulation of its own behavior between different interpretations.”* Gunther, 1968, p. 44

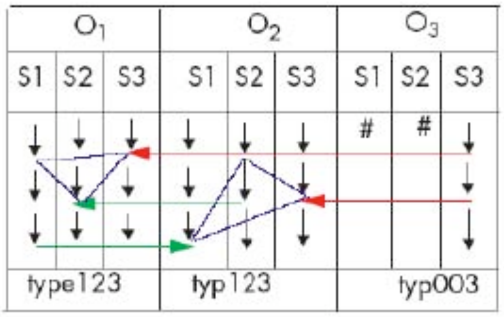
Hence, glued, i.e. buffered, interaction is unnecessarily cost-intensive even before any computation and coordination can happen.
There are conceptual costs, too. The identity approach, even in a 2-categorical setting, is conceptually restricting possibilities.

2.1.2. Polycontextural approach

A reflectional analysis of polysemy is an analysis of the semiotic actions of agents which are leading to the phenomenon of polysemy and its possible conflicts with other semiotic or logical principles. Therefore, such an analysis is more complex, because it has to describe the situation intrinsically, that is from the inside and not from the position of an external observer.
The aim of a polycontextural modeling of polysemy is to design the construction of polysemy in a finite and concrete manner.

The distinction of two fundamnetally different kinds of abstractions, the is-abstraction (identification) and the *as-abstraction* (thematization)*,* might help to model the reflectional situation. The formula of the as-abstraction “asa” is: *“X as Y is Z”*, hence, the is-abstraction “isa”, X is X, can be read as an abbreviation of: *“X as X is X"*.

The as-abstraction is introducing highest flexibility for modeling, designing and implementing complex *reflectional* situations. The as-abstraction is abstrating from the main principle of any computation and computing: the principle of *identity*.



PM	O_1	O_2	O_3
M_1	chair	□	□
M_2	□	head	□
M_3	$chair_{tok}$	person	token

PM	O_1	O_2	O_3
M_1	S_1	S_1	x
M_2	S_2	S_2	x
M_3	S_3	S_3	S_3

(see below §2 .3):

PM	O_1	O_2	O_3
M_1	$comm_1$	put_1	x
M_2	put_2	$comm_2$	x
M_3	get_3	get_3	$comm_3$

Mono-contextural "isa":
S1: Chair is part of a furniture ontology (Object),
S2: Chair is part of a department ontology (Person),
S3: Chair is part of a vocabulary ontology (Vocabulary).

Poly-contextural "isa as":
O1S1: Chair as such, that is, as an object (furniture) "Chair", $Chair_{obj}$,
O2S2: Chair as such, that is, as a person (head) "Chair", $Chair_{pers}$,
O3S3: Chair as such, that is, as the token (vocabulary) "Chair", $Chair_{token}$.

PM	O_1	O_2	O_3
M_1	[S_1]	x	x
M_2	x	[S_2]	x
M_3	x	x	[S_3]

Here, "as such" means, that the ontologies *Object, Person* and *Vocabulary* can be studied and developed for their own, independent of their interactivity to one another but placed and mediated in the constellation of their poly-contexturality, that is, their distribution over the 3 loci, O_1, O_2 and O_3.

Voc O3S3 in Furn O1S3 :
The token "Chair" as used to denote the object "Chair", $Chair_{token, obj}$
VocO3S3 in Dept O2S3 :
The token "Chair" as used to denote the person "Chair", $Chair_{token, Head}$
Chair O2S2 in Dept O1S2 :
The object Chair as used in the person ontology Dept, $Chair_{obj, Dept}$
Chair O1S1 in Furn O2S1 :
The person Chair as used in the object ontology Furn, $Chair_{pers, Furn}$

PM	O_1	O_2	O_3
M_1	S_1	[S_1]	x
M_2	[S_2]	S_2	x
M_3	[S_3]	[S_3]	S_3

Reflectional situations
Chair O2S2 in Dept O1S2:
System O1S1 has in its own domain space for a mirroring of O2S2. This space for placing the mirroring of O2S2 is the reflectional capacity realized by the architectonic differentiation of system O1. In other words, O1 is able to realize the distinction between its own data and the data received by an interacting agent. Data are therefore differentiated by their source, e.g. their functionality, and not only by their content.

PM	O_1	O_2	O_3
M_1	S_1	x	x
M_2	S_2	S_2	x
M_3	x	x	S_3

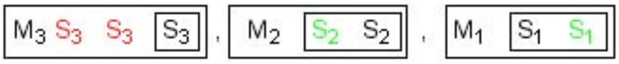
Chair O1S1, in Furn O2S1:
System O2S1 has in its own domain space for a mirroring of O1S1.

PM	O_1	O_2	O_3
M_1	S_1	S_1	x
M_2	x	S_2	x
M_3	x	x	S_3

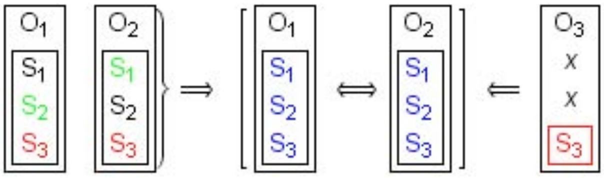
A (re)solution of the problem
The department Dept for itself has no conflict with polysemy. This conflict between Dept and Furn is mediated by the Voc. That is, the Person of the Dept as Chair is a person and nothing else.
The furniture Furn for itself has no conflict with polysemy. This conflict between Furn and Dept is mediated by the Voc. That is, the Chairs as objects of the Furn are chairs and nothing else.
The vocabulary Voc for itself, containing "Chair", has no conflict with the polysemy between Dept "Chair" and Furn "Chair".

The *meaning* of the polysemic situation is realised by
Meaning of (O3S3) = interaction of (O1S3, O2S3)

PM	O_1	O_2	O_3
M_1	S_1	S_1	x
M_2	S_2	S_2	x
M_3	S_3	S_3	S_3



The conditions for a conflict arises excactly between
O1 (S1,2,3) and O2 (S1,2,3) mediated by O3S3 as visualized by the blue triangles.



Both Furn and Dept are using Voc and both are using the string Chair of Voc. Both are different and are mapping the Voc differently relative to their position, thus the Voc has to be distributed over different places according to its use or functionality. The Voc used by Furn is in another functionality than the Voc used by Dept.

Up to now we have not yet produced a contradiction but only a description of the situation of *polysemy*, that is, the necessary conditions for a possible ontological, semantic and logical contradiction.

A user-oriented approach to the modelling of polysemy has to ask *"For whom is there a conflict?"* Therefore, we have additionally to the semantic and syntactic modelling of the situation to introduce some pragmatic instances. In our example this could be the user of a *Query* which is answering in a contradictory manner.

Query´s contradictory answer
Now we have to deal with the contextures: (Query, Voc, Furn, Dept).
In the classic situation, the Query answers with a logical conjunction of Chair as Person *and* Chair as a Department member, which both are logico-semantically excluding each other and therefore producing for the user of the Query a conflicting or contradictory answer if put into a single logical conjunction.

Logic comes into the play also for the polycontextural modelling, but here *conjunctions* too, are distributed over different contextures. And therefore, a contradiction occurs only if we map the complex situation all together onto a single contexture.

(O_4M_4) = Query

PM	O_1	O_2	O_3	O_4
M_1	S_1	S_1	x	□
M_2	S_2	S_2	x	□
M_3	S_3	S_3	S_3	□
M_4	□	□	□	S_4

Logic:

$$\text{Person} \bigcap \text{Object} \bigcap \text{Vocabulary} = \emptyset,$$

$$\text{Person}(\text{Chair}) \equiv \neg \text{Furniture}(\text{Chair}) \equiv \neg \text{Vocabulary}(\text{Chair})$$

$$\text{Chair} \in \text{Department} \Longrightarrow (\text{Chair} \notin \text{Furniture} \wedge \text{Chair} \notin \text{Vocabulary})$$

Contradictory Answer:

$$(\text{Chair} \in \text{Department} \wedge \text{Chair} \in \text{Furniture}) \Longrightarrow (\text{Chair} \in \text{Department} \wedge \text{Chair} \notin \text{Department}) : \#$$

Polylogic

$$\text{Chair} \in (\text{Department}, \text{Furniture}, \text{Vocabulary})$$

$$\text{Person} \bigcap \text{Department} = \emptyset, \ \text{Person} \equiv \neg \text{Department}$$

$$\begin{bmatrix} \text{Chair} \in^1 \text{Department}, \text{Chair} \notin^1 \text{Furniture}, \text{Chair} \notin^1 \text{Vocabulary} \\ \text{Chair} \in^2 \text{Furniture}, \text{Chair} \notin^2 \text{Department}, \text{Chair} \notin^2 \text{Vocabulary} \\ \text{Chair} \in^3 \text{Vocabulary}, \text{Chair} \notin^3 \text{Department}, \text{Chair} \notin^3 \text{Furniture} \end{bmatrix}$$

Polylogic with Conjunctions: $\bigwedge^{(3)} \equiv (\bigwedge^1 \wedge^2 \wedge^3)$,

Negations: $\neg^i$, $i = 1, 2, 3$ and non-entailments: $\neg^i \in \ \equiv \ \notin^i$

$$(\text{Chair} \in^1 \text{Department} \wedge^{(3)} \text{Chair} \in^2 \text{Furniture}) \Longrightarrow (\text{Chair} \in^1 \text{Department} \wedge^{(3)} \text{Chair} \notin^2 \text{Department})$$

$$\begin{pmatrix} \text{Chair} \in^1 \text{Department} \wedge \text{Chair} \notin^1 \text{Department} \Longrightarrow \emptyset \\ \text{Chair} \in^2 \text{Furniture} \wedge \text{Chair} \notin^2 \text{Furniture} \Longrightarrow \emptyset \\ \text{Chair} \in^3 \text{Vocabulary}, \text{Chair} \notin^3 \text{Vocabulary} \Longrightarrow \emptyset \end{pmatrix}$$

$$\begin{pmatrix} \text{Chair} \in^1 \text{Department} \bigwedge^1 & \text{xxx} \\ \text{xxx} & \bigwedge^2 \text{Chair} \notin^2 \text{Department} \\ & \bigwedge^3 \text{Chair} \in^3 \text{Vocabular} \end{pmatrix} \Longrightarrow^{(3)} \begin{pmatrix} \text{Chair} \in^1 \text{Department} \bigwedge^1 & \text{xxx} \\ \text{xxx} & \bigwedge^2 \text{Chair} \in^2 \text{Furniture} \\ & \bigwedge^3 \text{Chair} \in^3 \text{Vocabular} \end{pmatrix}$$

Short: $(\text{Chair} \in^1 \text{Department} \wedge^{(3)} \text{Chair} \in^2 \text{Furniture} \wedge^{(3)} \text{Chair} \in^3 \text{Vocabulary})$.

If we give up all the introduced ontological distinctions of polycontexturality and reducing therefore our ontologies to a single mono-contextural general ontology we saved our famous contradiction again. But now, this contradiction is a product of a well established mechanism of reduction. And sometimes it isn't wrong to have it at our disposition.

The costs of the polycontextural approach lies in its novelty and its intrinsic complexity.

Observations about observations in another worldmodel

"Unlike observation of the first order, which sees all elements in the world as connected in one context with geometric symmetry, observation of the second order views a world that is poly-contextual (polykontextural). In the poly-contextual world, the values of social institutions may not all be in the same context.From such perspective, labeling two values in different contexts as a set of binary for comparison in the same matrix is deemed to generate inaccurate analysis.
This is why "many-valued-logic" (mehrwertige Logik) is so essential in the poly-contextual world.When the values in seemingly antithetical binaries are, in fact, of different contexts, for those values to be antithetical yet mutually complementary is no longer impossible. In fact, this antithetical yet complementary pattern that is unthinkable in the realm of the world of the observation of the first order is totally consistent with the many-valued-logic of poly-contextual settings.
According to Lin, this many-valued-logic is exactly the essence of traditional Chinese law and its legal system."
Review: Duan Lin. _Weibo Lun Zhongguo Chuantong Falu_(Weber's Analysis on Chinese Traditional Law: Critiques on Weber's Comparative Sociology)
Dr. Lin Duan has provided a profound study on the demerits of Weber's methodology. See Lin Duan, Rujia lunli yu falü wenhua: shehuixue guandian de tansuo (Confucian Ethics and Legal Culture: Exploration from Sociology), Beijing: Zhongguo zhengfa daxue, 2002, p. 93; 122.

2.2. Diamond modeling

2.2.1. General strategies

Diamond constructions are reducing the costs of interactivity by the fact that their operations are intrinsically interactive. That is, the interplay of the conditions of matching and the compositions themselves are reflected in the complementarity of categories and saltatories of diamonds.
Diamonds are offering more structural space to model and implement interactivity than categories and n-categories.

The general strategy to reduce the costs of interaction is to find a concept and apparatus that is offering a wider *logical scope* to model the dynamics of the differences between actors and their communication.
The other part of the strategy is to *separate* basic functions, like coordination/computation (Fiadeiro) or locality/connectivity (Milner). Both strategies are changing the priority of time in computing to the favor of space, *"metaphorical space"* (Milner) for the localization of separated aspects of (mobile) computation.

This enlargement of *"metaphorical space"* happens for computer science in different steps, all trying to capitalize on new and broader concepts from other disciplines or on developing own computer science specific concepts, like , e.g. Goguen's Institutions and Padawitz' Swinging Types.

Modal logics
Coalgebra
Category theory
Combining logics
n-Category theory

But there are some fundamental limitations occurring in this endeavour.

"Grammatologically, the Western notational system is not offering space in itself to place sameness and otherness necessary to realize interaction/ality. Alphabetism is not prepared to challenge the dynamics of interaction directly. The Chinese writing system in its scriptural structuration, is able to place complex differences into itself, necessary for the development and design of formal systems and programming languages of interaction. The challenge of interactionality to Western thinking, modeling and design interactivity has to be confronted with the decline of the scientific power of alpha-numeric notational systems as media of living in a complex world. " (Interactivity, 2008)

To overcome such limitations, the graphematic and trans-classical strategies of:
Polycontexturality,
Morpho- and Kenogrammatics,
Proemiality and Chiasm,
Diamond theory,
had been introduced as subversive and experimental interventions and realized in form of fragmentarism.

It seems that for each step there is a progressive extension of the possibilities of 'space-ing' interactionality and reflectionality of notational systems.

All known strategies have their own advantages and deficits. The main problems are not so much the limitations of the specific approaches but their applications to domains for which they are not specifically designed and are therefore inadequate albeit the compagning propaganda.

It surely would be an absurd misunderstanding, also quite typical for the 'quick-reading' ritual of censors, to believe that I am hallucinating a computer science based on Ancient Chinese hieroglyphs. (Kaehr, The Chinese Challenge).

2.2.2. Categorical composition

"Category Theory is advocated as a good mathematical structure for this integration precisely because it focuses on relationships and interactions! The focus that Category Theory puts on morphisms, as structure preserving mappings, is paramount for Software Architectures because it is the morphisms that determine the nature of the interconnections that can be established between objects (system components)." (Fiadeiro, 2002, p. 12)

Category oriented implementations are based on the concept of categorical *composition* of morphisms which in itself is neither interactive nor reflective. It is mentioned that morphisms are representing interactions and interconnections. This might be appropriate for non-reflectional interactions, like actions on objects (Bunge). Reflection, and especially social reflection, gets into conceptual trouble. Interaction might be represented by morphisms, but reflection would then be represented by 'morphisms of morphisms'.

This figure leads automatically to the question: Is a morphism of a morphism, i.e. a second-order morphism (cf. Fiadeiro's homomorphism!) conceptually a morphism or something else? If it is conceived as a morphism, we are back again to the first-order concept of morphisms, i.e. to morphisms without reflection. And hence, the second-order concept can be reduced to a first-order concept. Otherwise, if the construct of a morphism of a morphism isn't reducible to a first-order categorical concept, then it is violating the axioms of the framework of category theory, especially its axiom of identity.

Hence, the workspace of the categorical interpretation of interactivity is modeled by the metaphors of super glue, for categories, and of stretched glue, for n-categories.

Such *societal* and *reflectional* metaphors are reasonable for computer science only if they are explicitly declared as *weak* metaphors, not suitable for social and psychological adaption. Albeit the fact that interdisciplinary *confusions* are supporting academic marketing strategies, they are nevertheless boring and economically and politically dangerous.

2.2.3. Dissemination

The dissemination of categorical systems in the sense of polycontexturality is offering the possibility of interactionality and reflectionality. But still for a considerable price of interpretative, i.e. observer depending, delays. Interactivity between disseminated contextural systems is guided by the strategies and mechanism of proemiality and are realized as chiasm.

Each contexture of a contextural compound constellation is composed by sub-contextures, usually as elementary morphisms and is of trichotomic structure. Thus, polycontexturality is a dissemination of trichotomic structures, that are, in themselves, not prepared to reflect their interactionality. Interactionality and reflectionality are introduced polycontexturally as interactions between different contextures, i.e. as trans-contextural events.

2.2.4. Chiasm

From an observer theoretical point of view, chiasms between contextures, realizing interactivity, is observed from an external observer. Hence, their internal mechanism is still not yet glue-free, but involved in a kind of a dissolution of the adhesion of glue, i.e. the chiastic jump between disseminated contextures is still sniffing glue and sticking the elements of mediation together by some metaphysical or kenomic glue.

Chiastic glue is technically delivered mainly by the *coincidence* relations of chiasms. They are responsible for guaranteeing that the distributed order relations, which are themselves glued by definition (matching conditions), and their risky exchange relations interact in harmony with similitude. The fulfillment of the coincidence relations in a chiasm is establishing categorial similarity, i.e. family resemblance of categorial kind. Otherwise, chiastic glue would be over-stretched and loosing its ends.

In other words, for chiasms to work, their relationality has to be in categorial harmony of similitude. Chiastic concepts have to fit together by general mappings or morphisms. Chiastic jumps, possible in chiasms, are insured by harmony and traced back and ruled by the coincidence or similarity relations. This important restriction and complicity to similarity is necessary for chiasms to avoid empty statements, like *"everything is connected"*, of universal connectionism. It also prevents jumps into the void.

Super-glue and stretched super-glue interactions, realized as buffers and buffered systems, don't need observers. They are designed and conceived as observer-independent objective utilities and mechanisms.
Therefore, the observer-independent approaches might be adequate for *signal* and *information* processing (information interaction) but are missing the demands for interactional situations of *semantic* and *knowledge* modeling and computation (symbolic interaction).

2.2.5. Diamondization

Jumps without guaranteed security have to be realized by somersaults, i.e. by salti. Salti are in a strict discontextural opposition to glued connections and secured journeys involving jumps. Hence, their theoretical modeling happens with the saltisitions of saltatories. Saltatories are building together with categories diamonds. What has to be risked is to stage-manage the drama of glue-free interactions and reflections within the framework of diamond theory. But metaphors like "*stage*" and "*framework*" are misleading by their unbroken coherence.

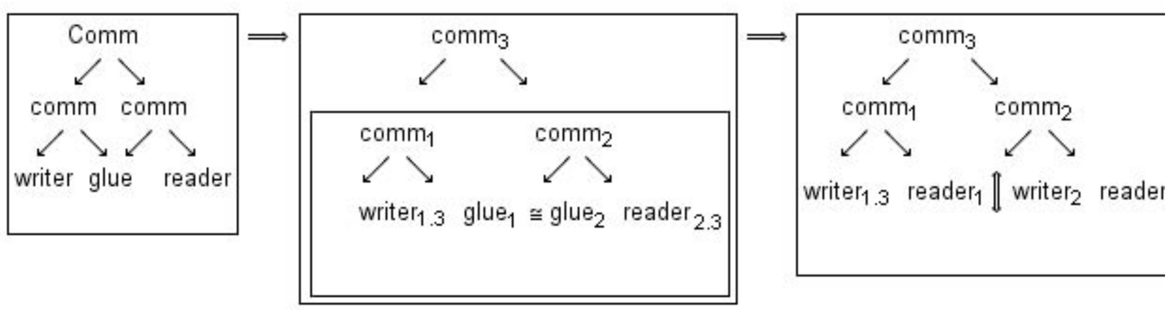
Categories: duality
Chiasms: guided complementarity or family resemblance (Wittgenstein)
Diamonds: discontecxtural complementarity

The epistemic tendencies "computation" and "coordination" (Fiadeiro) are thematized by the diamond strategies as complementary, antidromic and parallax.
Diamonds are offering a new kind of a "metaphoric space" to distribute both tendencies while keeping their autonomy untouched. With such a heterarchic order between computation and cordination there is no need to suppress the computational aspects of coordination and the coordinational aspects of commputation.

This is in contrast to the n-categorical modeling where there is no concept and no mechanism available on a principle level to implement the antagonistic epistemical directions of computation and coordination. Furthermore, both are set by the categorical approach into a *hierarchical* order: first computation, then coordination or the other way round: first coordination, then computation. Both are strictly excluding *"social life"*, i.e. interaction between computation and coordination.

2.3. Sketch of formal chiastic and diamond modeling

From hierarchy to heterarchy



Matrix modeling

PM	O_1	O_2	O_3
M_1	$comm_1 \rightarrow$	$write_1$	x
M_2	$write_2$	$\leftarrow comm_2$	x
M_3	$read_3 \rightarrow$	$read_3 \rightarrow$	$comm_3$

PM	O_1	O_2	O_3
M_1	$sender_1 \rightarrow$	$write_1$	x
M_2	$write_2$	$\leftarrow sender_2$	x
M_3	$read_3 \rightarrow$	$read_3 \rightarrow$	$receiver_3$

PM	O_1	O_2	O_3
M_1	$comm_1$	put_1	x
M_2	put_2	$comm_2$	x
M_3	get_3	get_3	$comm_3$

Communication (Sender, Receiver)

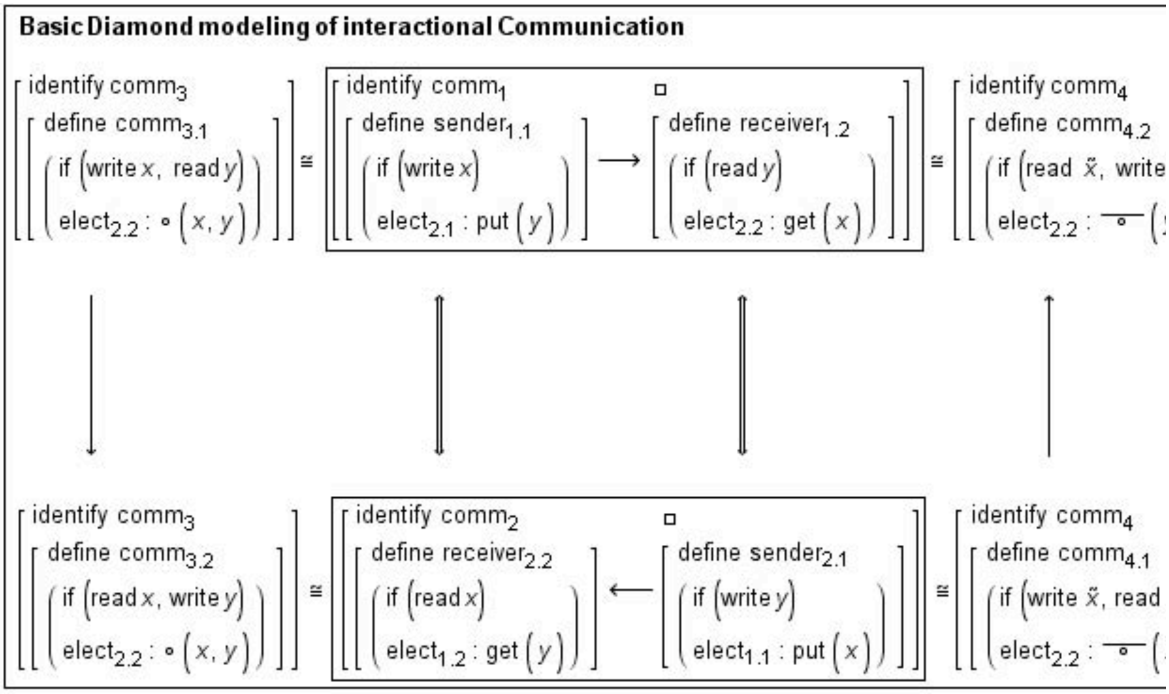
$comm_1$ as $sender_1 = put_1$ to O_2 @ $O_2\ M_1$
$comm_2$ as $sender_2 = put_2$ to O_1 @ $O_1\ M_2$
$comm_3$ as $(receiver_1 \circ receiver_2) = get_3$ from $O_1\ M_2$, get_3 from $O_2\ M_1$.

Contextural modeling in ConTeXtures

```
[ style interactional
[ topics symbolic          ;; < comm_i : symb_i --> symb_i, i = 1, 2, 3 >
[ thematize Comm^(3)       ;; < scenario : Comm^(3) = (com_1 , com_2 , com_3)
[                                                                     [ identify comm_3
[ identify comm_1              ][ identify comm_2              ][ [ define comm(sender_1 / receiver_2)
[ [ define sender_1           ]][ [ define receiver_2         ]][ ( lambda (x, y)
[ ( lambda (x)                 )][ ( lambda (y)                 )][ ( lambda (comm)
[ ( ( if (send x)              )][ ( ( if (receiv y)            )][ ( ( if (put x
[ ( ( elect_2 : put (y) ) ) ]][ ( ( elect_1 : get (x) ) ) ]][         get y)
[ (elect_3)                    ]][ (elect_3)                    ]][ ( ( ( (comm_3 : sender_1 receiver_2) ) ) ) ]
[                                                                     [ (elect_1 elect_2)              ]]]]
```

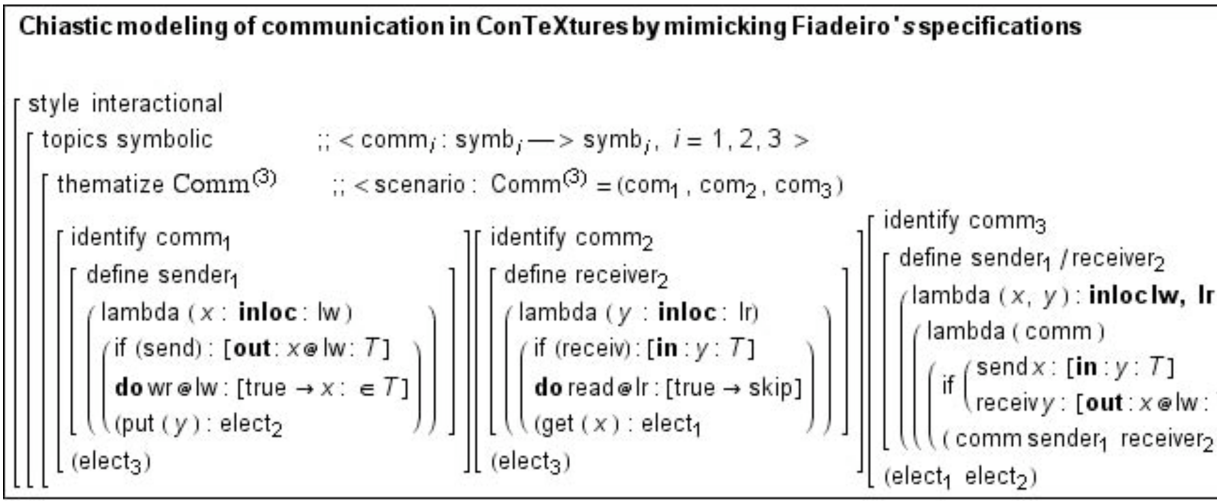
Matrix modeling in ConTeXtures of chiasm (send, receive, pos)

```
[                                                       ][ identify comm_2          □
[ identify comm_1                                       ][ [ define receiver_2.1   ][ define sender_2.2  ]
[ [ define sender_1.1   ][ define receiver_1.2     ]    ][ ( if (send x)           )( if (receiv y)      )
[ ( if (send x)         )( if (receiv y)           )    ][ ( elect_1.1 : put (y) ) ]( elect_1.2 : get (x) ) ]]
[ ( elect_2.2 : put (y) )]( elect_2.1 : get (x) )]      ]
```



Recursivity of communication

Communication, modeled as *a* chiasm is not only structural but also proceedural construction. contextural but distributed over different conxtures, hence trans – contextural.



2.4. Costs and resources

2.4.1. Conceptual analysis

There are many ways to measure costs, efficiency and resources of computational interaction and communication.

One quite direct way might be to show the parallel, or heterarchical structure of diamond modeling and implementation in contrast to hierarchical understanding of interaction and communication. With the presumption that “parallel” processing is more cost-efficient than non-parallel computing. And obviously, it has to be shown that polycontextural and diamond computation is at least conceptually “parallel".

Hence, using glue to organize hierarchically interaction, or even: the *‘social life’* of computing is, from the very beginning, more expensive and less efficient than heterarchical polycontextural and diamond computing.

Such considerations about efficiency and cost can be analyzed in detail for specific constellation.

Structual costs of deepness and broadness of a formula are not to be confused with the well-known complexity analyses based on compuptional time of big Omikron: O(n).

An analysis of the *system structures* of glued interaction and of chiastic and diamond interaction can be put into the simple results:
The kernel of *glue*-modeling consist of 3 unique instances (ports) at 1 locus (or nil), e.g. Comm = ($sender_0$, $glue_0$, $receiver_0$).
The kernel of *chiastic* modeling consist of 2 unique instances (roles) at 3 loci, e.g. $Comm^{(3)} = ($$sender_i$, $receiver_i$), i=1,2,3.
The kernel of *diamond* modeling consist of 2 unique instances (roles) at 4 loci, e.g. DiamComm $^{(4)} = ($$sender_i$, $receiver_i$), i=1,2,3,4.

The main difference between glued and mediated communication, i.e. between "Comm = ($sender_0$, $glue_0$, $receiver_0$)" and "$Comm^{(3)} = ($$sender_i$, $receiver_i$), i=1,2,3" and its mediation operation, is the fact that "$glue_0$" is a (coordinative) *instance* (object) for "($sender_0$, $receiver_0$)" while "*mediation*" is a (computational) *pocess* (action) between the positions pos_i, $i = 1, 2, 3$ of "$sender_i$, $receiver_i$".

Structural Costs
cost(X) = (n, m)
n: deepeness
m: broadness

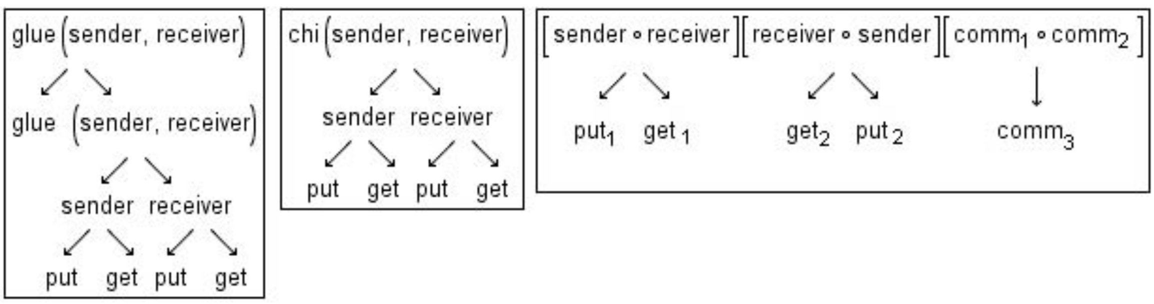
cost(glue) = (3, 1),
cost(chiasm) = (2, 3),
cost(diamond) = (2, 4)
In other terms, deepnes is representing computational complexity and broadness corresponds to the degree of polycontextural coordination and organization of disseminated computations.

2.4.2. Concept tree analysis

Concept and Kantarovic tree analyses for polycontextural constellations (formulas, etc.) can serve as a measure for structural complexity by the degree of *deepness* and *broadness* of the fundamental constructs of a situation. Broadness is the measure of the degree of dissemination, deepness is the conventional measure of the complexity of a formula. Because of their monocontextiurality, broadness is set to 1 for all classical formalizations.

Speed vs. directness
Speed of computation is crucial for closed non-interactional computational systems, i.e. for algorithmic computation. For interactional systems *directness* of reaction to change of the system/environment relationship are crucial. It is clueless to finish in high speed a calculation which has become obsolete in its premises because the situation has changed. The measure for interactional systems is directness of response and not speed of calculation. From the point-view of the calculation paradigm, higher directness appears as higher speed. But that's misleading. Directness means less steps to calculate and not higher speed.
Optimization for interactional computing systems means optimizing the architectural organization to support directness.
Speed for algorithmic systems is connected with computational time, i.e. with temporalization. Directness of interactional systems is connected with organizational space, i.e. with architectonics and topology. Organization (coordination) and computation are complementary as are directness and speed.

hierarchic **chiastic** **heterarchic modeling**



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The Category of Glue

Is there any glue to stop the decline of Western culture?

Rudolf Kaehr Dr.
ThinkArt Lab Glasgow

Abstract

A typology of different categories of glue (ordinary, super-, para-, proto-, trans-glue) are glued together with different strategies of gluing (set and category theory, combining logics, bi-category with (co)spans, polycontexturality and diamond theory). Interpretations of "interactional glue", "nerve glue", "logical glue" are sketched. Keywords of the dissemination of the concept of "glue" in history (Hegel, Marx, Lenin, Gunther, Derrida, Obama) and strategies (Glue, Opium, Mediation) of gluing them together under a general parapluie (ontology, society, solidarity, fear) are critically sketched.
The economical question is: *Can we still afford to glue interactions together?*
The category of glue isn't blue. Categories are clueless to interaction and are banking unsecured resources.

How good is Portuguese Glue?

The best quality of Portuguese Glue is accessible, for an affordable prize, at the Logic Shops for Combining Logics in Lisbon, Portugal.

Everything, that doesn't fit together by nature can be glued by categorical glue. Best selling products, at the time, are the *"(co)-span"* articles by José Luiz Fiadeiro.
Without doubt, José's glue, especially his *"interactional glue"*, is one of the most elaborated and purest form of glue on the market. Glue, today, is highly important. It always was. To feel save and gluish it is crucial to use only the finest glue available.

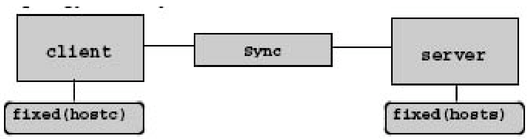
"We found out Portuguese glue is very good! LOL"
PlanetGeorge Forums
The Place George Michael Fans Call Home
http://planetgeorge.org/Forum/viewtopic.php?t=3552

Such a high quality has its own tradition of expertise.
Much was imported from the San Diego Zoo, California, USA. Other decisive work had been done by the scholars at place. They also had the opportunity to be guided by Brazilian specialists. As usual with success stories, there are hidden, well superseded sources, too.

Thus, the new product of combining and gluing is now available as the glue with the magic label "(co)span".
To span has a temporal aspect and *span* is has metric determination of an inter-space or gap.

> "In order to make interconnections independent of the nature of components involved, interaction protocols are formalized not in terms of morphisms (i.e. part-of relationships) but a generalized notion of (co-)span in which the arms are structured morphisms - the head (the glue of the protocol) and the hands (the interfaces of the protocol) belong to different categories, the category of glues being coordinated over that of the interfaces."
>
> "The "semantics" of the protocol is provided through a collection of sentences - what we call *interaction glue* - that establish how the interactions are coordinated. This may include routing events and transforming sent data to the

format expected by the receiver.



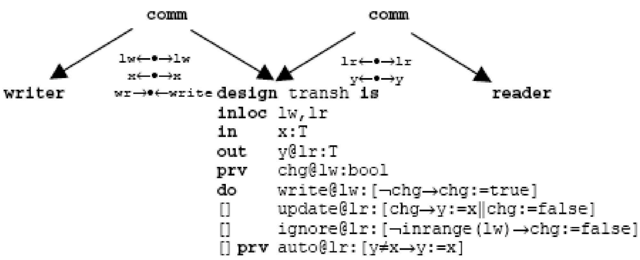
"In this way, it is specified that components *client* and *server* are bound statically to the network nodes identified by *hostc* and *hosts*, respectively. In what concerns the connector *Sync*, namely its *glue*, our design decision was to keep it location-transparent. This choice is justified by the fact that sync does not perform any *computation* but simply provides a pure *coordination* function just like an ideal, neutral “cable”. (ibid. p.10)

This form of transient sharing can be modelled through the binary coordination connector TranSh with roles writer and reader and the glue *transh*.

```
design writer  is                    design reader  is
inloc  lw                            inloc  lr
out  x@lw:  T                        in  y:  T
do  wr@lw:[true → x:∈T]              do  read@lr:[true →skip]
```

The roles define the behaviour required of the components to which the connector can be applied. For a writer, we require an action that models every kind of possible operation on x. For a reader, we require an action that models the access to the input variable y. This is because it is essential to know in which location this action is executed.

The glue ensures that updates to x are propagated to y whenever the reader and the writer are in contact with each other. Whenever the communication between the two components is possible, transh prevents the writer from writing x before the previous change of x has been propagated to y. In the other situations, lr is not in the range of lw and, hence, y remains with the value of x at disconnection time.
On re-connection, the value of x is sooner or later propagated to y. This is achieved through the execution of the action auto that is private to transh and, hence, subject to fairness requirements.” (p.12



On how Distribution and Mobility Interfere with Coordination
Antónia Lopes and José Luiz Fiadeiro
http://homepages.di.fc.ul.pt/~mal/papers/wadt02.pdf

Earlier on we had to do it with buffers. Buffers are definitively quite conservative. They are conserving messages, buffers them, to help to connect different processes or even one process only, connecting it with itself.

Buffers are not only conservative but passive too. Buffers, like glue, are not computational objects with own activity but computaionally inactive storage places.

Separation between computation and interaction
"This is why it is so important to put the notion of interaction at the centre of research in software-intensive system modelling, and to support methods and languages that *separate* interaction concerns from *computational* ones.” (p. 194)

"In the past, we developed a categorical framework supporting the *separation* between “*computation*” and “*coordination*” as architectural dimensions in software development [9].

Because we want the application of interaction protocols to be “agnostic” to the nature of the computations that are performed by the peers, we want that the protocol be based on the interfaces that components have available for

> interacting with each other, not on the computations that they perform locally. This suggests that the interactions should be established between objects of a category of interfaces, not between behaviours." (p. 195)
>
> "The extension is motivated by the fact that, whereas we want the interaction protocol to use a rich formalism to specify the coordination mechanisms superposed by the glue, its interfaces should be purely "syntactic" so as to avoid any assumption on the computations performed by the entities being interconnected." (p.207)

Like buffers, glues are important procrastinators. They stop the direct interaction between agents to secure message passing.

As in politics, everything has to be delayed, delegated to avoid collision of direct actions. Differences have to be overcome by respect and solidarity. Committees are organizing such 'generosity' of the ruling forces in power.

On the level of artificial interaction, say between computer systems, software supported services, etc., *direct* interaction has to be avoided. Computer scientists and administrators enjoy building walls and barriers between systems or agents who are considered to interact.
Such barriers are not only separating and slowing down communication, they are also actively or sometimes passively helping it to happen.
Buffers are one strategy, belonging to their world message passing.

Portuguese glue is another strategy. Much more modern, more general and better polished.

Nevertheless, the notion *glue* is not necessarily connected with the notions 'flexibility', 'dynamics', 'liveliness' of interaction and the autonomy of interacting systems.

So, what is the problem?

The question is: *Can we still afford to buy the glue?*

Glue is procrastinating, buffering, consuming time and resources.

Do we need glue? We surely need interaction.

> "This is why it is so important to put the notion of interaction at the centre of research in software-intensive system modelling, and to support methods and languages that separate interaction concerns from computational ones."

That's what the catalogs are telling us. And we agree. We have no chance to deny or reject the importance of *interaction* and *interactivity*. For social systems and for "soft-ware-intensive systems", too.

There are other, serious problems involved with the social glue strategy.
Not everything can be glued together. The parts to be glued need to be structurally similar to fit together. In a further metaphor, we cannot easily glue together water and steam or nerves and thoughts.

Also politicians want us all to glue together in the *"one world, one peace, one family"* eschatology. Others are gluing themselves together into the phantasm of *"one rationality, one reality, one formalism"*.

Hence, glue is not only resource-expensive but also leveling and eliminating differences. Glue is *homogenizing* heterogeneity.
This, easily, could be in conflict with the idea of social interaction.|
Hence, if, as in the glue paradigm, the interaction protocols of *role* A_P and *role* B_P are glued together with $glue_P$, then there is not much left for an interactional autonomy of A and B.

In fact, this scheme and strategy is what we are told since ever. E.g., communication theory or linguistics, semiotics, etc., for two agents to communicate, they have to share a common sign repertoire. Or, to avoid the danger of liveliness of multi-cultural life, you have to learn the official common language, i.e. the language of your political asylum.

I stopped to buy the story of the glue miracle.
There is simply no need to have glue in the head and between the fingers.

It’s good fun to have the head and the hands in different categories.
But why not jump?
Glue is a bad Ersatz for jumps. Satz, in German, also means jump. There is Ersatz-glue but no Ersatz-Satz.
Like the term “buffer”, “jump” sounds much too conventional. Worst case: jump from-to in Basic.
Hence, let such a trans-categorial/categorical jump be called “*saltition*" (sault, sauter, salto).

We have to learn to dance.
Therefore, the new service is not a product, like glue, but an *activity*. In fact, an *interactivity*, i.e a *strategy of interactionality*.

At first, lets learn to jump from the head of a category to the body of a saltatory. Use your hands! But there is no need for that. It might be adventures, but it isn’t dangerous.
For beginners, we could compromise to jump form one category to another category of a bi-category, then for advanced to a tri-category and more. This is safe. No abyss to overleap.
And, it’s not the size of the system, that counts but *“the number and intricacy on the interactions in which they will be involved”* into the game of *“social complexity"* (Fiadeiro).

The glue of social complexity

"The complexity involved in building the software components that will be deployed in such systems in not so much on the “size” of their code but on the number and intricacy on the interactions in which they will be involved, what in [6] we have called *social complexity*.

This is the motivation for studying the properties of structures of the form
<πA,G,πB>, which we call *structured co-spans*. More precisely, our aim is to define
and study the properties of a bicategory whose objects are signatures and whose 1-cells consist of interaction protocols."

Indeed, without the computational aspects of the *glue* it would not be possible to coordinate the interactions between n and m. That is, co-spans in SIGN are not expressive enough to formalize interaction protocols.

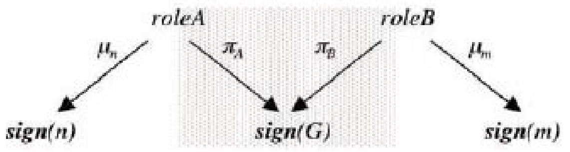
The basic difference is that it does not make sense to see software-intensive systems as being compositions, in an algebraic sense, of simpler components. There is not a notion of *whole* to which the *parts* contribute but, rather, a number of autonomous entities that interact with each other through external connectors.

Where we differ is in the idea that there is a *“system under consideration”*, conceived
a priori, that services crosscut. If we take one of the accepted meanings of ‘system’ - *a combination of related elements organised into a complex whole* - we can see why it is not directly applicable to SOC: services get combined at run time and redefine the way they are organised as they execute; no ‘whole’ is given *a priori* and services do not compute within a fixed configuration of a ‘universe’.

Whereas in CBD component selection is either performed at design time or programmed over a fixed universe of components, SOC provides a means of obtaining functionalities by orchestrating interactions among components that are procured at run time according to given (functional) types and service level constraints.(deduct, p.3)

According to [2], an architectural connector (type) can be defined by a set of roles and a *glue* specification. For instance, a typical client-server architecture can be captured by a connector type with two roles - client and server - which describe the expected behaviour of clients and servers, and a *glue* that describes how the activities of the roles are coordinated (e.g. asynchronous communication between the client and the server)." (Cat, p. 158)

A ***bicategory*** V consists of :
• A class | V | of objects (also called 0 – cells)
• For each pair < A, B > of *objects*, a *category* V (A, B) whose objects are called *arrows* (or 1 – cells) and whose morphisms are called 2 – *cells*
• For every triple < A, B, C > of objects, a *composition* law given by a (bi) functor
; A, B, C : V (A, B) × V (B, C) → V (A, C)
• For every object A an *identity* arrow $1_A : A \to A$.



Glue as a bureau d'exchange

Interaction, not composition

"There is not a notion of whole *to which the* parts *contribute but, rather, a number of autonomous entities that* interact *with each other through external* connectors*."*

How is are interactions connected?

"... a typical client-server architecture can be captured by a connector type with two roles - client and server - which describe the expected behaviour of clients and servers, and a glue *that describes how the activities of the roles are coordinated ..."*

interaction = {entities, connectors, glue}
What is interacting? Autonomous entities!
How are activities coordinates? Glue!
Which is the expected behavior? Connector!

Interaction is understood as a composition of actions hold together by glue. Interaction, hence, is not a basic term as it should be after the proclaimed intention, but action. And inter-action is a derivative concept build by the composition of actions.
This is in correspondance with nearly all approaches to interaction (Mario Bunge, Goguen, Kohout).

David Hestenes writes in the tradition of Mario Bunge:
"The properties of things are of two general types: *intrinsic* and *interactive*. Intrinsic properties belong to the thing by itself, while interactive properties are shared with other things.
The descriptors of interactive properties are called interaction variables or just
interactions. A thing that acts on another thing is called the *agent* of the action. Two things that act on one another are said to *interact*. Thus, *interactions* (Also called *connections*, *links*, *bonds*, or *couplings*) are mutual (or shared) properties of things. Interactions influence (change or constrain) the *object variables* of a thing according to natural laws."
David Hestenes, MODELING is the name of the game
http://modeling.asu.edu/R&E/ModelingIsTheName_DH93.pdf

Gluing things together

History of glue and gluing

Glue is universal. And the *gluons* are holding this universe together.
(Google offers 25,400,000 entries for glue.)

Glutination
Glue is a universal substance or even the substance of the universe. The activity connected to glue is gluing. Glutination is the category of gluing. Glutination is "The act of uniting with glue; sticking together."
http://www.thefreedictionary.com/Glutination

Agglutination
Agglutination is the clumping of particles. The word agglutination comes from the Latin agglutinare, meaning "to glue to."

Glue is very closely connected with the history of mankind. Glue glued not only groups together but glued things together to build tools to enable groups to glue together.

Glue is produced from nature: plants, animals, from human beings or synthetically.
Glue is a metaphor, a concept, a name, a programming language.
There are more than 26 million Google entries for Glue.

ultimate glue
"Why is sex the *ultimate glue*? Why is it so important in a romantic relationship? In a nutshell, sex is glue because it is the one that makes your romantic relationship unique from all the other relationships in your life."
http://drseth.blogspot.com/2008/10/why-sex-is-glue.html

Conceptual Glue (E. Margolis 1999)

glue - Collaborative International Dictionary of English v.0.48 :
Glue \Glue\ (gl[=u]), n. [F. glu, L. glus, akin to gluten, from
gluere to draw together. Cf. Gluten.]
A hard brittle brownish gelatin, obtained by boiling to a
jelly the skins, hoofs, etc., of animals. When gently heated
with water, it becomes viscid and tenaceous, and is used as a
cement for uniting substances. The name is also given to
other adhesive or viscous substances.
[1913 Webster]

Glue \Glue\, v. t. [imp. & p. p. Glued; p. pr. & vb. n.
Gluing.] [F. gluer. See Glue, n.]
To join with glue or a viscous substance; to cause to stick
or hold fast, as if with glue; to fix or fasten.
[1913 Webster]

"This cold, congealed blood
That glues my lips, and will not let me speak." - Shakespeare.
[1913 Webster]
http://onlinedictionary.datasegment.com/word/glue

Next to the material history of glue and the techniques of gluing there is also a short story of *conceptual* glue and gluing.

Hegel
"The 'glue' that binds the world together is, in Hegel's view of the matter, not the eternal falling apart of objects, but simply their necessary interconnectedness; if you attempt to separate them, they will not stay put. Nor is it that negation which disintegrates the universe that Hegel uses as the '*mortar*' to combine it; it is that

negation which, because it is as much positive as negative, does actually combine it. After all, it would appear that one is forced to admit that Hegel is more than a superficial thinker trying to palm off on a long-suffering public palpable absurdities."
http://www.gwfhegel.org/Books/TR3.html

Marx
"Religion is the sigh of the oppressed creature, the heart of a heartless world, and the soul of soulless conditions. It is the *opium* of the people."

Religion: The Glue That Binds Society Together".

Lenin
"Die Religion ist das Opium für das Volk. Die Religion ist eine Art geistigen Fusels, in dem die Sklaven des Kapitals ihr Menschenantlitz, ihren Anspruch auf ein auch nur halbwegs menschenwürdiges Dasein ersäufen."
http://www.vulture-bookz.de/marx/archive/quellen/Lenin~Opium_fuer_das_Volk.html

Sniffin' Glue: the Essential Punk Accessory
"The Baiti association says 98% of children living on the streets in Morocco are now addicted to *sniffing glue* and the number is growing."
http://news.bbc.co.uk/2/hi/africa/4113441.stm

Money
"Money as the medium of exchange is the glue of society, for society is sociated by human action, by human practice in living with one another. Monetary value is abstract usefulness which is understood by human understanding within the practice of trading, i.e. commodity exchange, and thus 'holds everything together'."
http://192.220.96.165/untpltcl.html

Heidegger
Heideggerian continental philosophy and naturalistic cognitive science need not be mutually exclusive and shows further that a Heideggerian framework can act as the *"conceptual glue"* for new work in cognitive science.
In _Reconstructing the Cognitive World_, Michael Wheeler
http://www.citeulike.org/user/TomQ/article/3444021

Gunther's Hide Glue
I haven't found any glue in Gunther's work. In a dialectical and cybernetical turn, Gunther calls what others would call glue "mediation" (Vermittlung). His *Theorie der Vermittlung* is realizing a tabular connection of complexity and complication of logical systems (place-valued and context-valued logics) with over-/under- and balanced constellations. The complexity of holding together is hidden by the *Hide Glue* and well fibred by *Fibrin Glue.*

Heinz von Foerster
"The question of applications in the social sphere was a problem to which I was attracted quite early on. I and my friends always regarded the social problem as having to do with the possibility of a linguistic connection. We saw language as the *glue* that forms a society. [...] Language makes second-order communication possible.

Interview with HvF, 26 November 1999.
http://bcl.ece.uiuc.edu/mueller/index.htm#fn45

Derrida:
Harold G. Coward, Toby Foshay, Jaques Derrida:Derrida and Negative Theology, 1992, § The Deconstruction of Buddhism
"It is because we see the world as a collection of discrete things that we superimpose causal relationships, to "*glue*" things together." David Loy, p. 247

Glue, a challenge
"*Tabbi*: Could you deconstruct Glue for me?
Ulmer: One of the first things that fascinated me about Derrida was the theory of the signature. The relation of the proper name to individual historical experience was an obvious place to test some of the poststructural claims about the place of chance, (non)motivation of language, and the like. The experiments that led to mystory and choragraphy began with the exploration of my own signature. Notions of fate have given way to the constructed subject; but still the proper name provides an anchor, a ground upon which identity may constellate.

"In Glas Derrida devotes considerable attention to the phenomenon of agglutination, and speaks of the gl and the glu.

"This theory resonated with the phrase used by peers to tease me in grade school (generating Elmer's Glue from Ulmer). My interest in arts using collage equaled my interest in theory: the art of collage has been defined as the art of gluing. One negative review described Applied Grammatology as sticking to Derrida like glue.

My initials are G.L.U. + the French silent e. The properties of glue are suggestive of my concern with group formation, with a certain kind of community creation. I have not done a full examination of the vehicle. I use glue online, usually in a MOO setting, but also in e-mail. Students often remind me about how glue is made. I have not thought about the implications of that yet."

Glas, 195-196b
by Jacques Derrida
"I forgot. The first verse I published: 'glu de l'étang lait de ma mort noyée' ('glue of the pool milk of my drowned death')."

And for poeple like us:
"Fear is the glue that binds these structures."

Moreover, since philosophy is the glue, the deep structure, that holds so many things together, its critique has ramifications that open up numerous areas of struggle. p.195

Web 2.0: Obama
Obama: "I'm rubber. You're Glue. Whatever you say...."
Rubber and Glue
Super Glue

Glue2.0
"You are summing up exactly the value of Glue - semantics is on the background doing its magic, but the really important thing is new way for people to connect - in the context, without friction."
http://www.zachbeauvais.com/archives/glue-sticks-stuff-together/
http://getglue.com/

More glue
glue - Free On-line Dictionary of Computing (26 May 2007) :
glue
<jargon> A generic term for any interface logic or protocol
that connects two component blocks.
For example, Blue Glu is IBM's SNA protocol, and hardware design
ers call any thing used to connect large VLSI's or circuit blocks
"glue logic" b(1999-02-22)
http://onlinedictionary.datasegment.com/word/glue

Like glue, too.
http://uk.youtube.com/watch?v=iLQ__fc7_jU

Semiotic glue
"... he plays well but, man!, with the WRONG technique
the piano is the most stationary of instruments, if one excepts the church organ
the chords played are just plain UGLY
the ladies morning musical club would be shocked
you need a kind of *semiotic glue*
i was lamenting about the fact that maybe the surface was TOO dissonant call it fauré, call it sorabji..."
http://pages.infinit.net/kore/contrepoint.html

A glue language is a programming language (usually a scripting language) used for connecting software components together.

Glue semantics. Glue logic.

Nerve glue.

Tautological glue
"The cultural *glue* that holds America together, **Bertrand Russell** said, is Americanism."

Cultural Glue
"Digital codes work as cultural glue through space and time."
http://www.nbi.dk/~emmeche/coPubl/91.JHCE/codedual.html

Glue or Cement?
"Normally, an organization consists of an architecture being the *cement*, or the *glue* between many agents. The levels of complexity of architectures and agents define the complexity level of the organization. Agent sorts can be discerned regarding the presence or absence of the following components: perception, interaction (including learning in the sense of habit formation), representation (including learning in the sense of chunking) and autonomy."
http://www.rug.nl/staff/h.w.m.gazendam/semiotics,_multi-agent_systems_and_organizations.pdf

From glue to gluons
"These forces, which "glue" the quarks together in "white" bundles, are mediated by field-quanta that are called *gluons*, which like photons are massless spin-1-particles. As a force between two quarks act between 3-3 colour-combinations, one should think there would be 9 different gluons, but it turns out that the photon is hiding among these combinations, so there are only 8 gluons."
http://www.library.utoronto.ca/see/SEED/Vol3-2/Christiansen_3-2.htm

Or simply: GLUE

Glue from animals, from human beings or synthetically. But there is also mental glue, conceptual, structural and algorithmic glue.

The glue of language. The language of glue. Language as glue and glue as language. The language GLUE.

And finally:
Glue is a novel by Scottish writer Irvine Welsh.
"Glue tells the stories of four Scottish boys over four decades, through the use of different perspectives and different voices. Glue addresses sex, drugs, violence, and other social issues in Scotland, mapping "the furious energies of working-class masculinity in the late 20th century, using a compulsive mixture of Lothians dialect, libertarian socialist theory, and an irresistible black humour." The title refers not to the abuse of adhesives, but the *metaphorical glue* holding the four together through changing times."

It's a never ending story. At least: There is no mankind without glue.
Scots are spelling glue as "glü" with a long "ü", hence "glüh". Like in German: "Glühbirne". There is without doubt a lot of linguistic glu(e) between glüh and glue. One is called Scottish Enlightenment.

The real story is here:
http://www.taz.de/1/zukunft/umwelt/artikel/1/verehrt-verraten-und-verglueht/

Glue as Leim
Nevertheless, the whole story of Glue would get a different turn if it would be told in German language, with the help of the key words: Leim, Schleim, Heim, Keim. Verbindung, Binder, Blinder and Kleber.

A strong linguistic neighbor of glue is "clue". The glue of cryptography.

"Glue everywhere. As Bertrand Russell cleverly put it in one of his treatises, glue is a very sticky business. "
http://judgemental.merseyblogs.co.uk/2007/08/someones_been_court_out.html

- **Typology of glue**

Glue is gluing. It glues things or persons or thoughts or whatever together. There are some degrees of distinctness in the process of gluing to observe.

Parts to be glued might belong to the same category of things. They might be of the same ontological species.

Such a species might be concrete, like classical objects, or it might be abstract, like classical concepts or models. Such an approach is well modeled by set theoretical concepts.

If the focus is more on the inter-relation between things and concepts and not as much on their content, category theoretical concepts and methods are well applied.

Both approaches, set and category theory, are belonging to mono-contextural thematizations. Both are based on simple dichotomies: element/set and object/morphism both belonging to one and only one universe or "conglomeration", hence mono-contextural.

If things are slightly more different and not automatically commensurable, bi-categories are at place. They still are at home in one universe. And their gluing power is restricted to glue together the two categories of bi-categories. Or later, for more complex constellations, n categories of n-categories. Still, under the umbrella of a unifying conglomeration. Hence, in fact, super-glued by the unique conglomeration.

Set theory and *category theory* are working with ordinary glue, called first-order glue. It would be unfair and misleading to call it super-market glue.

Because things, interactivity, are highly dynamic, different levels of consistency and coherency of the state of glue have to be considered:
hard vs. soft,
stable vs. elastic,
uniform vs. gasiform,
constant vs. scaling,
etc.

Interactivity is an ever-changing event, glue has to be able to adapt to the changing circumstances of gluing actions together.

The super-glue of n-categories is gluing together the gluing power of 1-categories.
For both, the rules of the gluing power are simple: No glue is a glued, and no glued is a glue. TND.

Advise how to use superglue
http://www.metacafe.com/watch/46663/super_glue

Polycontextural theories are more split and poly-phrenic. Awful things happens. Not only there is differentness between conglomerations to be glued, there is also no glue that could glue together such conglomerations, that are, as it was mentioned, the umbrellas for n-categories and their own gluing strategies. Much worse, what is gluing conglomerations together might turn out to be itself a conglomeration and conglomerations might play the role of glue. Such a non-glue glue isn't a superglue nor a crazy glue.
But a proto-glue, the glue before the glue, in general. What's before the glue is the abyss. And *proto*-glue is the jumping device to jump in-between the gaps of differentness, hence its real market-label is *trans*-glue, the glue beyond/between the glue.

Proto-glue
"Like Ozu's later *An Inn in Tokyo*, this one is at its best when it proceeds to indulge in moments of a proto-glue sniffing aesthetic, which is essentially my own term for gritty and surreal (think Herzog) moments of humor."
http://cinematalk.wordpress.com/2008/07/21/

In the case of diamond theory, not only the well known types of glue, like glue-glue, meta-glue, super-glue, proto- and trans-glue, are gluing glues together but, at once, antidromically, to each gluing, of whatever type, there is also an un-gluing gluing gluing.

First-Order Glue

"Glue has evolved significantly during the past decade."
"Glue (Dalrymple, 1999), a compositional semantics framework based on linear logic (Girard, 1987) has evolved over the years."
Miltiadis Kokkonidis, Glue as the Syntax-Semantics interface
http://users.ox.ac.uk/~lina1301/Kokkonidis06c_icttl_simplefog.pdf

Meta-level gluing

1st-order gluing:
Gluing of elements, sets and also gluing of categories shall be called 1-order gluing, i.e ordinary or real gluing.
2nd-order gluing:
N-categorical gluing shall be called complex gluing, i.e. gluing complexions of different categories, hence 2-order gluing.
3rd-order gluing:
Polycontextural gluing shall be called second-order gluing, i.e. trans-gluing, hence 3-order gluing. The French may call this stuff trance-glu(e).
4th-order gluing:
Diamond gluing shall be called splitting-gluing, i.e splitting the glue of gluing the glue, hence 4-order gluing.

Para-glue is the parapluie of all umbrellas without being itself an umbrella. It guarantees a state of gluey gluishness.

Theories of conceptual glue

Set theory

Set theory is working with ordinary glue. One big universe of sets is enough to glue all the sets together. Who, who wants more gets punished by antinomies. Others will have to climb the ladders of Bertrand Russell's escape strategy. In ordinary mathematical life ordinary glue is enough, it is doing its job of gluing things together properly.

Claude Shannon's glue
"Irish glue has the reputation with some persons."

With the mathematical theory of communication, the communication model needs a common sign set as a cut of the sign repertoires of two communicants, sender and receiver or source and target. Such a cut is representing the common ground of communication. It functions as the necessary communicational pool of pre-giveness without it no communication can be realized.

Mostly, this obvious triviality isn't mentioned at all. Communicants are supposed to communicate directly and successful against the disturbance of the channel by noise.

As an example of an explicit semiotic model of communication, which is considering a common pool of knowledge the following diagram may visualize the role of the pool. In fact, it is a set theoretic *intersection* between the pool of the source and the pool of the target.

Hence, without intersection (overlapping), no communication. Intersections are easily done, without any trouble, with the help of Cantor's glue.

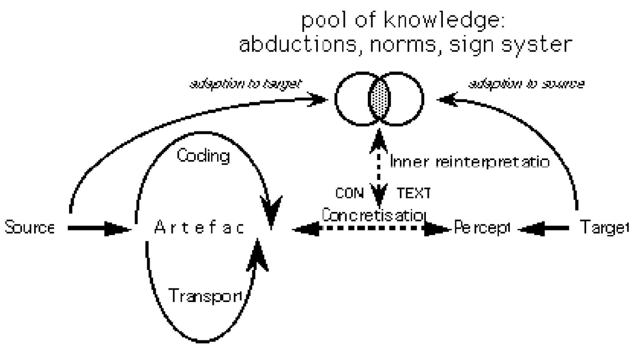


Fig. 5. General model of communication
"As the communication model is presented here (cf. Fig.5), it also incorporates modifications which do not stem from the Prague school. According to an idea, suggested both by Moles and Lotman, the

sender and receiver of any situation of communication start out with "codes" — or, as I would prefer to say, systems of interpretation —, which only partially *overlap*, struggling to homogenise the system of interpretation as the communication proceeds (cf. Sonesson 1995; 1997c).

The communicative act is said to be *sender-oriented*, to the extent that it is considered to be the task of the receiver to recover that part of the system of interpretation, which is not shared between the participants.

It will be *receiver-oriented*, to the extent that the task of recovering knowledge not held in common is assigned to the sender.
When sender and receiver fail to *negotiate* the parts of the interpretation system which they do not both possess, the resulting concretisation will be a *deformation*."

Göran Sonesson, The Limits of Nature and Culture in Cultural Semiotics
http://filserver.arthist.lu.se/kultsem/sonesson/CultSem2.html

■ Strategies to avoid glue: Hidden Glue

Strategies to avoid glue are well known.
One of them is best summarized for computing models and programming languages by the strategy *"EverythingIsa: Everything is a EverythingIsa."*
This method of gluing things together without getting wet and glued or agglutinated is applied for cosmological, social and biological theories too. It is the strategie of avoiding glue with the help of a hidden ultra-super glue, sometimes properly called *Deus Absconditus*.
The Class of all classes.
The Category of all categories.
The Module of all modules.
The Macro of all macros.
The ETC of all etc.

The other strategy, not yet well known in scientific circles and programming labs is : *Barr the barr*. Or equivalent: *Don' t barr the Barr*.
Albeit an ancient strategy, it is used and becoming fashion only recently.
The main Barr barrer is the thinker Martin Heidegger with his barred non-term-terms Sein and Ereignis from his writings≱.

"Insofern kann ‚Ereignis', bezogen auf
diese Systeme, nur *gebarrt* geschrieben werden: ~~Ereignis.~~ " (Peter Fuchs)

(FrameMaker offers the function "Strikethrough")

EverythingIsa: Everything is a EverythingIsa.
http://c2.com/cgi/wiki?EverythingIsa

Categories and Contextures
www.thinkartlab.com/pkl/lola/Categories-Contextures.pdf

Peter Fuchs: Ereignis, Welt und Weltereignis
http://www.fen.ch/texte/gast_fuchs_weltereignis.pdf

■ Category theory

Category theory, which doesn’t want to be involved with the internal’s of sets, is interested more into the inter-relationships between so called objects. Such inter-relating morphisms are building a *society of objects*. What is the glue of this society of objects?The glue of this society is not Opium but *coincidence*. Coincidence relations as *matching* condition are gluing morphism together. Without such glue there is no commutativity for the composition of morphisms. Compositionality as such remains an open question.

Category theory as the general glue of mathematical studies.

In other words, the conditions of the composition of morphisms, i.e. the coincidence between codomain (target) and domain (source), or the matching conditions of mappings for the ’object-free” category are not themselves defined by categorical notions.

This sounds trivial, because the matching conditions are defined in a logical meta-language. But the interactivity between the categorical object-language and the defining meta- or proto-language isn’t clear.

> *"For him [Jean-Yves Girard], category theory characterises objects in terms of their “social lives"".*
> José Luiz Fiadeiro, Categories for Software Engineering, p. 2

Combining: splitting/slicing
Where there is no glue there are bridges.
Combining logics

"By ‘*bridge principles’* we mean, in a wide sense, any interactions (i.e., derivations) among distinct logic operators which are not instances of valid derivations in the individual logics being combined.

"Therefore bridge principles are the result of reciprocal action or influence of the collective logics being combined, and not merely derived rules or theorems.”
http://www.cle.unicamp.br/cle30-ebl-slalm/TutorialEBL01.pdf

Glue logic
"Glue logic is a theory of ‘semantic assembly’, that is, the way in which information about the meaning that is provided by lexical items and grammatical constructions is put together to get a meaning for the whole utterance.”
http://www.als.asn.au/proceedings/als2003/andrews.pdf

Adhesive categories
Stephen Lack and Pawel Sobocinski: We introduce adhesive categories, which are categories with structure ensuring that pushouts along monomorphisms are well-behaved. Many types of graphical structures used in computer science are shown to be examples of adhesive categories. Double-pushout graph rewriting generalises well to rewriting on arbitrary adhesive categories.

"This provides further evidence of how pushouts behave in adhesive categories as well as making more precise the intuition that the pushout operation “glues together” two structures along a common substructure. As a corollary, it follows that in an adhesive category the lattices of subobjects are distributive.

"Definition 5.2 (**Gluing Conditions**). Given a production p as in (1), a *match* in C is a morphism $f : L \longrightarrow C$. A match f satisfies the *gluing conditions* with respect to p precisely when there exists an object E and morphisms $g : K \longrightarrow E$ and $v : E \longrightarrow C$ such that

$$\begin{array}{ccc} L & \overset{l}{\longleftarrow} & K \\ f\downarrow & & \downarrow g \\ C & \underset{v}{\longleftarrow} & E \end{array}$$

is a pushout diagram."(p. 15/16)
http://www.maths.usyd.edu.au/u/stevel/papers/adhesive.html
http://www.brics.dk/RS/03/31/BRICS-RS-03-31.pdf

Bi-category theory

From glu(e) to (co)span.
A span, even a co-span isn't yet a salto.

> Typically, in a category of systems, morphisms capture a "component-of" or "sub-system" relationship. As already motivated, in software intensive systems it does not make sense to talk about "component-of" relationships in an algebraic way." (CALCO'07, p.195)

Glue is a crucial term in the work of José Luiz Fiadeiro.
Also the term "glue" isn't honored in the index of his *"Categories and Communities"* eBook. The term "glue" nevertheless occurs 36 times at strategic positions.

IGLU, as a white box of gluish mediation

> *"The other structure that is important for interaction protocols is that of the glues; we assume that glues can themselves be organised in a category **IGLU** and that a functor **sign:IGLU->SIGN** returns, for every glue, the structure of interactions (signature) that are being coordinated by the protocol. As a consequence, a morphism σ:G1->G2 of glues captures the way G1 is a sub-protocol of G2, again up to a possible renaming of the interactions and corresponding parameters. That is, σ identifies the glue that, within G2, captures the way G1 coordinates the interactions sign(G1) as a part of sign(G2). In fact, because we need to be able to compose interaction protocols, we assume that **IGLU** is also a finitely co-complete category.* (J.L. Fiadeiro, Schmitt, p.200)
>
> *"That is, sources of morphisms in diagrams in **IGLU** are, essentially, signatures, which is why we decided to work with structured morphisms in interaction protocols. (ibd. 201)*

Nevertheless, things are highly intriguing:

> "More precisely, given a coordinated category **sign:IGLU→SIGN**, using **cospan(SIGN)** for interconnections is too poor because it does not support the definition of coordination mechanisms, and using **co-span(IGLU)** is too strong because the interfaces involve computational aspects. This is why we proposed to work over an algebraic structure co-span(sign) that is based instead on sign-structured morphisms." (p.14, CALCO'07)

There is a big and a small iglu included: **iglu:SIGN→IGLU.**

■ Polycontextural logic

Ferdinand de Saussure: glue/clue

Mediation of different contextures is glue-less. Glue is clueless to mediation. Glue, as we know it from category theory is not a mechanism, it is a non-mechanism of suggestiveness. It suggests solutions where there are nothing more than desires. Glue turns out to be a universal blank, a fashionable legerdemain of domains and codomains.

Gotthard Gunther tried in his early philosophical attempts to get rid of the hallucinogenic, glutinous, adhesive sizziness of the ultimate clamminess of social-ontological considerations.

First, in the early 30s he discovered the conceptual mechanism of mediation in Hegel's Logic. Then he tried to glue together his discoveries with the then arising mathematical logic, especially the early work of Alfred Tarski.

After his emigration to the USA, he tried to work out the logical mechanism of mediation. First, as a multi-valued *place-value system* of logic culminating in his general logical theory of mediation (*Vermittlungstheorie*) of different trans-classic types of logics. Then as a morphogrammatically based quindecimal system of mediation.

There was still a lot of glue necessary to let the mechanism of mediation run. But because of its imminent processuality, mediation isn't to fix by any glue.

Only a brand new procedure of evaporating such mediating glue led to a more clamminess-free running of the mechanism. The sacrifice was enormous and radical: he had to eliminate any kind of conceptual and apparative lubricant of onto-logical heritage. This glue-free mechanism, called *morphogrammatics*, enabled a kind of a first run of clean and pure mediation of logical systems as a basic framework for cybernetic, cognitive and volitive, conceptual designs.

Where there are no objects and no inter-relating morphisms in the play there is also nothing which could be glued.

Such morphogrammatic mechanisms are based on the inscriptions of emptiness, called *kenograms*. The situation established is not specifically gluish, there is not much academic gossip possible about and of the ultimate but structured emptiness of the void; but the mechanism is working in its dry silence.

It may still be an open question if such sacrifices are strictly necessary to get rid of the self-fumigation of current glue-strategies in computer science not to mention the bulk of social theories in sociology and informatics.

■ Diamond theory

Agglutination, inversion, chiasm: “gl” and “lg”.

Complementarity of categories and saltatories is interplaying in a glue-free game of jumps. Categories might be glued. Saltatories are not gluing their gaps. Complementarity between categories and saltatories happens in a glue-free interplay of bridging salti.

In other words, how can we glue things together without getting hassled by the clamminess of our glue and still being able to enjoy the gluishness of its intoxication?

The answer to this paradox is given by the jump-operation of saltisitions.

Saltisitions and hetero-morphisms are characterized by antidromic orientations. Hence, it would be natural to think of them as products of inversion, i.e. as inverted morphisms. But that’s not a solution. The inversion of “glue” is “ugly”, and there is no doubt that glue is fundamentally ugly and a categorial member of ugliness.

A combination of the ag- “gl” and de-glutional “lg” to “gl-lg” is discovering a tiny *chiasm* in the very concept of the ugliness of agglutination (GLAS, Derrida). This phenomenon probably was the very reason that let to the misleading hope that the mechanism and strategy of *inversion* and *dislocation* of (semiotic) gluton will help to avoid the crash of the evaporating glue of togetherness.

Saltisitions are inscribing the conditions of the possibility of categorical compositions. Compositions in category theory are glued together by the matching conditions. Their clamminess might be avoided by a jump from category to diamond theory.

First,
the categorical gluing operation is mirrored in its complementary hetero-morphism. Hetero-morphisms are reflecting, complementarily, the compositions of categories by keeping their concept of compositionality while avoiding their clamminess.

Second,
by the complementarity of categories and saltatories the clamminess of the matching conditions for compositions gets its complementary counter-part in saltatories as the glue-less jumping operation between hetero-morphisms. Saltitions are freed of any resemblance to glueness. The glue has evaporated into the joy of jumps.

Third,
Diamond theory is thematizing the activity of the composition operator not as a morphogram but as a complementarity to the operator, implemented as a hetero-morphism.
Diamonds are thematizing the basic operation of category theory as such: the operation of composition. The thematization is modeled into the hetero-morphisms.
In a general setting of graphematic analysis of composition, the morphogrammatics of the operator "composition" has to be taken into account, too. That is, the neither-nor gesture of categorical object and morphism has a double face: *hetero-morphism* and *morphogram* of composition.
http://www.thinkartlab.com/CCR/2007/07/complementary-blog-diamond-strategies.html

Forth,
Composition-free interplay.
In a kenomic play (Fink, Derrida) there is no composition, neither any “gl” nor “lg” mispend.

Diamond theory of interactivity

This part of the Short Study *“Category of Glue”* will be glued together as soon as I have found the “gl"/"lg"-free glue of ugliness and shall be published as part II of “Category of Glue".

Luhmann's secret diamonds

New entries for the Zettelkasten

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Abstract

A kind of a similarity between Luhmann's concepts of sign, system, difference and re-entry and the main figures of diamond theory is observed.

1. Where are Luhmann's diamonds?

1.1. Citations

"When a communication constitutes a previous communication as a communication, it simultaneously distinguishes it from all those other things in the world that are not communication. In this sense, all operations of autopoietic systems always constitute the difference between the system and its environment.

"Distinctions, however, are observations that constitute a difference between two sides and thereby relate these sides to each other. Observations, which are thus the application of distinctions, `open' the system for conditions of the environment, but as internal operations they `close' the system by distinguishing it from its environment (1997a page 92 ^ 108).
An observation relates and differentiates: it is a unity of difference.

"Its main cornerstones are a conception of space *as the possibility of drawing distinctions, and an analytical focus on accessibility systems and organisations."*

Martin Gren, Wolfgang Zierhofer: The unity of difference: a critical appraisal of Niklas Luhmann'stheory of social systems in the context of corporeality and spatiality
http://www.envplan.com/epa/fulltext/a35/a35280.pdf

http://www.cjsonline.ca/pdf/luhmann.pdf

Self-referentiality of distinction
Social systems are constituted as ``self-referential objects. We can observe and describe these as systems only if we accept that they refer to themselves in every operation‘ (Luhmann, 1995, page 437).

Components of distinction: indication and distinction
"However, there is a fourth point that will occupy us at least for a short while. I have already alluded to it. Spencer Brown's theory design contains a well-hidden paradox. It is constituted by re-entry itself or--if we refer to the beginning of the calculus, the first injunction 'Draw a distinction!'*--by the fact that the distinction must be and is drawn merely in order to distinguish one side. Thus, every distinction contains two components:* indication *and* distinction. *The distinction contains itself, but apparently in a very specific form: namely as the distinction between distinction and indication, and not merely some juxtaposition such as of large and small, or anything else that could be conceived of as a distinction."* Luhmann, p.19

Consecutivity as a reason for time and memory
"When a system constitutes itself, it draws a difference between system and environment by carrying out two subsequent operations: distinction *and* indication *(Rasch and Wolfe, 2000, page 36).*
First it distinguishes two sides and then it indicates one of these sides as the system (or the observer). As these operations are consecutive *they constitute or `consume' time. All observations are thus temporal: one cannot be on both sides of a distinction at the same time. Introducing a difference in time is one of the operations that autopoietic systems use in order*

to `unfold' (or solve) the paradox of reentering distinctions (or the self-application of a code)." (Martin Gren, Wolfgang Zierhofer, ibd)

``To cope with these consequences of a re-entry of the internal/external difference in itself, the system needs and constructs time. It needs a memory function to discriminate forgetting and remembering. Its past is given a highly selected present and, in this sense, a reality" (Luhmann, in Rasch and Wolfe, 2000, page 37).

Binary codes
"From the point of view of the system, its binary code is universally valid because it may be applied to all its objects: for example, to all statements (science) or to all actions (law). Binary coding excludes third possibilities, and, as soon as the code is applied to itself, either tautologies (`true is true') or paradoxes (`true is not true') are produced. Therefore, the binary code must not regulate its own application. This is instead the task of programs (Luhmann, 1989, pages 37, 39 ^ 40, 45)." p.5

1.2. Interpretation

It seems to be more fruitful today to thematize and formalize Luhmann's distinctions with the help of *diamond* theory instead of the *Calculus of Indication* of George Spencer Brown.

A key notion in Niklas Zettelkasten, obviously, is *self-reference*.
The other crucial notion is the self-referential concept of *difference*.

With that all kind of connections to logical, methodological and epistemological considerations are provoked. A strange connection to Spencer-Brown was inaugurated, mainly by the influence of Heinz von Foerster. The re-entry figure became a machina creativa, albeit nobody had a training in formal languages at all.

Difference and relation; *différance*
But Luhmann's work is about social theories and not about logic. Neither is Luhmann's theory of social systems a semiotic or semiological theory. This point ios not yet well understood. Semiotics, but the french "sémiologie" too, are based on *relations*, triadic for semiotics and dyadic for semiology. But Luhmann's concept of a self-referential and "therefore", paradoxical concept of difference isn't based on relations but on difference (Unterscheidung). Relations are presupposing difference, and are thus secondary to the paradox concept of difference. Relations are logical and not paradoxical.

Derrida has given strong deconstruction of the semiological and semiotic sign concept and its relational foundations in logocentrism. With his radicalized interpretation of de Saussure's semiology, he transformed the concept of difference to the paradoxical non-concept of *différance*. The difference of the difference, the *différance*, is not in a relationship to relations.

Similar, Gotthard Gunther's non-concept of *proemial relationship*.

Hence, Luhmann's insistence on self-reference might well be reformulated in different ways. One, which I proposed for many years, is interpreting self-reference and its circularity in the framework of a polycontextural understanding of *chiasms*, i.e., technically, as proemial relationships.

Now, after this chiastic theory got some maturity, albeit not much recognition, it is time to introduce the *diamond* approach to difference and circularity of system and environment . Diamond strategies are a further radicalization of the earlier approach of polycontextural chiasm.

Also Luhmann's work is not well known in the Anglo-Saxon world, it isn't a wrong feeling to observe that also the themes and topics, and their highly reflected treatment by Luhmann, has no real existence in the world-leading sociological literature of the super-power theoreticians.

2. Supplementing the Zettelkasten

It doesn't seem too risky to risk an interpretation of Luhmann's theoretizations out-side or beyond second-order cybernetic figures and metaphors.
In other words, is there a strict necessity to understand Luhmann's adventure in terms of his entries of his own Zettelkasten?
Is it possible to 're-construct' his constructivism and re-enter into it without its terminology and jargon of difference, distinctions, re-entry and self-referentiality?
Luhmann's theory is self-referential, thus it could refer to itself in different terminological modi, and still

keeping its adventures strategies and networks of constructing a de/constructive theory of social systems alive.

Hence, I will take the risk to supplement the Zettelkasten by smuggeling some non-contents of diamond boxes into this, now electronic, Zettelkasten.

By re-reading the passage with its introduction of the difference of *system* and *environment*, I think that I'm observing, or as I prefere to say, hallucinating some features not yet been recognized and mentioned, neither explicitly by Luhmann nor by his followers.

Self-referentiality without referentiality?

The rhetoric figures of Luhmann's texts are not necessarily determined by the frameworks of the used technical weaponry. The cage of the jargon is not necessarily incarcerating the dynamics of the gesture.

Technically, I try to understand Luhmann's theory of social systems from the viewpoint of polycontextural and diamond systems. Hence, I try to avoid to go into the litany of second-order cybernetics, systems theory and Spencer-Brown's Calculus of Indication and its extensions.
Even more technically, my interpretation of Luhmann's gestures with the introduction of his rhetoric figures is due to a morphogrammatic subversion, abandoning any jargon and terminological content, as crucial as it might be, and conceiving the dynamics of the pattern, only.

After this new diamond approach is introduced, experienced and further developed, a renewed lecture of Luhmann's work as involved with the above mentioned second-order trends, might happen again.

The term "diamond" refers to itself, only. There is no reference to exposed marketing labels necessary.

2.1. Binaries

Communication/distinctions
system/environment
Observations: open/close
relate/differentiate
space
Open/close are inverse operations

First, the system is the difference between system and environment.
Second, the system can be defined through a single mode of operation.
Third, every (social) system observes internally (i.e. within the system) its own system/environment distinction; there is a re-entry of the system/environment distinction into the system.
Fourth, every social theory is part of the social domain and as such part of what it describes.

Systems exist.
Obviously, this is a paradoxical formulation. And only academic blindness can deduce that it is a confession for ontological realism.

2.2. Uncovering Luhmann's diamonds

Statement
"When a communication constitutes a previous communication as a communication, it simultaneously distinguishes it from all those other things in the world that are not communication. In this sense, all operations of autopoietic systems always constitute the difference between the system and its environment

How can this happen? If an operation of an autopoietic systems is producing by its action, i.e. operation, both, the intended operation and at the same time, the operation of distinguishing the system of the first operation from its environment, then it *"constitute[s] the dfference between the system and its environment"*. How is an autopoietic operation simultaneously operating in its domain (system) and producing an environment of the domain? Or in other words, how is an operation operating that it is able to operate and thereby by such operation constituting (operating) its own environment?

The first answer, which might be given by Luhmann is the hint to Spencer Brown's Calculus of Indication: "Draw a distinction!" With this distinction, the 'world' is 'divided', i.e. 'distinguished' into two parts, the *inside* and the *outside* of the 'world' or 'space'.
But what is given by the CI? Two 'equations'.
In this formulation, no world appears. The world or space is presuposed and realized by a sheet of paper or

another medium of inscription. This might be interpreted cognitively by a user of the CI. And this interpretation will become a meta-theoretical environment of the calculus. But nevertheless no part of the calculus in question.

Again, *"When a communication constitutes a previous communication as a communication, it simultaneously distinguishes it from all those other thing in the world that are not communication."*

Interpretation
"When a communication constitutes a previous communication as a communication"
This is involving several procedures:
1. *"communication constitutes a previous communication"*, this might be naturally understood as a *composition* of two communications.
2. *"as a communication"* means, that the composition has to be realized as a composition of communications and nothing else. But this condition is exactly what is called the '*matching conditions* for compositions'.
4. With this formulation we get a clue to understand what could be meant by the consequence: "*it simultaneously distinguishes it from all those other thing in the world that are not communication.*"
This consequence of the composition of communications is following *consecutively* the 'assumption' of the operation of composition albeit it states its *simultaneity*.

Diamondization
Luhmann's communicational statement, the 'axiom' of communication, interpreted as a categorical composition of communications offers a natural introduction of the otherness of communication, i.e. the simultaneous environment of communication by the saltatorical hetero-morphisms.
It needs two communications to realize communication and its environment as the singular otherness of communication. This asymmetry is directly covered by the saltatories od diamond theory, which are complementary to the categories of communication.

Because of the operativity of the diamond interpretation of Luhmann's conception of communication, communication might now be studied operatively on all levels of complexity and complication necessary, together with their interplay.

This diamond interpretation is not reducible to the indicational calculus and its use for autopoietic and communicational systems.

Again, what are the conditions for communication? Communications have to be "*anschlussfähig*", i.e. they have to fulfil the conditions of connectivity.
In category and diamond theory, such conditions are exactly the *matching conditions* of composition.

Now, there are two possibilities opened up.
One insists that the conditions of the possibility of something are not identical with such a conditional something.
The other position could take a highly formalistic turn towards self-referentiality and postulate that there is no logical difference between the conditions of something and such a something.
Without doubt, the latter position leads quite directly to logical paradoxes. But who cares?
Why should we use logic? And which logic anyway?
It also could be mentioned that the comparison itself is too much restricted by logic and alternativity.

The first position sounds harmless if we take the statement in a hierarchical way, i.e. if we postulate a sequential order between the conditions and the entity. But why should we accept this decision as the only working possibility?
The diamond approach, obviously is postulating a simultaneity of both thematizations, the conditions of the possibility and the characteristics of the entity.

It might be a question of taste which of both positions has to be considered as more crazy: the ultra-formalistic or the diamond approach.

Re-entry and in-sourcing
``*To cope with these consequences of a re-entry of the internal/external difference in itself, the system needs and constructs time."* (Luhmann)

Again, in-sourcing:
"The idea of in-sourcing the matching conditions into the definition of diamonds seems to be in correspondence with the two main postulates of "Chinese Ontology", i.e., the permanent change *of things and the finiteness or* closeness *of situations. That is, diamonds should be designed as structural explications of the* happenstance *of compositions and not as a succession of events (morphisms)."*

The figure of re-entry tries to correspond to the device to include *"the internal/external difference in itself"*. This happens in "*consequences*" and needs/constructs time.
Hence, the idea of a simultaneous realization of the difference of system and its environment gets lost in the infinit delirium of self-reference.

In-sourcing the matching conditions of composition is a finite and simultaneous constellation of categories and saltatories. It is the interplay of both, categories and saltatories of a diamond constellation, which is realizing the figure of re-entry in a finit and differential manner.
Both strategies, the re-entry and the in-sourcing, seems to correspond to a similar gesture.

3. Diamonds

3.1. Constellations

The advantage to enter the adventure of diamonds is twofold: ultra-paradox and trans-operative, at once.

Diamonds are paradox and pataphysical.

The disadvantage of the calculus of indication is its low paradoxality and its hermetic narrowness towards operativity.

The relationship between difference-theory and the calculus of indication, Laws of Form, is parasitic.
Difference theory is not contributing anything to the development of the calculus. It is solely interpreting some simple situations and transferring some terms into its jargon and Zettelkasten.
There is no direct *modeling* between difference theory and indicational calculation.

The situation appears radically different in the case of diamond theory. Diamonds are per se defining the differential relation of system and environment immanently and intrinsically to their basic constructions (terms, notions, operations). Diamonds are build as an interplay of categories and saltatories.

Luhmann's difference theoretic relationship to the CI is interpretative. There is no operative counterpart in the CI which is directly and operatively supporting his difference theoretical interpretation.
Historically, it was notorious, that the Bielefelder had been chasing prefaces to the Laws of Form.
With an operative correspondence, this wouldn't have motivated so many chaser.

3.2. Diamond features

"The diamond modeling of the otherness of the others is incorporating the otherness into its own system. An external modeling of the others would have to put them into a different additional contexture. With that, the otherness would be secondary to the system/environment complexion under consideration. The diamond modeling is accepting the otherness of others as a "first class object", and as belonging genuinely to the complexion as such.

"Again, it seems, that the diamond modeling is a more radical departure from the usual modal logic and second-order cybernetic conceptualizations of interaction and reflection. The diamond is reflecting onto the same (our) and the different (others) of the reflectional system.

"In another setting, without the "antropomorphic" metaphors, we are distinguishing between the system, its internal and its external environment. The external environment corresponds the rejectional part, the internal to the acceptional part of the diamond. Applied to the diamond scheme of diamondized morphisms we are getting directly the diamond system scheme out of the diamond-object model.
Thus, a diamond system is defined from its very beginning as being constituted by an internal and an external environment." (Kaehr, Diamond 2007)

CI

The repetition of a distinction, ⌉⌉ = ⌉, is a *composition*, hence, like all composition, the CI composition has to fulfil some matching conditions.
They are well hidden, because there is not much content involved inn the CI composition.
But as I have shown in a slightly cabaret performance, there are matching conditions to be accepted.
⌉⌉:
Draw a distinction, mark it! Fine. Now, the same again! Draw a distinction, mark it!

But *where* should I draw and mark it? Behind the blackboard, at the scribble board of the toilet? *When* should I mark it? *How* should I mark it, with a different color upon the first mark? And so on!

Obviously, it is supposed that I mark the mark bravely, one after the other on a strict line of a writing scheme. Therefore, the matching conditions are internalized in the mind of the distinction drawer and not yet inscribed at the blackboard (B. Brecht).

When I played this game with Luhmann 1993 at a research seminar in Hamburg, I didn't have the simple technical term of *"matching conditions"* at hand. But the message was clear and I got the feeling that the figure was well received.
But then, what's next? Waiting for a new preface from George?

Interplay of categories and saltatories
For diamond theory, the identity of objects of a category is defined by the hetero-morphisms of a saltatory. And complementary, the morphisms of a saltatory are defined by the objects of a category. Hence both distinctions, objects and morphisms, as basic starting concepts of category theory, have to be introduced at once. Both have, for their introduction, to be considered as being in a heterarchic order. This can be done without circularity only if there is conceptual space for the distribution of the concepts "object" and "morphism" accessible.

For diamond theory, the second-order concept of self-referentiality is deconstructed to the interplay of categories and saltatories in diamonds. Such an interplay isn't involved with relations and relational logic.

The system-paradox of *"A system is a system and its environment"* is transposed to *"A diamond is an interplay of categories and saltatories. And saltatories and categories are a diamond of an interplay of categories and saltatories."*

A system (diamond) is a system (category/saltatory) and an environment (saltatory/category) of a system.

3.3. What's the sacrifice

Diamonds are doubled, split and antidromic from the very beginning, which in itself is doubled and therefore neither a beginning nor an origin.

As a consequence, the hegemony of singular identity has to be given up. Such an identities is derivational. Singularity of identity, its uniqueness, hence, is a gravitational obstacle for flexible, metamorphic and heterarchic thinking and acting.

"*First it distinguishes two sides and then it indicates one of these sides as the system (or the observer). As these operations are* consecutive *they constitute or `consume' time. All observations are thus temporal: one cannot be on both sides of a distinction at the same time."*

Such a necessity for consecutivity appears as a relict of old-European tradition, i.e. Western culture. This obsolete hegemony is the source of the Western concept of time.

A in a diamond world, there is no need for such consecutivity, and therefore for an understanding of time based on it.

This achievement of Western culture, its hegemony and its blind spot, has to be sacrificed. It anyway always was an illusion/allusion.

Because the operations are consecutive in time, because "one cannot be on both sides of a distinction *at the same time*", time is based on the identity of observations and distinction is based on the temporality of observation. Time is timing itself.

If such circularity is accepted, why not to accept simultaneity? Neither circularity nor simultaneity has a privileged relationship with absurdity.

Take a risk! Draw a distinction!
The risk concept of the theory of social systems is not only risky but adventurous, and, as it becomes more obvious, deemed to cause catastrophes.

4. Beyond economy

Time production as a result of a singularity of observation, which needs and produces, produces and consumes, time and memory. For diamonds, no such implications have to be observed.

Hence, diamonds are more observer independent, are getting more radically rid of anthropological, egological and subjectivistic inheritences of old-Europaen philosophy.
Hence, diamonds are deliberating difference-analytical limitations for system-thinking and are more open for a general theory of (social) systems.

There is no time for time production and consumption.

It, therefore, can be stipulated, that the time-structure of diamonds with its interplay of dromic and antidromic horizons, is independent of the economy of production and consumption.

"The idea of in-sourcing the matching conditions into the definition of diamonds seems to be in correspondence with the two main postulates of *Chinese Ontology*, i.e., the permanent change of things and the finiteness or closeness of situations. That is, diamonds should be designed as structural explications of the *happenstance* of compositions and not as a succession of events (morphisms). More exactly, diamonds are contemplating the interplay of acceptional and rejectional thematizations. Thus, morphisms with their matching conditions and composability are in fact of secondary order for the understanding of diamonds."

5. Graphematics: From difference to différance

If a sign is defined, introduced and characterised by differences and by relations, i.e. differentially and not relationally, and difference/différance is staged on an ontology-free arena as an interplay between categorical and saltatorical gestures, then diamonds are designing and inventing not semiotic but graphematic horizons (systems).

Semiotic systems appear in such situations as frozen diamonds.

6. Diskussion of the traditional view

6.1. Comments

There is, despite the massive multitude, a kind of an established view on sign theory, semiotics or sémiology, especially in its meta-theoretic formulations, conceptualizations and jargon.
First, a sign is a *relational object*: dyadic for Saussure, triadic for Peirce, tetradic for some other semioticians. As a consequence, signs are constituting a *system* of signs. Hence, signs tend to be characterized as systems and not as (relational) objects.

Second, despite all the onto-logical topics and inherited problems, signs are *self-referential*, they are able to refer to themselves by definition.

Third, there are many other characteristics for signs, like sign classes, sign relations, sign thematics, etc.

Fourth, signs are iterable. Independent of specifique forms of identity and linearity of whatver media, the iterability of signs is the main characteristics of semiosis.

Endless
"Thus there is a virtual endless series of signs when a sign is understood; and a sign never understood can hardly be said to be a sign."
6 - v. 1902 - MS 599 -Reason's rules .
http://www.cspeirce.com/menu/library/rsources/76defs/76defs.htm

6.1.1. Relationality

Charles Sanders Peirce
One is: Peirce' semiotics is tradic-trichotomic and relational, hence based on triadic relations.

What is a sign? A sign is a triadic relation.
As every student of math knows: Each n-adic relation might be represented by a succession of dyadic relations. Hence, n-adicity is reducible to dyadicity.
Well done! What is a relation? A relation is a subset of a set, i.e. a product set, a Cartesian set. Therefore, a sign is a subset of a set.

But what we are not told is that a relation is understood as a set and a relation as a set is a representation of a set. Hence, there are no genuine relations, because all relations are sets.
Furthermore, a sign is a representation. And a relation is a representation of a set, hence a sign is a relation of a relation. A sign is not simply a relation, triadic or else, but a second-order relation, i.e. a relation of a relation over a set. And therefore a set of a set.

More technically, after the Kuratowski-Wiener intervention, a relation is an ordered set. And an ordered set is based on the *pair axiom*, which is guaranteeing ordered pairs, necessary for the definition of relations as sets of ordered pairs, which are themselves sets. But the idea of order, i.e. an ordered set, is a relational idea, based on a relational intuition, and is not presuposing the notion of sets.

Sign
"My definition of a representamen is as follow:
A REPRESENTAMEN is a subject of a triadic relation TO a second, called its OBJECT, FOR a third, called its INTERPRETANT, this triadic relation being such that the REPRESENTAMEN determines its interpretant to stand in the same triadic relation to the same object for some interpretant."
20 - 1903 - C.P. 1-541 - Lowell Lectures: Lecture III, vol. 21, 3d Draught .
http://www.cspeirce.com/menu/library/rsources/76defs/76defs.htm

Thirdness
"[...] In its genuine form, thirdness is the triadic relation existing between a sign, its object, and the interpreting thought, itself a sign, considered as constituting the mode of being of a sign. A sign mediates between the interpretant sign and its object. Taking sign in its broadest sense, its interpretant is not necessarily a sign. [...]
A sign therefore is an object which is in relation to its object on the one hand and to an interpretant on the other, in such a way as to bring the interpretant into a relation to the object, corresponding to its own relation to the object. I might say similar to its own for a correspondence consist in a similarity; but perhaps correspondence is narrower."
28 - 1904 - C.P. 8-832 - Letter to Lady Welby dated "1904 Oct.12 .
http://www.cspeirce.com/menu/library/rsources/76defs/76defs.htm

Triadomany
"I fully admit that there is a not uncommon craze for trichotomies. I do not know but the psychiatrists have provided a name for it. If not, they should. "Trichimania," [?] unfortunately, happens to be preëmpted for a totally different passion; but it might be called *triadomany*. I am not so afflicted; but I find myself obliged, for truth's sake, to make such a large number of trichotomies that I could not [but] wonder if my readers, especially those of them who are in the way of knowing how common the malady is, should suspect, or even opine, that I am a victim of it."
('On trichotomies', CP 1.568, 1910)
http://www.helsinki.fi/science/commens/dictionary.html

Ferdinand de Saussure

The other approach is: Semiology in the sense of Saussure is *dichotomic* and *relational*, hence based on binary relations, which are building together a *system*.

A sign is the basic unit of language (a given language at a given time). Every language is a complete system of signs. Parole (the speech of an individual) is an external manifestation of language."
"In language there are only differences, and no positive terms

"L'idée fondamentale de Saussure est que le langage est un système clos de signes. Tout signe est défini par rapport aux autres, par pure différence (négativement), et non par ses caractéristiques propres ("positives") : c'est pourquoi Saussure parle de "*système*". (WiKi, fr)

A relation, especially a dyadic or binary relation, is a relation between entities, i.e. between positive terms. Again, a relation is a set and a set consists of elements which are positive terms.
What's about Saussure's *kenome (*Kénôme)?
Why should a difference be a relation? And why should a difference be dyadic?
There are a lot of dyads and dichotomies in Saussure's semiology.

http://www.revue-texto.net/Saussure/Saussure.html

6.1.2. Self-reference of signs

"Such an object (or referent) of the sign can be a sign itself, and in this sense, self-reference becomes possible as a mode of a sign referring to a sign." (Peirce)

If a sign is defined as a triad of (object, representant, representamen), i.e. as a triadic relation, then there is no definition given and no operator introduced, which would define such a self-application of a sign as referring to a sign, hence to itself. What is missing is a definition of an operation which is ruling the composition of signs.

Semiotics thus is not simply about signs but about the *composition* of signs.

Bense, and later Toth, introduced 3 modes of primary compositions for signs: the *iterative*, the *adjunctive* and the *superivative.* None of them are well defined, especially the matching conditions for the composition of signs is left in the dark.

6.1.3. System theory of signs

Signs are not appearing as entities but as elements of a system. To understand signs, which themselves need to be understood to be signs, a theory of systems is required.

"Every language is a complete system of signs." (Saussure)

Hence, to understand signs we have to understand systems. And without surprise, to understand systems we have to understand signs. And as there are hundreds of definitions for systems there are even more definitions for signs. And vice versa.

6.2. Citations

Niklas Luhmann: 'Sign as Form'
A Comment
By Nina Ort and Markus Peter

Abstract: 'Sign as Form' is Niklas Luhmann's attempt to combine systems theory with sign theory by trying to integrate George Spencer-Brown's 'Laws of Form'. Systems theory operates with two sorts of metaphors representing either the meaning of inside and outside of a form (asymmetry) or both sides of an complementary couple (symmetry) which determine the subsequent arguments. The integration of an included third term, that would complete a semiotic sign, cannot be achieved by operating with dyadic distinctions however. This contribution discovers difficulties that arise from that combination and tries to show how the use of n-valued logic helps to overcome these problems."

http://www.imprint.co.uk/C&HK/vol6/v6_3-ort.html

"Even though self-reference is the topic of the present study, its basic assumption is neither a naïve theory of reference nor the structuralist or constructivist theory of the signs that have no referents. Our study will be based on Charles S. Peirce's semiotics, in the framework of which reference is the relationship of the sign to its object. However, the object to which a sign refers back is not a piece of the so-called real world, but something which precedes and thus determines the sign in the process of semiosis as a previous experience or cognition of the world. Such an object (or referent) of the sign can be a sign itself, and in this sense, self-reference becomes possible as a mode of a sign referring to a sign."

Winfried Nöth, Self-Reference in the Media1

http://www.uni-kassel.de/iag-kulturforschung/projektbeschreibung.pdf

"Luhmann's systems theory is based on Spencer-Brown's dualistic philosophy of differences. This seems to make it incompatible with American pragmatic semiotician C. S. Peirce's triadic semiotics that seems to offer that trans-disciplinary theory of meaning and signification that the cybernetic functionalistic informational approaches are missing. But in his seminal work A calculus of [for, rk.] self-reference, *Varela sees that the necessity of a third element in autopoiesis theory and second-order cybernetics has been overlooked and adds that to the system in a way that makes it compatible with Peirce's semiotics and still keeps the connection to cybernetics and autopoiesis."*

Søren Brier, CBS, Cybersemiotics

http://www.brier.dk/SoerenBrier/DoctoralSummary.pdf

Polycontexturality of Signs?

Are there signs anyway?

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Abstract

How to read polycontextural sign matrices? Are there such constructs like *polycontextural signs*? It is argued that there are in fact no entities or processes in the "real-world" like signs in the sense of semiotics at all. Semiotic signs are logocentric constructs realized by semioticians and defined by identity principles. This might be appropriate for a mono-contextural world-view but it is not sufficient for the experiences in a polycontextural world.
An example is given, how to construct and read a polycontextural configuration as a texteme. Also composition/decomposition of sign classes are presented.

1. Do polycontextural signs exist?

1.1. Toth's question

The semiotician Alfred Toth is asking:
"Are there polycontextural signs?" (Toth, p. 1, 26.04.2009)

Is the lack of identity a sign for polycontexturality (of signs)?

"However, representatives of polycontextural theory have often pointed out that semiotics is clearly a monocontextural system in which the logical Law of Identity (and the other 2-3 fundamental laws of classical thinking) are valid without restrictions." (ibd., p.1)

Distributed over some polycontextural notions, Toth shows *"that the classical semiotics has no identity and thus is polycontextural"*. Even if we could agree with this demonstration, the question remains: Is it enough for a semiotic system to be non-identical to be polycontextural?

Obviously, the first obstacle in the process to be declared as polycontextural on the base of a lack of identity is the concept of *negation* involved in the "non" of non-identical. Negation is too much in complicity with the system it is negating.

Toth gives his analysis an interesting turn in inverting the starting question "*Are there polycontextural signs?"*

*"So, from here, the question should not be if there are polycontextural signs, but if there are monocontextural signs.
In classical semiotics, polycontexturality is hidden in the triadic-trichotomic structure of a seeming monocontexturality."* (ibd., p.2)
http://www.mathematical-semiotics.com/pdf/Polyc.%20signs%20question.pdf

1.2. Polysemy as polycontexturality

Instead of denying identity, a complementary gesture might manage to introduce a multitude of identities as a refutation of the dominance and hegemony of the logical and semiotic principle of identity.

Independent of the complexity of the semiotic matrix, the sign classes and their sign relations are always separated by their identity.
Traditionally there are 10 sign classes recognized. All of them are properly distinguished from each other.

A dicent is a dicent and not a rhema or a symbol or a legi-sign, etc.

In an *iterative* sense, some complications might be produced for the 10 sign classes. Following Bense, we get, e.g., an argumentative-symbolic legi-sign as the semiotic modeling of formal languages (Kalkülsprachen), or a dicentic-symbolic sin-sign for "epic languages" and a dicentic-indexical legi-sign for "programming languages".

To all those isolated sign classes and sign relations, semioticians are delivering more or less convincing linguistic, media-related or physical examples.
Such identity constructs are reasonable for traffic systems and other unambiguous sign-related situations.

The intention to focus on identity of definitions and examples is not necessarily a semiotic action but an action, i.e. modeling, guided and ruled by the interests of identity, i.e. of identical identification.

With identity and identification, a specific form of rationality is supposed.
Similar restrictions are introduced by Chomsky's grammar. There is a strict distinction between meaningful and meaningless sentences. Nevertheless, all meaningless sentences are easily domesticated in a game which is opening up meaning for all.

My thesis therefore is: To all examples and to all distinctions there are always overlapping other distinctions involved that are suppressed, denied and rejected by such an act of identification.

As an example, the concept of "natural number" might be mentioned. Even for such an elementary concept like the "natural numbers" there is no identitive definition available. Most definitions (introductions, postulations) are circular or lost in the abyss of non-foundedness.

Hence, identification in the mode of identity is an ontological and epistemological procedure and follows not semiotic or sign theoretical necessity. Again, semiotics in a general sense, thematized as an identity system, is ruled by non-semiotic decisions.

In other words, semiotics as an identity system of whatever complexity is dominated by logocentric preconditions, in fact by linguistic and logical notions.

Therefore, semiotic distinctions in a polycontextural paradigm are not governed by the ontological is-abstraction but are involved into the free interplay of actional as-abstractions.

From a polycontextural point of view, signs are results of actions and actions are not necessarily reducible to single agents but might be realized as interactions between a multitude of mediated actor-systems. Each semiotic action is simultaneously involved and coupled with its environment, which contains itself a multitude of agents.

It turns out that classical semiotic systems are not actional but structural or relational and are based on a singular epistemic instance, i.e. interpretant.

1.3. Semiotic Matrix

A closer analysis of sign processes makes it obvious that signs are always intrinsically interwoven and overlapped with other signs. Signs as representamens are representing entities of a given world; polycontexturality is opening up worlds. Thus, signs in polycontextural situations are not simply representational but evocational. Such evocativeness of signs is not yet well studied. It is also not grasped by Bense's creativity function of signs.

But this connectedness of signs in polycontextural situations is not the classical statement of the system-dependency of signs, i.e. the statement that signs are not occurring in isolation but necessarily in or as a system.

Semiotics in the sense of Bense and Toth is build on the base of the so called semiotic matrix, i.e. the Cartesian product of the sign components.

"Ein Zeichen ist danach eine triadische Relation, genannt, Interpretantenbezug, welche eine dyadische Relation, genannt Objektbezug, und eine monadische Relation, genannt Mittelbezug enthält. Da eine Relation eine Teilmenge eines kartesischen Produktes ist, kann man auch sagen, das Zeichen sei eineTeilmenge von Teilmengen von kartesischen Produkten." (Toth, Bühler, p.2, 2009)

Then, sign classes are interpreted as parts of the matrix. These parts are disjunct and well separable from each other. There is no overlapping or penetration from and by other sign classes, i.e. classical semiotics is based on disjunctively separable sign classes.
As a reasonable result, such a kind of semiotics is not dealing with the whole matrix but only with its parts, i.e. the sign classes.

Transclassical semiotics, in the sense of polycontextural semiotics, is not in such a comfortable situation. Polycontexturality is not understood simply as a multitude of semiotic contextures but by its interactivity, reflectionality and interventionality between a plurality of contextures.

It is no surprise that, e.g. in polycontextural logic, the overwhelming majority of logical functions are not uniformly separable into cleanly defined sub-systems but are highly interwoven. That is, junctions, like disjunctions, conjunctions and implications, are a minority compared to the bulk of transjunctional logical functions.

It has to be stipulated therefore, that the same situation holds for a polycontexturally conceived semiotics.

From a strict terminological point of view it might be obsolete and confusing to still call this construction semiotics. A more appropriate title would be a mediation-system for interacting semiotics.

Hence a reading of a polycontextural semiotic matrix with the aim to collect sign classes doesn't work anymore as a separation of mono-contextural sign constellations, like (3.1 2.1 1.1).
Such a reading of a polycontextural constellation obviously is producing "wild" beasts, "verwilderte Matrizen" (Toth), of, probably, not much use.

Polycontextural semiotics is forced to accept the semiotic tissue as a whole, i.e. as a game of interplaying contextures and their semiotic operations.
Hence, polycontextural semiotics is not reducible to separable sign classes. It always has to deal, at least for triadic semiotics, with the whole matrix. For more complex semiotics, a new kind of separability has to be studied, i.e. the separability of the general matrix into its overlapping sub-matrices.

Toth's question *"Are there polycontextural signs?"*, thus has, at first, to be denied. By definition and tradition, signs are not polycontextural. (And there is obviously no such a thing like a "keno-sign", too.)

1.4. Toth's interpretation

In contrast to the just mentioned 'holistic', i.e. poly-contextural interpretation of arbitrary semiotic matrices as interplaying sub-matrices, Thot is giving an interpretation of polycontextural matrices by the means of classical strategies.
Even a *pentadic* matrix gets its *separated* sign classes, i.e. 5-Zkl = (a.b c.d e.f g.h i.j) with a, ..., j ∈{1, ..., 5}, this time not triadic but pentadic. But, as Toth is observing correctly, highly wild situations are disturbing such isolative interpretations.

Arbitrarity
"3. Kaehrs komponierte pentadische Matrix suggeriert grösstmögliche Arbitrariät bei der Kompositionen n-adischer und m-adischer zu (n+m-1)-adischer Matrizen und umgekehrt zur Dekomposition (n+m)-adischer Matrizen in (n-1-m)-adische und/oder (n-m-1)-adische Matrizen. Die einzige Anforderung an die "Richtigkeit" der komponierten Matrix wäre dann, dass die zueinander inversen Subzeichen die gleichen kontexturellen Indizes bekommen (z.B. (2.3) und (3.2), (1.5) und (5.1), etc.). In letzter Instanz führt diese Arbitrarität also dazu, dass in Übereinstimmung mit einer obigen Festellung die abstrakte Form einer pentadischen Zeichenklasse als

5-Zkl = (a.b c.d e.f g.h i.j)
mit a, ..., j ∈{1, ..., 5}

anzusetzen ist. Da ferner die triadischen Hauptwerte a, c, e, g, i nicht mehr paarweise verschieden sein müssen, kann jede x-beliebige Folge von 6 Ziffern natürlicher Zahlen als pentadische Zeichenklasse interpretiert werden.'" (Toth, Interakt Sem1Sem2, p. 3, 2009)

This arbitrarity might lead to a wild and nonsensical use of the concept of sign classes. Toth gives a possible solution for a reduction of the "wilderness" of the pentadic situation for "Haupt-Zeichenklassen" and "Neben-Zkln".

"Haupt-Zeichenklassen"
"4. Eine Einschränkung für diese völlig *verwilderte* Menge von Zeichenklassen könnte man daraus entnehmen, dass man wie bei der triadischen Matrix die Reihen der pentadischen Matrix als "Haupt-Zeichenklassen" interpretiert und aus den pentatomischen Pentaden der dyadischen Subzeichen Regeln zur Komposition von Zeichenklassen ableitet." (ibd., p.3)

"Neben-Zkln".
2.1 5-Zkl = (5.a 4.b 3.c 3.d 1.e):
(a = 1) --> b = 1, c = 5, d = 4, e = 1
(a = 2) --> b = c = d= 2, e = 4
(a = 5) --> b = c = d = e = 5
2.2. (5-Zkl = (3.a 3.b 3.c 2.d 3.e)) --> a = e = 5, b = 4, c = d = 3

Die Reihen der Matrizen enthalten triadische Sprünge und Wiederholungen [...]." (ibd., p. 3/4)

Thus, a sign class with "strange-values" is producing some kind of *jumps* and *gaps* in the arithmetics of semiotics. Toth mentions the examples:
(1-3-3-4-5) of the colum (1),
(3-2-3-4-4) of the colum (2) and
(3-2-3-3-3) of the colum (3) of the matrix below.
The colums (4) and (5) are not disturbed by "stange-values".
The same happens to the rows of the matrix.

$$\text{inter}\left(\text{Sem}^{(5,3,2)}\right) = \begin{bmatrix} MM & 1 & 2 & 3 & 4 & 5 \\ 1 & 1.1_1 & 3.4_2 & 3.5_2 & 1.4 & 1.5 \\ 2 & 3.4_2 & 2.2_1 & 2.3_1 & 2.4 & 2.5 \\ 3 & 3.5_2 & 3.2_1 & 3.3_{1.2} & 3.4_2 & 3.5_2 \\ 4 & 4.1 & 4.2 & 3.4_2 & 4.4_2 & 4.5_2 \\ 5 & 5.1 & 5.2 & 3.5_2 & 5.4_3 & 5.5_2 \end{bmatrix}$$

However whatever kind of restrictions are introduced for a reasonable handling of complex sign classes, with m>=3, the strategy of selecting a single isolated chain of *"prime signs"* out of the matrix remains the same.

Without doubt, there might be some interesting insights possible with this approach, but there will still be an overwhelming majority of situations excluded, i.e. not interpreted as reasonable semiotic constellations. With that, a new, meta-semiotic problem occurs: What are the criteria of exclusion of the non-semiotically interpreted constellations? In other words, a criterion to distinguish between acceptable and non-acceptable constellations has to be introduced.

Again, as most junctional "value-sequences" in polycontextural logic are disturbed by external values, Gunther's "Fremdwerte", which are getting a logical meaning only in the context of an interplay between different mediated logical systems, the "Fremdwerte" of semiotic sign classes incur the same destiny: they are members of other sign classes interacting together in the complex semiotic game.

Therefore, polycontextural semiotics has to study, at least, both directions of the interplay: the interactional (reflectional, interventional) aspect between contextures, i.e. the transcontextural interplay, and the intra-contextural aspect of the disturbance by transcontextural interpenetrations.

2. From signs to textemes

Instead of excluding "strange" sign classes or to stretch adventures interpretations about gaps and jumps in the chain of prime signs, their origin in the complexity of polycontextural semiotics has to be considered first.

Because such situations are fundamentally different from semiotic approaches, the idea of *textemes* had been introduced. From the position of the idea of textemes, signs in a semiotic sense, are reductions of textemes.

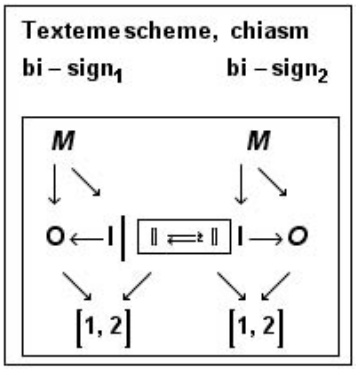
Therefore, a first step to a general theory of interactional semiotics on the base of the new concept of *textemes*, i.e. bi-sign systems or anchored diamonds, consisting of the semiotic intra-kernel and the semiotic internal/external environments, and its interplay, is proposed.

A *texteme* consists of two diamondized anchored signs, i.e. bi-signs, inter-playing together by their mediated external environments. Hence, a texteme is an interplay of two bi-signs. A bi-sign is a diamondized anchored sign, i.e. a sign with intrinsic environments and its anchor.

This is a kind of bottom up introduction. Because we know signs and have not yet experienced textemes, this way of building up textemes is legitimate. But nevertheless, it works only because we know how to construct textemes out of signs which are not able to offer any of the principles of textemes, that are needed to realize such a construction, like their chiastic interplay between the environments of signs and the anchoring of signs.

As we know well enough, signs lack environments, there is no chance to construct out of signs in a sign-theoretical sense a semiotic environment of the sign concept. And obviously, there is no such mechanism as a chiasm in the sense of proemiality for signs. And again, semiotics is not offering any insight and mechanism for anchoring signs. Hence, neither environments, internal and external, nor interactions between signs based on their environments and their anchoring are conceivable.

These statements are surely in conflict with the well established interactional socio-, bio-, zoo-semiotic programs as well as with the advances in computational semiotics. From the point of view of polycontextural and diamond theoretical approaches to sign theory, those programs have to be seen as *applications* of classical, *a priori* non-interactional semiotics, onto semiotics, and not as anything else. Their merits are to be communicable in a society of traditionally trained knowledge-mongers.



Hence, a decomposition chain might clarify the concept of texteme:
A *texteme* is decomposable to its interacting *bi-signs* by excluding its chiastic interactivity.
A semiotic *diamond* is a bi-sign, de-rooted from its *anchor*,.
A single *bi-sign* is disconnected from its neighbor bi-sign, hence it is a bi-sign without interaction but realizing an anchored semiotic diamond with its isolated, and hence restricted, *environment*.
A *sign* is a semiotic diamond, depraved from its *environment* and its *anchor*.

This decomposition from the texteme to the sign has no reverse: There is no semiotic mechanism per se to construct out of semiotics the concept of textemes.

The *complexity* of a basic texteme is 12, i.e. 2x3 for its "*signs*", 2x2 for its *anchors*, and 2x1 for its *environments*.

It might be asked if such matrices exists as semiotic matrices and not merely as mathematical matrices. A possible answer might be given with a semiotic interpretation of the texteme construction.

Sign_1 of $\text{bi} - \text{sign}_1$ shall be interpreted by the semiotic matrix $\text{Sem}_1^{(3,2)}$ and $\text{Sem}_1^{(3,2)}$ shall correspond to the sign_2 of $\text{bi} - \text{sign}_2$□. Such 3x3-matrices are well accepted as semiotic matrices. Therefore, the step to compose such matrices to a 5x5-matrix shouldn't cause to many problems. Semiotic matrices are occurring as numeric matrices and as matrices over M, I, O.

The idea of non-identical but polycontextural and multi-layered semiotic constellations seems to be accessible for formal treating within such a construction of interplaying semiotics. Hence, such inconsistent situations, where a dicent appears at once as a rhema, including all other kinds of overlapping and metamorphosis, are getting a formal framework for their interplay.

With that in mind, the semiotic interpretation of the texteme below follows naturally.

It is an example, how to interpret matrixes for polycontexturally conceived sign-complexions, i.e. textemes.

Procedure

The (OMI)-matrix $SCI^{(3,2)}$ is translated into the corresponding numeric matrix $MM^{(3,2)}$, with its 2 – subsystem indeces. For both, a matrix-composition with environments and anchors are presented. The result of this composition, with the matching conditions (overlapping) $3.3_1 \equiv 3.3_2$, is written as $Texteme^{(5,3,2)}$. The environments of Sem_1 and Sem_2 are reduced to $\overline{3.3}_1 \leftrightarrow \overline{3.3}_2$ corresponding to the overlapping conditions, and omitting the micro-environments of the 2-sub-systems of the triadic matrices.

3 – contextural semiotic matrix

$$Sem^{(3,2)} = \begin{pmatrix} MM^{(3,2)} & .1_{1.3} & .2_{1.2} & .3_{2.3} \\ 1_{1.3} & \mathbf{1.1_{1.3}} & \mathbf{1.2_1} & \mathbf{1.3_3} \\ 2_{1.2} & \mathbf{2.1_1} & \mathbf{2.2_{1.2}} & \mathbf{2.3_2} \\ 3_{2.3} & \mathbf{3.1_3} & \mathbf{3.2_2} & \mathbf{3.3_{2.3}} \end{pmatrix}$$

Matrix notation for [1, 2] – anchored texteme

$$\frac{\Pi^{(3,2,2)}_{\substack{\text{Type}\\ \text{texteme}\\ [1,2]\text{–anch}}} \begin{pmatrix} M & & \square \\ \downarrow & \searrow & \\ I \rightarrow & O \longrightarrow 1 & \end{pmatrix}}{\underbrace{\begin{pmatrix} O.O_{1.3} & O.M_1 & O.I_3 \\ M.O_1 & M.M_{1.2} & M.I_2 \\ I.O_3 & I.M_2 & I.I_{2.3} \end{pmatrix} \Bigg| \left(\overline{I.I} \rightleftarrows \overline{I.I}\right) \Bigg| \begin{pmatrix} O.O_{1.3} & O.M_1 & O.I_3 \\ M.O_1 & M.M_{1.2} & M.I_2 \\ I.O_3 & I.M_2 & I.I_{2.3} \end{pmatrix}}_{[1,2]}}$$

Matrix notation for [1, 2] – anchored texteme, without 2 – subsystems

$$\frac{\Pi^{(3,2,2)}_{\substack{\text{Type}\\ \text{texteme}\\ [1,2]\text{–anch}}} \begin{pmatrix} M & & \square \\ \downarrow & \searrow & \\ I \rightarrow & O \longrightarrow 1 & \end{pmatrix}}{\underbrace{\begin{pmatrix} 1.1_1 & 1.2_1 & 1.3_1 \\ 2.1_1 & 2.2_1 & 2.3_1 \\ 3.1_1 & 3.2_1 & 3.3_1 \end{pmatrix} \Bigg| \overline{3.3}_1 \leftrightarrows \overline{3.3}_2 \Bigg| \begin{pmatrix} 3.3_2 & 3.4_2 & 3.5_2 \\ 4.3_2 & 4.4_2 & 4.5_2 \\ 5.3_2 & 5.4_2 & 5.5_2 \end{pmatrix}}_{[1,2]}}$$

Matrix notation for [1, 2] – anchored texteme, with 3 – subsystems

$$\underset{\substack{\textbf{Type}\\ \textbf{texteme}\\ \textbf{[1,2]–anch}}}{\Pi^{(5,3,2)}} \begin{pmatrix} M & & \square \\ \downarrow \searrow & & \\ I \rightarrow & & O \longrightarrow 1 \end{pmatrix}$$

$$\begin{bmatrix} 1.1_1 & 1.2_1 & 1.3_1 & 1.4 & 1.5 \\ 2.1_1 & 2.2_1 & 2.3_1 & 2.4 & 2.5 \\ 3.1_1 & 3.2_1 & 3.3_{1,2} & 3.4_2 & 3.5_2 \\ 4.1 & 4.2 & 4.3_2 & 4.4_2 & 4.5_2 \\ 5.1 & 5.2 & 5.3_2 & 5.4_2 & 5.5_2 \end{bmatrix} \Bigg\| \left[\overline{3.3}_1 \leftrightarrows \overline{3.3}_2\right]$$

[1, 2]

$$\textbf{Texteme}^{(5,3,2)} = \begin{bmatrix} MM & 1 & 2 & 3 & 4 & 5 \\ 1 & 1.1 & 1.2 & 1.3 & 1.4 & 1.5 \\ 2 & 2.1 & 2.2 & 2.3 & 2.4 & 2.5 \\ 3 & 3.1 & 3.2 & 3.3 & 3.4 & 3.5 \\ 4 & 4.1 & 4.2 & 4.3 & 4.4 & 4.5 \\ 5 & 5.1 & 5.2 & 5.3 & 5.4 & 5.5 \end{bmatrix} \Bigg\| \left[\overline{3.3}_1 \leftrightarrows \overline{3.3}_2\right] \| [1,\ 2]$$

$\textbf{Texteme}^{(5,3,2)} =$

$0.\ \mathrm{Sign}^{(3,2)} = \mathrm{Sem}^{(3,2)}$

$1.\ \mathrm{Sem}^{(5,3,2)} =$

$\quad \mathrm{Sem}^{(3,2)} \circ \mathrm{Sem}^{(3,2)} \setminus MC = \{(3.3)_1 \equiv (3.3)_2\},$

$2.\ \mathrm{diam}\left(\mathrm{Sem}^{(5,3,2)}\right) =$

$\quad \mathrm{Sem}^{(3,2)} \diamond \mathrm{Sem}^{(3,2)} =$

$\quad \mathrm{Sem}^{(3,2)} \circ \mathrm{Sem}^{(3,2)} \| \left(\overline{3.3}_1 \Longleftrightarrow \overline{3.3}_2\right)$

$3.\ \mathrm{anchor}\left(\mathrm{diam}\left(\mathrm{Sem}^{(5,3,2)}\right)\right) =$

$\quad \mathrm{diam}\left(\mathrm{Sem}^{(5,3,2)}\right) \| \left[1,\ 2\right]\Big] =$

$\quad \left[\mathrm{Sem}^{(5,3,2)} \| \overline{3.3}_1 \Longleftrightarrow \overline{3.3}_2 \| \| [1, 2]\right] =$

$\quad \textbf{Texteme}^{(5,3,2)}.$

More *precisely*, the matching conditions for the composition of overlapping matrices are not the matching conditions of *concatenational* composition of morphisms but *overlapping* conditions of different complexity, in this case of length 1. Hence, concatenational matching conditions for morphisms are overlapping in zero elements, while the proposed composition of matrices is overlapping in one element, here $3.3_1 \equiv 3.3_2$.

Again, 3.3_1 is not the codomain of Sem_1 and 3.3_2 is not the domain of Sem_2 of the composition $\mathrm{Sem}_1 \circ \mathrm{Sem}_2$. Thus, it is just an abbreviation to call those "matching conditions”, MC, simply matching conditions and not *overlapping* matching conditions. Such overlapping conditions had been called “mediation conditions”, MC, in the “Matrix-paper".

From a logical perspective, overlapping is producing contradictions. Therefore, a proper treatment of overlapping patterns has to consider its *morphogrammatic* foundation.

2.1. Again then, what is a polycontextural sign?

The term “polycontextural sign” is an *abbreviation* for the wording: *“Signs in polycontextural constellations”*. This means that signs and sign systems are conceived as distributed and mediated, i.e. disseminated over the kenomic matrix.
In the same sense, as there is no poly-Lambda Calculus, but lambda calculi disseminated over the kenomic matrix, and the label “poly-Lambda Calculus” is a simple, maybe misleading, abbreviation, polycontextural semiotics or poly-semiotics, are abbreviations for the dissemination of semiotics.

This is a *conservative* interpretation because it is conserving the original concept of signs, i.e. its triadic-trichotomic structure, and is not changing or deconstructing it towards another conception. But the experiences with conservative expansions are enabling new decisions which are deliberating from the acceptance of the basic system and its restrictions.

From the point of view of polycontextural dissemination, even classical semiotics appear as polycontextural, simply because it is distributed too, albeit only over a single place and not able to be aware of it. What's called its blind spot.

More exercises
For the classical matrix $MM^{(3,3)}$ the disjunctivity of the sign classes holds.

$$MM^{(3,3)} = \begin{bmatrix} O.O_1 = \text{Index} & O.M_1 = \text{Icon} & O.I_1 = \text{Symbol} \\ M.O_1 = \text{Qualisign} & M.M_1 = \text{Sinsign} & M.I_1 = \text{Legisign} \\ I.O_1 = \text{Rhema} & I.M_1 = \text{Argument} & I.I_1 = \text{Dicent} \end{bmatrix}$$

Sign matrix build from $MM^{(3,3)}$: $MM^{(5,3,2)} = MM^{(3,3)} + MM^{(3,3)} / I.I_1 \equiv O.O_2$

$$MM^{(5,3,2)} = \begin{bmatrix} O.O_1 & O.M_1 & O.M_1 & 1.4 & 1.5 \\ M.O_1 & M.M_1 & M.I_1 & 2.4 & 2.5 \\ I.O_1 & I.M_1 & I.I_1 \equiv O.O_2 & O.M_2 & O.I_2 \\ 4.1 & 4.2 & M.O_2 & M.M_2 & M.I_2 \\ 5.1 & 5.2 & I.O_2 & I.M_2 & I.I_2 \end{bmatrix}$$

[id]	O_1	O_2
M_1	sem_1	x
M_2	x	sem_2

Only the two matrices MM_1 and MM_2 are considered, the rest is omitted.

Symmetrical case of interaction :

$$interact_{sym} - MM^{(5,3,2)} =$$

$$\begin{bmatrix} O.O_1 & O.M_2 & O.M_1 & 1.4 & 1.5 \\ M.O_2 & M.M_1 & M.I_1 & 2.4 & 2.5 \\ I.O_1 & I.M_1 & I.I_1 \equiv O.O_2 & O.M_2 & O.I_2 \\ 4.1 & 4.2 & M.O_2 & M.M_2 & M.I_2 \\ 5.1 & 5.2 & I.O_2 & I.M_2 & I.I_2 \end{bmatrix}$$

[bif]	O_1	O_2
M_1	sem_1	x
M_2	$trans_2$	sem_2

Asymmetrical case for interaction $(O.M_2 / M.O_2) \longrightarrow (O.M_1 / M.O_1)$:

$$\text{interact}_{asym} - MM^{(5,3,2)} =$$

$$\begin{bmatrix} O.O_1 & O.M_2 & O.I_2 & 1.4 & 1.5 \\ O.M_2 & M.M_1 & M.I_1 & 2.4 & 2.5 \\ O.I_2 & I.M_1 & I.I_1 \equiv O.O_2 & O.M_2 & O.I_2 \\ 4.1 & 4.2 & M.O_2 & M.M_2 & M.I_2 \\ 5.1 & 5.2 & I.O_2 & I.M_2 & I.I_2 \end{bmatrix}$$

[bif]	O_1	O_2
M_1	sem_1	x
M_2	$trans_{2.3}$	sem_2

All sign classes composed out of the matrix $MM^{(5,3,2)}$are *homogeneous*, i.e. clean of "strange values", i.e. id($MM^{(5,3,2)}$) = $MM^{(5,3,2)}$.

All combination of the original matrix $MM^{(5,3,2)}$ are produced by *operations* on the matrix. A set of important operators had been introduced as the super-operators, sop={id, perm, iter, repl, red, bif}. The above interactions are based on the interactional operation "bif".

Because there is no space offered by the notation of the matrix $MM^{(5,3,2)}$for iterations and replications, the presentation is still a short for the full scheme, as it was introduced in previous papers.

The sign class ($O.O_1$ $O.M_2$ $O.M_1$) with its contextural type (1, 2, 1) is obviously disturbed by the "strange" value $O.M_2$ of contexture 2, its meaning isn't easy to determine as an isolated event. Studied as the result of an interaction between the two matrices, the meaning is well defined as a *rejectional* value from the position of the first matrix and as a *penetrational* value from the position of the second matrix.

Composition and decomposition

Polycontexturality is mainly about *composition* (mediation) and *decomposition* of systems.

Hence, a sign class like 5-Zkl = (5.a 4.b 3.c 3.d 1.e) with a, ..., e $\in$ {1, ..., 5} has to be studied in a double way: *globally*, as the whole pattern, i.e. (5.a 4.b 3.c 3.d 1.e), and *locally*, as a composition of sub-systems, here classical semiotic systems or sign classes 3-Zkl.

Hence, the sign class 5-Zkl is understood as a composition of the sign class 3-Zkl, which is representing the Peircean semiotics distributed over 10 different places in the sign class 5-Zkl.

The advantage of the decomposition method is clear. Each sign class m-Zkl is decomposable into its sub-systems, the distributed sign class 3-Zkl.

There is no need to invent infinite many different semiotics for arbitrary m-Zkl.

On the other hand, the distribution method is not restricted to triadic-trichotomic semiotics. For each n<m, there is a distribution of n-Zkl in m-Zkl.

Morphograms for sign classes

If we insist that 3-Zkl is defined as <3.x 2.y 1.z>, then obviously, patterns like <3.x 4.y 5.z> or <1.x 4.y 5.z> are not defining Peircean sign classes at all.

The distribution of 3-Zkl over the places offered by m-Zkl is forcing an *abstraction* from the concrete value set {1, 2, 3}. What is distributed then is the *pattern* of the 3-Zkl and not the concrete singular 3-Zkl with the values 1, 2, 3. This pattern is called the *morphogram* of 3-Zkl.

With that, it is natural to introduce the remaining 4 morphograms for m=3 as *reductions* of the original sign class, thus: <3.x 2.y 2.z>, <3.x 3.y 1.z>, etc.

morph($ZR^{(3,2)}$) = {MG_1, MG_2, MG_3, MG_4, MG_5}

The values of x, y, z ∈ {1, 2, 3} have not been considered.

This exercise was done with my paper "*Interactional operators in diamond semiotics*" and "*Matrix*" for the case of complexity 3 and 4. Hence, again for 4 and further results for 5.

4 – ZkI = $ZR^{(4,2)}$:

$$4 - ZkI = ZR^{(4,2)} = (4.a\ 3.b\ 2.c\ 1.d) \text{ with } a, \ldots, d \in \{1, \ldots, 4\}$$

$$ZR^{(3,2)} = \langle 3.x,\ 2.y,\ 1.z \rangle \text{ with } x, y, z \in \{1, 2, 3\}$$

$$\text{decomp}(ZR^{(4,2)}) =$$

$$\left(ZR_1^{(3,2)}, ZR_2^{(3,2)}, ZR_3^{(3,2)}, ZR_4^{(3,2)}\right) = \begin{bmatrix} 3.x, & 2.y, & 1.z, & -- \\ --, & 3.x, & 2.y, & 1.z \\ 3.x, & 2.y, & --, & 1.z \\ 3.x, & --, & 2.y, & 1.z \end{bmatrix}$$

Matching conditions for $ZR^{(4,2)}$

$$(3.x)_1 \cong (3.x)_3 \cong (3.x)_4$$
$$(2.y)_1 \cong (3.x)_2 \cong (2.y)_3$$
$$(1.z)_1 \cong (2.y)_2 \cong (2.y)_4$$
$$(1.z)_2 \cong (1.z)_3 \cong (1.z)_4.$$

5 – Zkl = $ZR^{(5,2)}$:

$5 - Zkl = ZR^{(5,2)} = (5.a\ 4.b\ 3.c\ 2.d\ 1.e)$ with $a, ..., e \in \{1, ..., 5\}$

$ZR^{(3,2)} = <3.x, 2.y, 1.z>$ with $x, y, z \in \{1, 2, 3\}$

$decomp(ZR^{(5,2)})$:

$ZR_1^{(3,2)} = <3.x, 2.y, 1.z, --, -->: (1, 2, 3)$

$ZR_2^{(3,2)} = <--, --, 3.x, 2.y, 1.z>: (3, 4, 5)$

$ZR_3^{(3,2)} = <3.x, 2.y, --, 1.z, -->: (1, 2, 4)$

$ZR_4^{(3,2)} = <3.x, 2.y, --, --, 1.z>: (1, 2, 5)$

$ZR_5^{(3,2)} = <3.x, --, 2.y, 1.z, -->: (1, 3, 4)$

$ZR_6^{(3,2)} = <3.x, --, 2.y, --, 1.z>: (1, 3, 5)$

$ZR_7^{(3,2)} = <3.x, --, --, 2.y, 1.z>: (1, 4, 5)$

$ZR_8^{(3,2)} = <--, 3.x, 2.y, 1.z, -->: (2, 3, 4)$

$ZR_9^{(3,2)} = <--, 3.x, 2.y, --, 1.z>: (2, 3, 5))$

$ZR_{10}^{(3,2)} = <--, 3.x, --, 2.y, 1.z>: (2, 4, 5)$

Generalizations

The method of composition/decomposition holds generally for n-adic constellations of sign classes too.

m-$ZR^{(m,n)}$ = $ZR^{(m,2)}$ o $ZR^{(m,2)} \circ ... \circ ZR^{(m,2)}$, m, n>=3.

Further readings:

http://www.thinkartlab.com/pkl/lola/Transjunctional%20Semiotics/Transjunctional%20Se
http://www.thinkartlab.com/pkl/lola/Matrix/Matrix.html
http://www.thinkartlab.com/pkl/lola/Abacus.pdf